4.3 Decision Maths: Shortest Paths and Route Inspection: 30-minute Pack

Total Marks

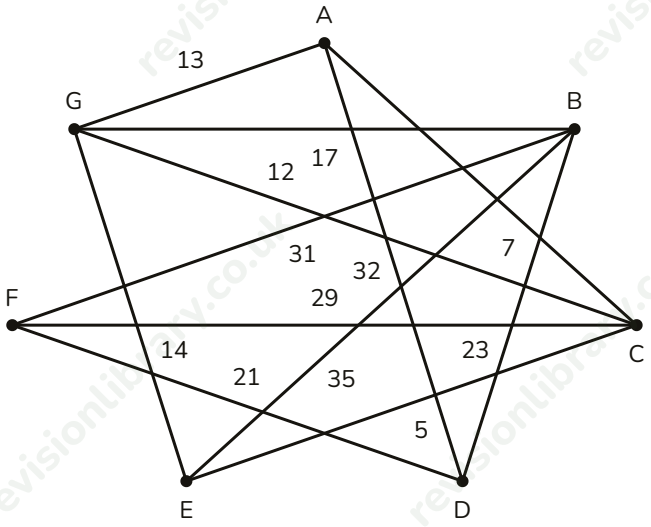
EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AC = 7, AD = 32, AG = 13, BD = 23, BE = 35, BF = 31, BG = 17, CE = 5, CF = 29, CG = 12, DF = 21, EG = 14.

Diagram NOT accurately drawn



(a) Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

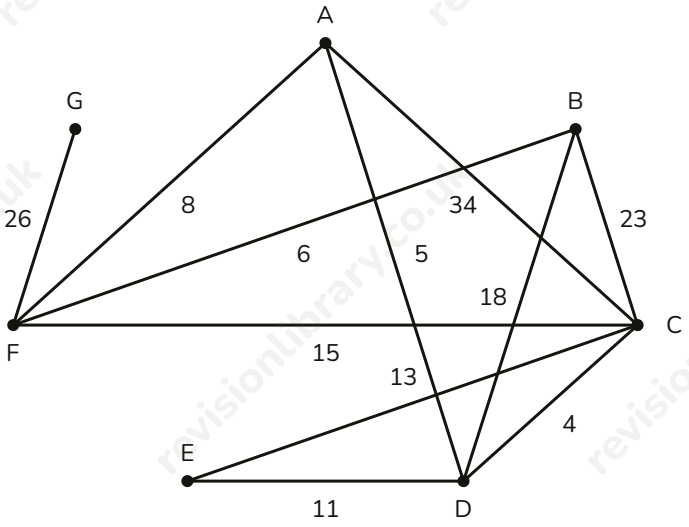
(b) State the shortest route from A to G and its length. **(2)**

(Total for Question 1 is 5 marks)

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2 A network has vertices A, B, C, D, E, F, G and the following weighted arcs, which must all be traversed: AC = 34, AD = 5, AF = 8, BC = 23, BD = 18, BF = 6, CD = 4, CE = 13, CF = 15, DE = 11, FG = 26. The total weight of all the arcs is 163.

Diagram NOT accurately drawn



(a) Identify the vertices of odd degree. **(2)**

(b) Find the shortest distance between each pair of odd vertices, and hence find the combination of repeated arcs of least total weight that makes the network Eulerian. **(3)**

(c) Find the length of a shortest closed route that traverses every arc at least once. **(2)**

(Total for Question 2 is 7 marks)

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3 A driver must plan a tour visiting four towns, Marchford (M), Cavendish (C), Elton (E) and Brindle (B), before returning to the start. The distances between each pair of towns (in miles, every pair directly connected) are: MC=150, MB=320, ME=405, CE=380, CB=245, EB=210.

(a) By deleting town M and finding a minimum spanning tree for the remaining network of C, E, B, find a lower bound for the length of the optimal tour. **(4)**

(b) Apply the nearest neighbour algorithm, starting at M, to find an upper bound for the length of the optimal tour. **(3)**

(c) There are only three distinct routes that visit each town once and return to M. List all three routes and find the length of each. Hence state the length of the optimal tour. **(4)**

(d) Verify that the length of the optimal tour found in part (c) lies within the bounds found in parts (a) and (b). **(2)**

(Total for Question 3 is 13 marks)

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Mark scheme · 4.3 Decision Maths: Shortest Paths and Route Inspection: 30-minute Pack

Question 1

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), C (7), E (12), G (13), B (30), D (32), F (36) **A1**

(a) Answer: A (0), C (7), E (12), G (13), B (30), D (32), F (36)

(b) traces back from the end vertex through the labels **M1**

(b) route AG, length 13 **A1**

(b) Answer: AG, length 13

Question 2

(a) finds the degree of every vertex **M1**

(a) the odd vertices are A, B, C, G **A1**

(a) Answer: A, B, C, G

(b) finds the shortest path for each pair of odd vertices: AB = 14, AC = 9, AG = 34, BC = 21, BG = 32, CG = 41 **M1**

(b) totals the weight of each of the three pairings and compares them **M1**

(b) optimal pairing AC, BG, repeating AD, DC, BF and FG with extra weight 5 + 4 + 6 + 26 = 41 **A1**

(b) Answer: repeat AD, DC, BF and FG, extra weight 5 + 4 + 6 + 26 = 41

(c) total weight of the network plus the extra weight **M1**

(c) 163 + 41 = 204 **A1**

(c) Answer: 204

Question 3

(a) network on C,E,B has arcs CE=380, EB=210, CB=245; sorted ascending: EB(210), CB(245), CE(380) **M1**

(a) EB accepted (210); CB accepted (245), connecting all of C, E, B with 2 edges (CE not needed, and would form a cycle); minimum spanning tree weight = 210+245 = 455 **M1**

(a) two least-weight arcs from M are added: MC=150 and MB=320 (the two smallest of MC=150, MB=320, ME=405) **A1**

(a) lower bound = 455 + 150 + 320 = 925 (miles) cao **A1**

(a) Answer: Lower bound = 925 miles.

(b) from M, nearest unvisited is C (150); from C, nearest unvisited is B (245, compared with CE=380) **M1**

(b) from B, only E remains (210); return from E to M (405); tour: M-C-B-E-M **A1**

(b) length = 150+245+210+405 = 1010 (miles) cao **A1**

(b) Answer: Upper bound = 1010 miles (tour M-C-B-E-M).

(c) Route 1: M-C-E-B-M = 150+380+210+320 = 1060 **M1**

(c) Route 2: M-C-B-E-M = 150+245+210+405 = 1010 **M1**

(c) Route 3: M-E-C-B-M = 405+380+245+320 = 1350 **A1**

(c) optimal tour = Route 2, M-C-B-E-M, length 1010 (miles), the shortest of the three cao **A1**

(c) Answer: Optimal tour: M-C-B-E-M, length 1010 miles.

(d) 925 (lower bound, part (a)) ≤ 1010 (optimal tour, part (c)) **B1**

(d) 1010 (optimal tour, part (c)) ≤ 1010 (upper bound, part (b)); in fact the nearest neighbour tour found in part (b) is itself the optimal tour here cao **B1**

(d) Answer: 925 ≤ 1010 ≤ 1010, so the optimal tour does lie within the bounds; here the upper bound found by nearest neighbour is exactly optimal.

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