4.2 Decision Maths: Graphs, Trees and Minimum Spanning Trees: 30-minute Pack

Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A network has vertices A, B, C, D and edges AB=3, AC=3, BC=5, BD=4, CD=4 (weights in km).

(a) Apply Kruskal's algorithm to find the total weight of a minimum spanning tree for this network. **(3)**

(b) State two different edge sets that both give a minimum spanning tree of total weight 10, and explain why both are valid minimum spanning trees. **(2)**

(c) State whether Prim's algorithm, started at vertex A, could also produce either of the two trees found in part (b). Give a reason. **(1)**

(Total for Question 1 is 6 marks)

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2 A network has vertices A, B, C, D, E, F and the following weighted arcs: AB = 3, AC = 34, AE = 8, BC = 37, BF = 25, CD = 28, CE = 27, CF = 13.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 2 is 7 marks)

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3 The table below shows the direct distances, in km, between six market towns A, B, C, D, E, F that a regional council wishes to connect with a new fibre-optic network, following existing roads only where a distance is given (a dash indicates no direct road).

	A	B	C	D	E	F
A	-	12	-	9	-	14
B	12	-	7	-	11	-
C	-	7	-	6	10	8
D	9	-	6	-	5	-
E	-	11	10	5	-	4
F	14	-	8	-	4	-

(a) State the number of edges required in a minimum spanning tree connecting all 6 towns. **(1)**

(b) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. Show, at each stage, the town added to the tree and the length of road used to connect it. **(9)**

(c) State the total length of cable required for the minimum spanning tree found in part (b). **(1)**

(d) Verify your answer to part (c) by applying Kruskal's algorithm to the same network and comparing the total weight obtained. **(2)**

(Total for Question 3 is 13 marks)

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Mark scheme · 4.2 Decision Maths: Graphs, Trees and Minimum Spanning Trees: 30-minute Pack

Question 1

(a) edges sorted into ascending order: AB(3), AC(3), BD(4), CD(4), BC(5) **M1**

(a) AB and AC both correctly accepted, then either BD or CD correctly accepted to complete a 3-edge tree **M1**

(a) 10 (km) cao **A1**

(a) Answer: 10 km

(b) {AB, AC, BD} correctly identified as one minimum spanning tree of weight 10 **B1**

(b) {AB, AC, CD} correctly identified as a second minimum spanning tree of weight 10, with explanation that BD and CD have equal weight (4), so either may be chosen without changing the total weight (oe) **B1**

(b) Answer: {AB, AC, BD} and {AB, AC, CD}, both of weight 10.

(c) Yes; since BD and CD both have weight 4, Prim's algorithm could select either edge when connecting D, so it could produce either of the two trees (oe) **B1**

(c) Answer: Yes; the tie between BD and CD means either tree could result.

Question 2

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: AB (3), AE (8), CF (13), BF (25), CD (28) **A1**

(a) Answer: AB (3), AE (8), CF (13), BF (25), CD (28)

(b) 77 **B1**

(b) Answer: 77

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AB (3), AE (8), BF (25), CF (13), CD (28) **A1**

(c) Answer: AB (3), AE (8), BF (25), CF (13), CD (28)

Question 3

(a) 5 cao **B1**

(a) Answer: 5

(b) AD=9 selected, D added (rejecting AB=12, AF=14) **M1**

(b) DE=5 selected, E added (comparing AB=12, AF=14, DC=6, DE=5) **M1**

(b) EF=4 selected, F added **M1**

(b) DC=6 selected, C added (rejecting EC=10, EB=11, FC=8) **M1**

(b) BC=7 selected, B added (rejecting AB=12, EB=11) **A1**

(b) correct order of towns added stated: D, E, F, C, B **A1**

(b) correct final edge set stated: {AD, DE, EF, DC, BC} **A1**

(b) correct method shown of eliminating (crossing out) the column of any town already in the tree when searching for the next minimum entry (oe) **B1**

(b) tree confirmed to use 5 edges connecting all 6 towns with no cycles **B1**

(b) Answer: Order of addition: D, E, F, C, B. Tree = {AD, DE, EF, DC, BC}.

(c) 31 (km) ft their tree from part (b) **B1**

(c) Answer: 31 km

(d) Kruskal's algorithm correctly applied: EF(4), DE(5), CD(6), BC(7) accepted in order; CF(8) rejected as it would form a cycle; AD(9) accepted **M1**

(d) total = 31 (km), matching part (c) and confirming the minimum spanning tree weight cao **A1**

(d) Answer: Kruskal's algorithm also gives 31 km, confirming part (c).

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