# 4.3 Decision Maths: Shortest Paths and Route Inspection: Test Pack

Total Marks

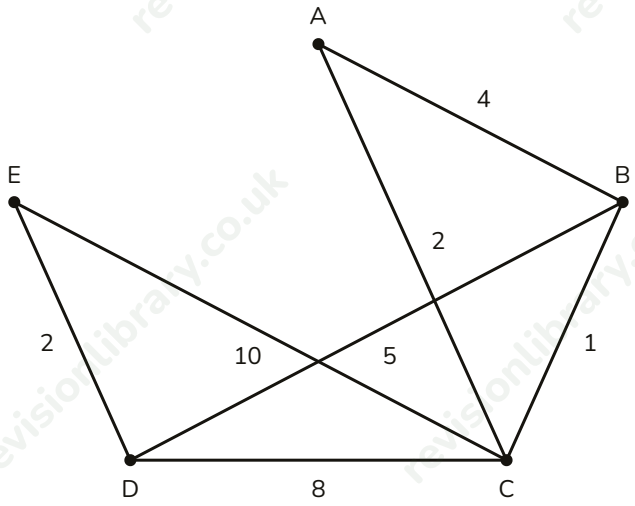
EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

**1** The network below shows the cycle paths connecting five villages: Ashfield (A), Barrowby (B), Cotgrave (C), Denwick (D) and Elsdon (E). The number on each arc is the length of that path, in km: AB = 4, AC = 2, BC = 1, BD = 5, CD = 8, CE = 10, DE = 2.

Diagram NOT accurately drawn




**(a)** State one condition on the arc weights of a network that Dijkstra's algorithm requires in order to be guaranteed to find the shortest path. **(1)**

**(b)** Use Dijkstra's algorithm, starting at A, to find the shortest path from A to E. Show clearly the order in which each vertex receives its permanent label, and the working values considered at each stage. **(5)**

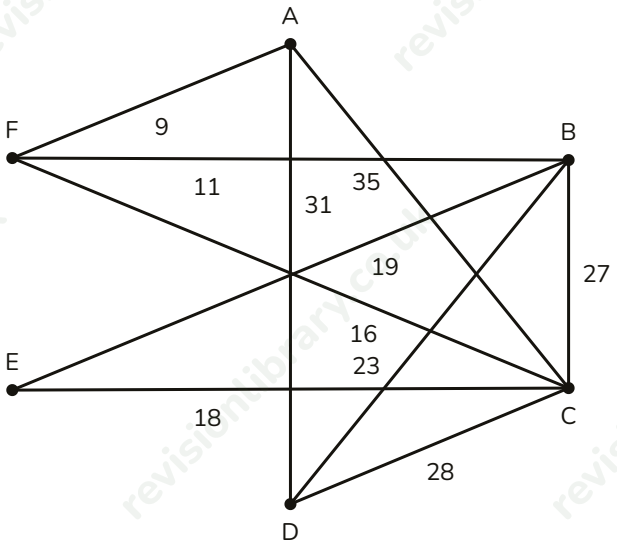
**(c)** Hence state the shortest path from A to E, and its length. **(2)**

**(Total for Question 1 is 8 marks)**


© Revision Library | revisionlibrary.co.uk

**2** A network has vertices A, B, C, D, E, F and the following weighted arcs: AC = 35, AD = 31, AF = 9, BC = 27, BD = 23, BE = 19, BF = 11, CD = 28, CE = 18, CF = 16.

Diagram NOT accurately drawn




**(a)** Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

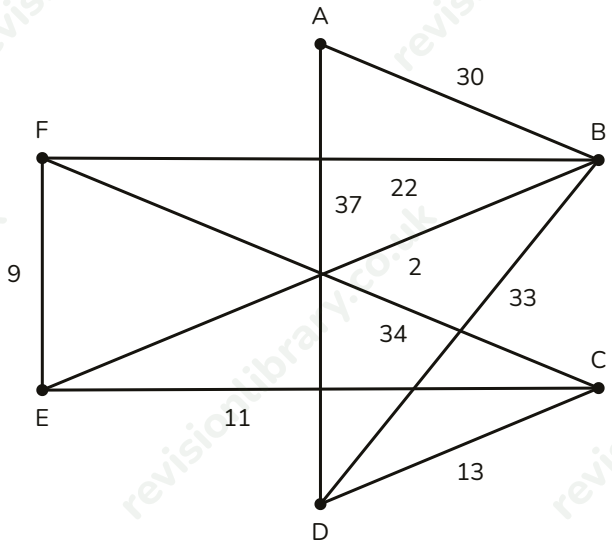
**(b)** State the shortest route from A to F and its length. **(2)**

**(Total for Question 2 is 5 marks)**


© Revision Library | revisionlibrary.co.uk

**3** A network has vertices A, B, C, D, E, F and the following weighted arcs: AB = 30, AD = 37, BD = 33, BE = 2, BF = 22, CD = 13, CE = 11, CF = 34, EF = 9.

Diagram NOT accurately drawn



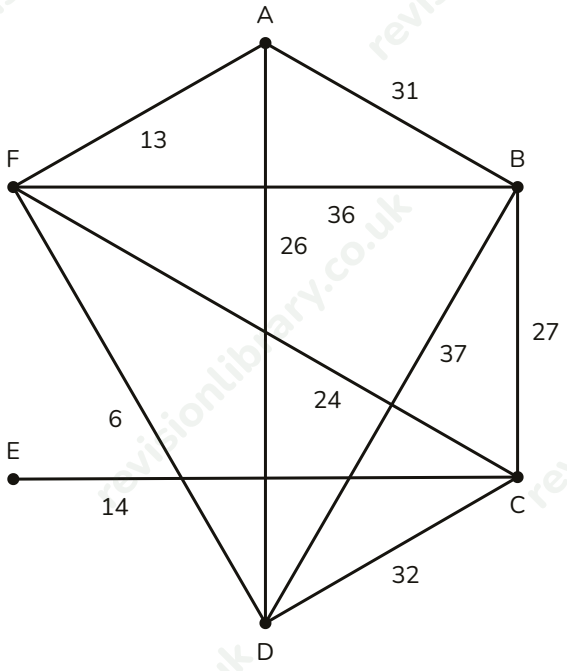

**(a)** Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

**(b)** State the shortest route from A to F and its length. **(2)**

**(Total for Question 3 is 5 marks)**


© Revision Library | revisionlibrary.co.uk

**4** A network has vertices A, B, C, D, E, F and the following weighted arcs, which must all be traversed: AB = 31, AD = 26, AF = 13, BC = 27, BD = 37, BF = 36, CD = 32, CE = 14, CF = 24, DF = 6. The total weight of all the arcs is 246.




**(a)** Identify the vertices of odd degree. **(2)**

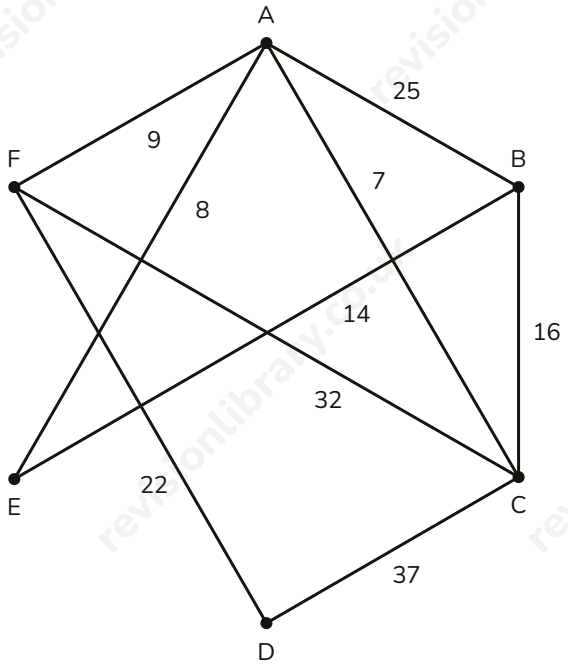
**(b)** Find the shortest distance between each pair of odd vertices, and hence find the combination of repeated arcs of least total weight that makes the network Eulerian. **(2)**

**(c)** Find the length of a shortest closed route that traverses every arc at least once. **(2)**

**(Total for Question 4 is 6 marks)**


© Revision Library | revisionlibrary.co.uk

**5** A network has vertices A, B, C, D, E, F and the following weighted arcs: AB = 25, AC = 7, AE = 8, AF = 9, BC = 16, BE = 14, CD = 37, CF = 32, DF = 22.



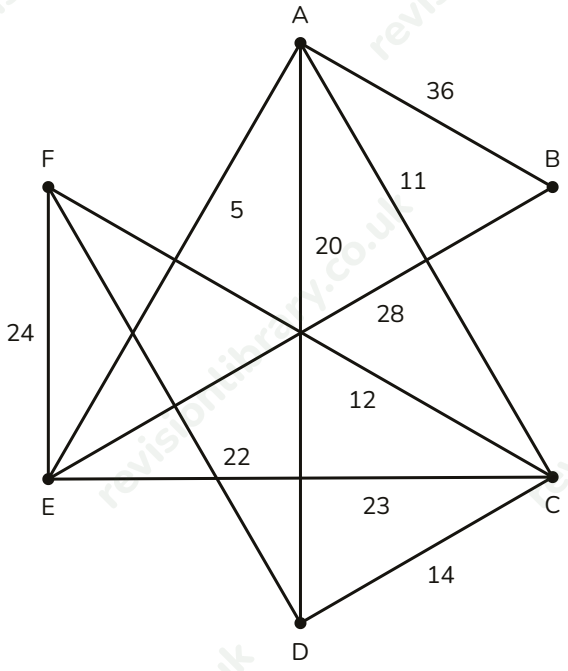

**(a)** Use Dijkstra's algorithm to find the shortest distance from A to every vertex, listing the vertices in the order in which they become permanently labelled, with their final labels. **(3)**

**(b)** State the shortest route from A to F and its length. **(2)**

**(Total for Question 5 is 5 marks)**


© Revision Library | revisionlibrary.co.uk

**6** A network has vertices A, B, C, D, E, F and the following weighted arcs, which must all be traversed: AB = 36, AC = 11, AD = 20, AE = 5, BE = 28, CD = 14, CE = 23, CF = 12, DF = 22, EF = 24. The total weight of all the arcs is 195.




**(a)** Identify the vertices of odd degree. **(2)**

**(b)** Find the shortest distance between each pair of odd vertices, and hence find the combination of repeated arcs of least total weight that makes the network Eulerian. **(2)**

**(c)** Find the length of a shortest closed route that traverses every arc at least once. **(2)**

**(Total for Question 6 is 6 marks)**


© Revision Library | revisionlibrary.co.uk

**7** A one-way road system linking a supermarket's warehouse (S) to a delivery hub (T) is modelled by a directed network with vertices S, A, B, C, T. Every arc may only be travelled in the direction stated: S to A = 5, S to B = 9, A to B = 2, A to C = 6, B to C = 1, B to T = 7, C to T = 3 (weights are journey times, in minutes; there is no arc in the reverse direction unless separately listed).

**(a)** Explain why the direction of each arc must be taken into account when applying Dijkstra's algorithm to this network. **(1)**

**(b)** Use Dijkstra's algorithm on this directed network, starting at S, to find the shortest time from S to T. Show the order in which each vertex is permanently labelled and the working values considered. **(6)**

**(c)** State the shortest route from S to T, and the time it takes. **(2)**

**(d)** Verify that the shortest route found in part (c) does not use the direct arc from S to B, and explain what this shows about B's permanent label. **(2)**

**(Total for Question 7 is 11 marks)**


© Revision Library | revisionlibrary.co.uk

# Mark scheme · 4.3 Decision Maths: Shortest Paths and Route Inspection: Test Pack

## Question 1

(a) all arc weights (lengths) must be non-negative (oe) **B1**

**(a) Answer: All arc weights must be non-negative.**

(b) A permanently labelled with 0; working values found: B = 4 (via A), C = 2 (via A) **M1**

(b) C permanently labelled second, with value 2; B updated to 3 (via C, since 2 + 1 = 3 < 4), D = 10 (via C), E = 12 (via C) **M1**

(b) B permanently labelled third, with value 3; D updated to 8 (via B, since 3 + 5 = 8 < 10) **M1**

(b) D permanently labelled fourth, with value 8; E updated to 10 (via D, since 8 + 2 = 10 < 12) **A1**

(b) E permanently labelled fifth (last), with final value 10; order of labelling correctly stated as A, C, B, D, E **A1**

**(b) Answer: Order of permanent labelling: A(0), C(2), B(3), D(8), E(10).**

(c) A-C-B-D-E (oe, traced back through predecessors from E) **B1**

(c) 10 (km) cao **B1**

**(c) Answer: Shortest path: A-C-B-D-E, length 10 km.**

## Question 2

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), F (9), B (20), C (25), D (31), E (39) **A1**

**(a) Answer: A (0), F (9), B (20), C (25), D (31), E (39)**

(b) traces back from the end vertex through the labels **M1**

(b) route AF, length 9 **A1**

**(b) Answer: AF, length 9**

## Question 3

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), B (30), E (32), D (37), F (41), C (43) **A1**

**(a) Answer: A (0), B (30), E (32), D (37), F (41), C (43)**

(b) traces back from the end vertex through the labels **M1**

(b) route ABEF, length 41 **A1**

**(b) Answer: ABEF, length 41**

## Question 4

(a) finds the degree of every vertex **M1**

(a) the odd vertices are A, E **A1**

**(a) Answer: A, E**

(b) finds the shortest path A-F-C-E between the odd vertices: AE = 13 + 24 + 14 = 51 **M1**

(b) repeats AF, FC and CE with extra weight 51 **A1**

**(b) Answer: repeat AF, FC and CE, extra weight 13 + 24 + 14 = 51**

(c) total weight of the network plus the extra weight **M1**

(c) 246 + 51 = 297 **A1**

**(c) Answer: 297**

## Question 5

(a) labels A with 0 and updates the working values of its neighbours **M1**

(a) repeatedly makes permanent the vertex with the smallest working value and updates its neighbours **M1**

(a) final labels: A (0), C (7), E (8), F (9), B (22), D (31) **A1**

**(a) Answer: A (0), C (7), E (8), F (9), B (22), D (31)**

(b) traces back from the end vertex through the labels **M1**

(b) route AF, length 9 **A1**

**(b) Answer: AF, length 9**

## Question 6

(a) finds the degree of every vertex **M1**

(a) the odd vertices are D, F **A1**

**(a) Answer: D, F**

(b) finds the shortest path for each pair of odd vertices: DF = 22 **M1**

(b) optimal pairing DF with extra weight 22 **A1**

**(b) Answer: repeat DF, extra weight 22**

(c) total weight of the network plus the extra weight **M1**

(c) 195 + 22 = 217 **A1**

**(c) Answer: 217**


© Revision Library | revisionlibrary.co.uk

## Question 7

(a) an arc can only be used in the direction shown; a vertex's working value may only be updated using an arc that points towards it, so travelling the 'wrong way' along a one-way arc is not permitted (oe) **B1**

**(a) Answer: Arcs may only be used in the direction given, so a vertex can only be reached via arcs pointing into it.**

(b) S permanently labelled with 0; initial working values A = 5 (via S), B = 9 (via S) **M1**

(b) A permanently labelled second, with value 5; B updated to 7 (via A, since 5 + 2 = 7 < 9), C = 11 (via A, since 5 + 6 = 11) **M1**

(b) B permanently labelled third, with value 7 (via A); C updated to 8 (via B, since 7 + 1 = 8 < 11) **M1**

(b) C permanently labelled fourth, with value 8 (via B); T updated to 11 (via C, since 8 + 3 = 11) **M1**

(b) T permanently labelled fifth (last), with value 11 **A1**

(b) full order of labelling correctly stated as S, A, B, C, T **A1**

**(b) Answer: Order of permanent labelling: S(0), A(5), B(7), C(8), T(11).**

(c) S-A-B-C-T (oe, traced back through predecessors from T) **B1**

(c) 11 (minutes) cao **B1**

**(c) Answer: S-A-B-C-T, 11 minutes.**

(d) the route S-A-B-C-T uses arc A to B, not the direct arc S to B, so the direct arc SB is not used cao **B1**

(d) this shows that B's permanent label of 7 was achieved via A (5 + 2 = 7), which is less than the direct arc's weight of 9, so the direct arc is not always part of the shortest route through a vertex (oe) **B1**

**(d) Answer: The route uses arc A-B, not S-B directly; B's shortest working value (7, via A) beats the direct arc S-B (9).**


© Revision Library | revisionlibrary.co.uk