4.3 Decision Maths: Shortest Paths and Route Inspection: 1-hour Pack

Total Marks

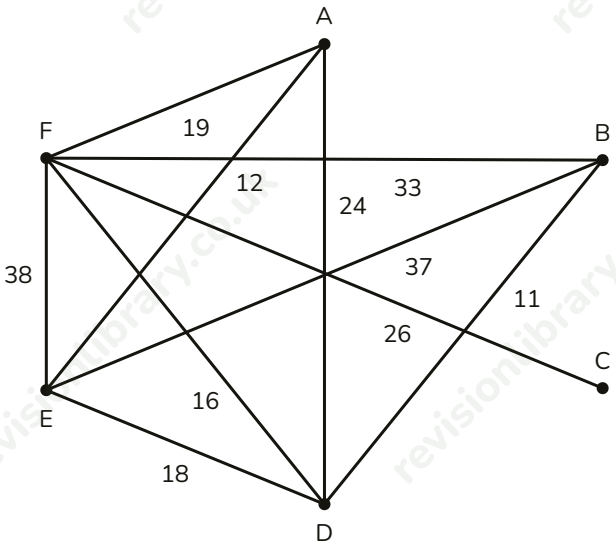
EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A network has vertices A, B, C, D, E, F and the following weighted arcs, which must all be traversed: AD = 24, AE = 12, AF = 19, BD = 11, BE = 37, BF = 33, CF = 26, DE = 18, DF = 16, EF = 38. The total weight of all the arcs is 234.

Diagram NOT accurately drawn



(a) Identify the vertices of odd degree. **(2)**

(b) Find the shortest distance between each pair of odd vertices, and hence find the combination of repeated arcs of least total weight that makes the network Eulerian. **(3)**

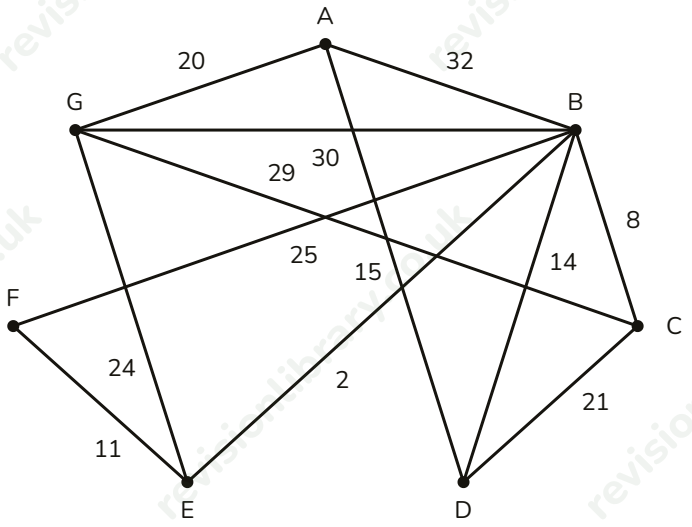
(c) Find the length of a shortest closed route that traverses every arc at least once. **(2)**

(Total for Question 1 is 7 marks)

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2 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AB = 32, AD = 15, AG = 20, BC = 8, BD = 14, BE = 2, BF = 25, BG = 30, CD = 21, CG = 29, EF = 11, EG = 24.

Diagram NOT accurately drawn



(a) Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

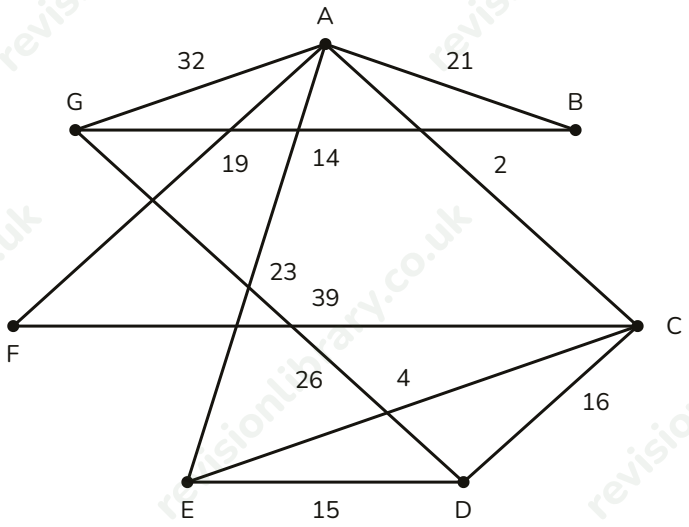
(b) State the shortest route from A to G and its length. **(2)**

(Total for Question 2 is 5 marks)

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3 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AB = 21, AC = 2, AE = 23, AF = 19, AG = 32, BG = 14, CD = 16, CE = 4, CF = 39, DE = 15, DG = 26.

Diagram NOT accurately drawn



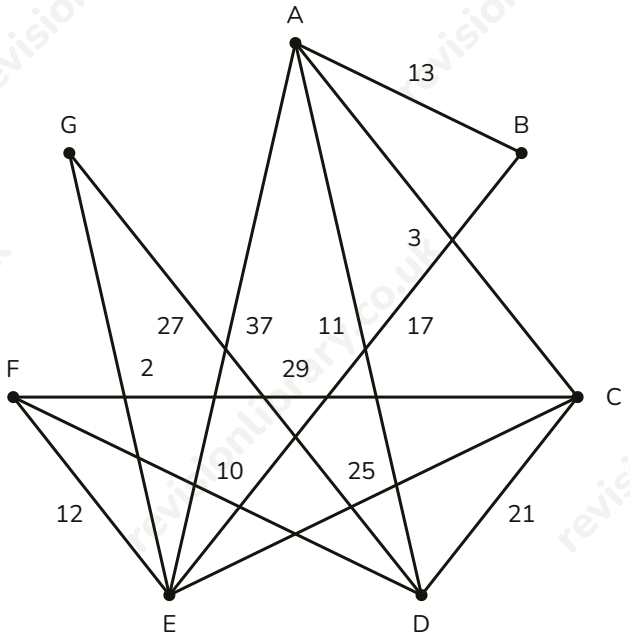
(a) Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

(b) State the shortest route from A to G and its length. **(2)**

(Total for Question 3 is 5 marks)

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4 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AB = 13, AC = 3, AD = 11, AE = 37, BE = 17, CD = 21, CE = 25, CF = 29, DF = 10, DG = 27, EF = 12, EG = 2.



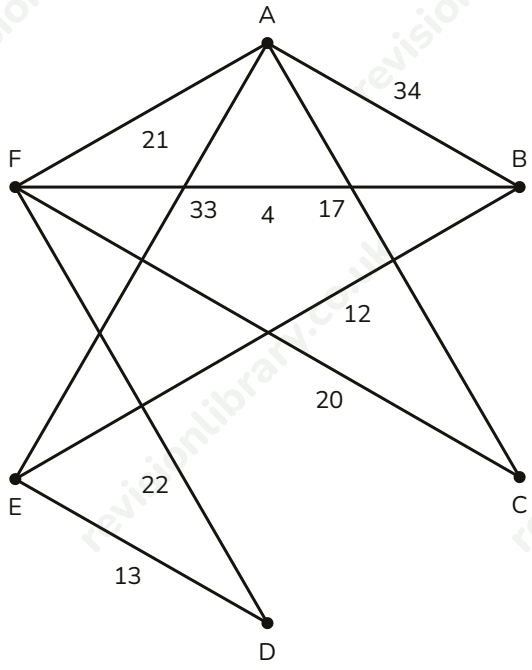
(a) Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

(b) State the shortest route from A to G and its length. **(2)**

(Total for Question 4 is 5 marks)

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5 A network has vertices A, B, C, D, E, F and the following weighted arcs: AB = 34, AC = 17, AE = 33, AF = 21, BE = 12, BF = 4, CF = 20, DE = 13, DF = 22.



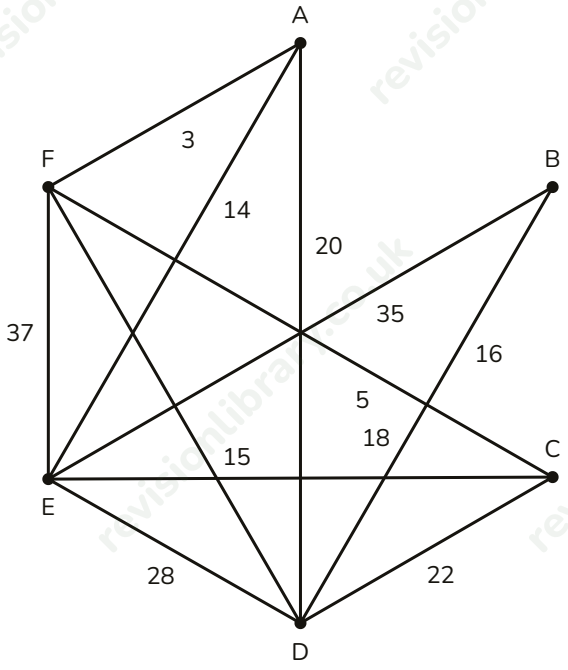
(a) Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

(b) State the shortest route from A to F and its length. **(2)**

(Total for Question 5 is 5 marks)

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6 A network has vertices A, B, C, D, E, F and the following weighted arcs: AD = 20, AE = 14, AF = 3, BD = 16, BE = 35, CD = 22, CE = 18, CF = 5, DE = 28, DF = 15, EF = 37.



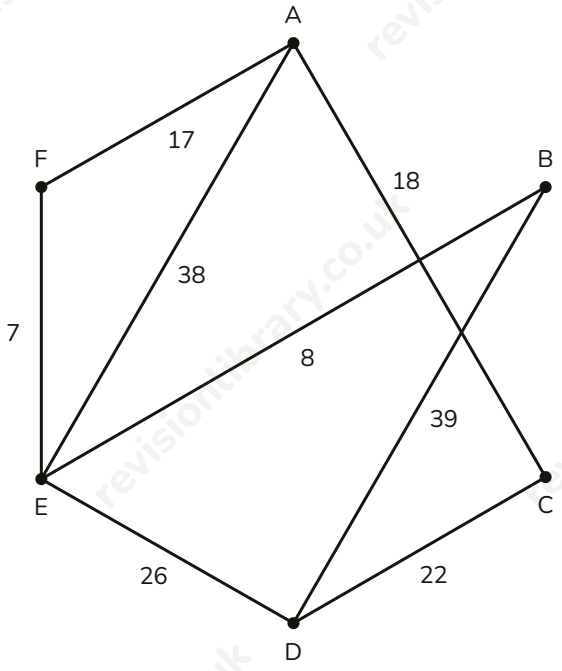
(a) Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

(b) State the shortest route from A to F and its length. **(2)**

(Total for Question 6 is 5 marks)

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7 A network has vertices A, B, C, D, E, F and the following weighted arcs, which must all be traversed: AC = 18, AE = 38, AF = 17, BD = 39, BE = 8, CD = 22, DE = 26, EF = 7. The total weight of all the arcs is 175.



(a) Identify the vertices of odd degree. **(2)**

(b) Find the shortest distance between each pair of odd vertices, and hence find the combination of repeated arcs of least total weight that makes the network Eulerian. **(2)**

(c) Find the length of a shortest closed route that traverses every arc at least once. **(2)**

(Total for Question 7 is 6 marks)

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8 A network of junctions used by a fleet of delivery vans is modelled by the vertices A, B, C, D, E, F, G. The arcs represent direct roads between junctions, and their weights are journey times, in minutes: AB = 6, AC = 4, AD = 9, BC = 2, BE = 7, CD = 3, CE = 5, CF = 8, DF = 4, EF = 1, EG = 6, FG = 3.

(a) State the rule used to decide which temporarily labelled vertex is made permanent at each stage of Dijkstra's algorithm. **(1)**

(b) Use Dijkstra's algorithm, starting at A, to find the shortest time from junction A to junction G. Show the order in which each vertex is permanently labelled, and the working values considered. **(6)**

(c) State the shortest route from A to G, and the time it takes. **(2)**

(d) State the shortest route from A to F, and the time it takes. **(2)**

(e) The council is considering closing the direct road CF for resurfacing. Explain, with reference to your working in part (b), whether this would change the shortest route (and its time) from A to G. **(2)**

(Total for Question 8 is 13 marks)

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Mark scheme · 4.3 Decision Maths: Shortest Paths and Route Inspection: 1-hour Pack

Question 1

(a) finds the degree of every vertex **M1**

(a) the odd vertices are A, B, C, F **A1**

(a) Answer: A, B, C, F

(b) finds the shortest path for each pair of odd vertices: AB = 35, AC = 45, AF = 19, BC = 53, BF = 27, CF = 26 **M1**

(b) totals the weight of each of the three pairings and compares them **M1**

(b) optimal pairing AB, CF, repeating AD, DB and CF with extra weight 24 + 11 + 26 = 61 **A1**

(b) Answer: repeat AD, DB and CF, extra weight 24 + 11 + 26 = 61

(c) total weight of the network plus the extra weight **M1**

(c) 234 + 61 = 295 **A1**

(c) Answer: 295

Question 2

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), D (15), G (20), B (29), E (31), C (36), F (42) **A1**

(a) Answer: A (0), D (15), G (20), B (29), E (31), C (36), F (42)

(b) traces back from the end vertex through the labels **M1**

(b) route AG, length 20 **A1**

(b) Answer: AG, length 20

Question 3

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), C (2), E (6), D (18), F (19), B (21), G (32) **A1**

(a) Answer: A (0), C (2), E (6), D (18), F (19), B (21), G (32)

(b) traces back from the end vertex through the labels **M1**

(b) route AG, length 32 **A1**

(b) Answer: AG, length 32

Question 4

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), C (3), D (11), B (13), F (21), E (28), G (30) **A1**

(a) Answer: A (0), C (3), D (11), B (13), F (21), E (28), G (30)

(b) traces back from the end vertex through the labels **M1**

(b) route ACEG, length 30 **A1**

(b) Answer: ACEG, length 30

Question 5

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), C (17), F (21), B (25), E (33), D (43) **A1**

(a) Answer: A (0), C (17), F (21), B (25), E (33), D (43)

(b) traces back from the end vertex through the labels **M1**

(b) route AF, length 21 **A1**

(b) Answer: AF, length 21

Question 6

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), F (3), C (8), E (14), D (18), B (34) **A1**

(a) Answer: A (0), F (3), C (8), E (14), D (18), B (34)

(b) traces back from the end vertex through the labels **M1**

(b) route AF, length 3 **A1**

(b) Answer: AF, length 3

Question 7

(a) finds the degree of every vertex **M1**

(a) the odd vertices are A, D **A1**

(a) Answer: A, D

(b) finds the shortest path A-C-D between the odd vertices: AD = 18 + 22 = 40 **M1**

(b) repeats AC and CD with extra weight 40 **A1**

(b) Answer: repeat AC and CD, extra weight 18 + 22 = 40

(c) total weight of the network plus the extra weight **M1**

(c) 175 + 40 = 215 **A1**

(c) Answer: 215

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Question 8

(a) the temporarily labelled vertex with the smallest working value is made permanent at each stage (oe) **B1**

(a) Answer: The vertex with the smallest current working value is selected and made permanent.

(b) A permanently labelled with 0; initial working values B = 6 (via A), C = 4 (via A), D = 9 (via A) **M1**

(b) C permanently labelled second, with value 4; D updated to 7 (via C, since 4 + 3 = 7 < 9), E = 9 (via C), F = 12 (via C); B unchanged at 6 (4 + 2 = 6 is not an improvement) **M1**

(b) B permanently labelled third, with value 6 (via A); no improving updates result from B **M1**

(b) D permanently labelled fourth, with value 7 (via C); F updated to 11 (via D, since 7 + 4 = 11 < 12) **M1**

(b) E permanently labelled fifth, with value 9 (via C); F updated to 10 (via E, since 9 + 1 = 10 < 11), G = 15 (via E) **A1**

(b) F permanently labelled sixth, with value 10 (via E); G updated to 13 (via F, since 10 + 3 = 13 < 15); G permanently labelled seventh (last) at 13; full order stated as A, C, B, D, E, F, G **A1**

(b) Answer: Order of permanent labelling: A(0), C(4), B(6), D(7), E(9), F(10), G(13).

(c) A-C-E-F-G (oe, traced back through predecessors from G) **B1**

(c) 13 (minutes) cao **B1**

(c) Answer: A-C-E-F-G, 13 minutes.

(d) A-C-E-F (oe, traced back through predecessors from F) **B1**

(d) 10 (minutes) cao **B1**

(d) Answer: A-C-E-F, 10 minutes.

(e) correct conclusion: no change results (oe) **B1**

(e) valid justification, e.g. F's temporary value of 12 obtained via CF was superseded first by 11 (via D) and then by 10 (via E), so arc CF was never used to obtain F's final permanent label, or any part of the shortest route to G (oe) **B1**

(e) Answer: No change; the shortest route already avoids arc CF.

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