4.2 Decision Maths: Graphs, Trees and Minimum Spanning Trees: 1-hour Pack

Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The graph G has vertices A, B, C, D, E and edges AC, AE, BC, BD, BE, CD.

(a) Find the degree of each vertex, and verify the handshaking lemma for G. **(3)**

(b) Classify G as Eulerian, semi-Eulerian or neither, giving a reason. **(2)**

(c) The complete graph K_6 has 6 vertices, each joined to every other. Find its number of edges, and state whether it is Eulerian. **(3)**

(Total for Question 1 is 8 marks)

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2 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AB = 16, AF = 23, BC = 36, BD = 38, BE = 33, BG = 24, CE = 10, DE = 5, FG = 19.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 2 is 7 marks)

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3 The graph G has vertices A, B, C, D, E, F and edges AC, AD, AE, BD, BE, BF, CD, CE, EF.

(a) Find the degree of each vertex, and verify the handshaking lemma for G. **(3)**

(b) Classify G as Eulerian, semi-Eulerian or neither, giving a reason. **(2)**

(c) The complete graph K_7 has 7 vertices, each joined to every other. Find its number of edges, and state whether it is Eulerian. **(3)**

(Total for Question 3 is 8 marks)

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4 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AB = 23, AE = 16, AG = 4, BG = 15, CE = 5, CF = 18, CG = 39, DE = 27, DF = 12, DG = 35, EF = 11.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 4 is 7 marks)

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5 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AB = 9, AC = 5, AE = 38, AG = 27, BC = 35, BG = 13, DE = 15, DF = 18, EF = 36, EG = 12.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 5 is 7 marks)

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6 The graph G has vertices A, B, C, D, E, F, G and edges AB, AG, BE, CD, CE, CF, CG, DE, EF, FG.

(a) Find the degree of each vertex, and verify the handshaking lemma for G. **(3)**

(b) Classify G as Eulerian, semi-Eulerian or neither, giving a reason. **(2)**

(c) The complete graph K_7 has 7 vertices, each joined to every other. Find its number of edges, and state whether it is Eulerian. **(3)**

(Total for Question 6 is 8 marks)

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7 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AD = 11, AE = 18, BC = 10, BE = 30, BG = 35, CF = 23, CG = 33, DF = 39, DG = 6, EF = 29, FG = 5.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 7 is 7 marks)

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Mark scheme · 4.2 Decision Maths: Graphs, Trees and Minimum Spanning Trees: 1-hour Pack

Question 1

(a) counts the edges at each vertex **M1**

(a) degrees: A 2, B 3, C 3, D 2, E 2 **A1**

(a) sum of degrees = 12 = 2 x 6 edges **A1**

(a) Answer: A 2, B 3, C 3, D 2, E 2; sum 12

(b) counts the vertices of odd degree: B, C **M1**

(b) semi-Eulerian (has an Eulerian trail but no cycle) **A1**

(b) Answer: semi-Eulerian (has an Eulerian trail but no cycle)

(c) 6(6 - 1)/2 **M1**

(c) 15 edges **A1**

(c) each vertex has degree 5, which is odd, so K_6 is not Eulerian **B1**

(c) Answer: 15 edges; not Eulerian

Question 2

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: DE (5), CE (10), AB (16), FG (19), AF (23), BE (33) **A1**

(a) Answer: DE (5), CE (10), AB (16), FG (19), AF (23), BE (33)

(b) 106 **B1**

(b) Answer: 106

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AB (16), AF (23), FG (19), BE (33), DE (5), CE (10) **A1**

(c) Answer: AB (16), AF (23), FG (19), BE (33), DE (5), CE (10)

Question 3

(a) counts the edges at each vertex **M1**

(a) degrees: A 3, B 3, C 3, D 3, E 4, F 2 **A1**

(a) sum of degrees = 18 = 2 x 9 edges **A1**

(a) Answer: A 3, B 3, C 3, D 3, E 4, F 2; sum 18

(b) counts the vertices of odd degree: A, B, C, D **M1**

(b) neither Eulerian nor semi-Eulerian **A1**

(b) Answer: neither Eulerian nor semi-Eulerian

(c) 7(7 - 1)/2 **M1**

(c) 21 edges **A1**

(c) each vertex has degree 6, which is even, so K_7 is Eulerian **B1**

(c) Answer: 21 edges; Eulerian

Question 4

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: AG (4), CE (5), EF (11), DF (12), BG (15), AE (16) **A1**

(a) Answer: AG (4), CE (5), EF (11), DF (12), BG (15), AE (16)

(b) 63 **B1**

(b) Answer: 63

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AG (4), BG (15), AE (16), CE (5), EF (11), DF (12) **A1**

(c) Answer: AG (4), BG (15), AE (16), CE (5), EF (11), DF (12)

Question 5

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: AC (5), AB (9), EG (12), BG (13), DE (15), DF (18) **A1**

(a) Answer: AC (5), AB (9), EG (12), BG (13), DE (15), DF (18)

(b) 72 **B1**

(b) Answer: 72

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AC (5), AB (9), BG (13), EG (12), DE (15), DF (18) **A1**

(c) Answer: AC (5), AB (9), BG (13), EG (12), DE (15), DF (18)

Question 6

(a) counts the edges at each vertex **M1**

(a) degrees: A 2, B 2, C 4, D 2, E 4, F 3, G 3 **A1**

(a) sum of degrees = 20 = 2 x 10 edges **A1**

(a) Answer: A 2, B 2, C 4, D 2, E 4, F 3, G 3; sum 20

(b) counts the vertices of odd degree: F, G **M1**

(b) semi-Eulerian (has an Eulerian trail but no cycle) **A1**

(b) Answer: semi-Eulerian (has an Eulerian trail but no cycle)

(c) 7(7 - 1)/2 **M1**

(c) 21 edges **A1**

(c) each vertex has degree 6, which is even, so K_7 is Eulerian **B1**

(c) Answer: 21 edges; Eulerian

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Question 7

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: FG (5), DG (6), BC (10), AD (11), AE (18), CF (23) **A1**

(a) Answer: FG (5), DG (6), BC (10), AD (11), AE (18), CF (23)

(b) 73 **B1**

(b) Answer: 73

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AD (11), DG (6), FG (5), AE (18), CF (23), BC (10) **A1**

(c) Answer: AD (11), DG (6), FG (5), AE (18), CF (23), BC (10)

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