4.1 Decision Maths: Algorithms, Sorting and Bin Packing: 30-minute Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The items with the following sizes are to be packed into bins of capacity 20: 11, 15, 12, 14, 7, 11, 5, 5, 4.

(a) Find a lower bound for the number of bins needed. **(2)**

(b) Use the first-fit algorithm, taking the items in the order given, to pack the items into bins. State the contents of each bin. **(3)**

(c) Use the first-fit decreasing algorithm to pack the items. State the number of bins used and the contents of each bin. **(3)**

(Total for Question 1 is 8 marks)

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2 A courier drives to eight different addresses in one shift. The distances, in miles, driven to reach each address, in the order the deliveries were planned, are:
34, 12, 55, 8, 41, 27, 63, 19

(a) Using the first item of each sublist as the pivot, perform a quicksort on this list to sort it into ascending order. At each stage, state the pivot used and the two resulting sublists (items less than the pivot, and items greater than or equal to the pivot). Continue until every sublist contains at most one item, then write out the final sorted list. **(5)**

(b) State the total number of comparisons used to complete the sort, given that each non-pivot item in a sublist is compared exactly once with that sublist's pivot. **(2)**

(c) State one advantage and one disadvantage of always choosing the first item of a sublist as the pivot, compared with other pivot-selection methods. **(2)**

(Total for Question 2 is 9 marks)

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3 Freya, an art technician at a secondary school, has eight tins of paint with the following volumes, in litres, to store in crates that can each hold a maximum of 10 litres:
8, 2, 7, 3, 6, 4, 5, 5

(a) Calculate the theoretical lower bound for the number of crates needed to hold all eight tins. **(2)**

(b) Use a full-bin method: starting with the largest unallocated tin each time, try to find one or more other tins whose volumes sum to exactly fill the remaining space in that crate. Show the contents of each crate you form. **(4)**

(c) Compare your answer to part (b) with your lower bound in part (a), and state what this shows about this particular set of tin volumes. **(1)**

(Total for Question 3 is 7 marks)

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Mark scheme · 4.1 Decision Maths: Algorithms, Sorting and Bin Packing: 30-minute Pack

Question 1

(a) total size = 84 **M1**

(a) 84/20 = 4.20, so at least 5 bins **A1**

(a) Answer: 5

(b) places each item in the first bin with enough room, in the order given **M1**

(b) first bins correct **A1**

(b) bins: (11, 7); (15, 5); (12, 5); (14, 4); (11) **A1**

(b) Answer: (11, 7); (15, 5); (12, 5); (14, 4); (11)

(c) sorts the items into descending order **M1**

(c) applies first-fit to the sorted list **M1**

(c) 5 bins: (15, 5); (14, 5); (12, 7); (11, 4); (11) **A1**

(c) Answer: 5 bins: (15, 5); (14, 5); (12, 7); (11, 4); (11)

Question 2

(a) first pivot = 34 identified, with correct sublists {12, 8, 27, 19} (< 34) and {55, 41, 63} (≥ 34) **M1**

(a) correct second pivot and split on {12, 8, 27, 19}: pivot 12, giving {8} and {27, 19} **A1**

(a) correct second pivot and split on {55, 41, 63}: pivot 55, giving {41} and {63} **A1**

(a) correct third-level split on {27, 19}: pivot 27, giving {19} and { } (empty) **A1**

(a) correct final sorted list 8, 12, 19, 27, 34, 41, 55, 63 cao **A1**

(a) Answer: Final sorted list: 8, 12, 19, 27, 34, 41, 55, 63.

(b) correct method: sums the number of non-pivot items compared at each level, 7 + 3 + 2 + 1 **M1**

(b) 13 comparisons cao **A1**

(b) Answer: 13 comparisons.

(c) advantage: it is quick and simple to apply, since no extra comparisons are needed to choose the pivot (oe) **B1**

(c) disadvantage: if a sublist happens to be already sorted (or reverse sorted), this choice gives very unbalanced splits, leading to the algorithm's worst-case order of $O(n^2)$ (oe) **B1**

(c) Answer: Advantage: quick to select, no extra comparisons needed. Disadvantage: gives poor (unbalanced) splits, and hence poor efficiency, on lists that are already sorted or nearly sorted.

Question 3

(a) total volume = 8+2+7+3+6+4+5+5 = 40 litres **M1**

(a) lower bound = ceiling(40/10) = 4 cao **A1**

(a) Answer: Lower bound = 4 crates.

(b) correctly pairs the largest remaining tin, 8, with the 2-litre tin to exactly fill a crate: {8, 2} **M1**

(b) correctly pairs the next largest remaining tin, 7, with the 3-litre tin: {7, 3} **A1**

(b) correctly pairs the next largest remaining tin, 6, with the 4-litre tin: {6, 4} **A1**

(b) correctly places the two remaining 5-litre tins together to exactly fill the last crate: {5, 5} **A1**

(b) Answer: Crates: {8,2}, {7,3}, {6,4}, {5,5}, each exactly full (10 litres).

(c) all 4 crates are exactly full with no wasted space, so the lower bound of 4 found in part (a) has been achieved: this packing is optimal (oe) **B1**

(c) Answer: The full-bin method achieves exactly 4 crates, matching the lower bound, so this is an optimal packing with no wasted capacity.

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