4.4 Decision Maths: Critical Path Analysis and Linear Programming: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A project consists of activities A, B, C, D, E, F, G, H. Each activity is listed with its duration in days and the activities that must be completed before it can start: A (duration 9, after none); B (duration 2, after A); C (duration 3, after A, B); D (duration 7, after A); E (duration 8, after D); F (duration 2, after A, D); G (duration 7, after B, E); H (duration 3, after A).

(a) Carry out a forward pass to find the earliest start time of each activity, and state the minimum completion time of the project. **(3)**

(b) Carry out a backward pass and hence find the critical activities. **(3)**

(c) Find the total float of activity B. **(2)**

(d) Find a lower bound for the number of workers needed to complete the project in the minimum time, if each activity needs one worker. **(2)**

(Total for Question 1 is 10 marks)

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2 A bakery makes x batches of rolls and y batches of loaves each day. The constraints are $x + 4y \leq 32$, $x + 3y \leq 25$, $x \geq 0$ and $y \geq 0$. The profit, in hundreds of pounds, is $P = 5x + 2y$, and the profit is to be maximised.

(a) Draw the feasible region and find the coordinates of each of its vertices. **(3)**

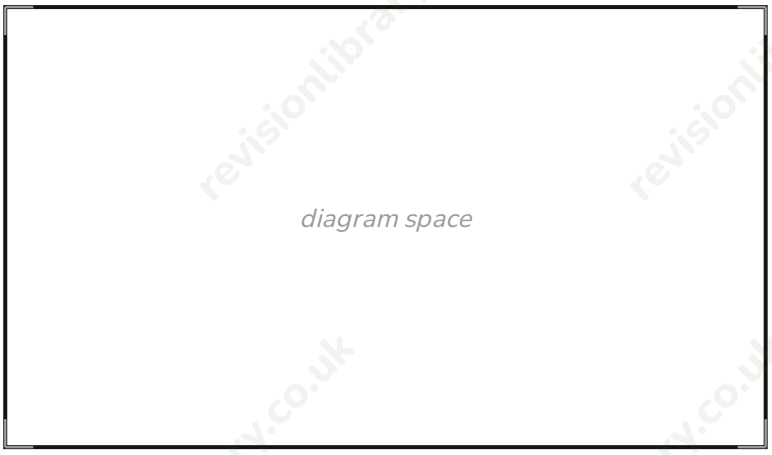
(b) Evaluate P at each vertex and hence find the maximum profit and the values of x and y which give it. **(3)**

(Total for Question 2 is 6 marks)

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3 Priya is planning a small kitchen renovation. The table below shows the activities involved.

Activity	Task	Duration (days)	Immediate predecessors
A	Strip out the old units	2	-
B	Order the new worktop (delivery lead time)	1	-
C	First-fix the plumbing and electrics	3	A
D	Fit the new worktop	2	A, B



(a) Explain why a dummy activity is required when drawing the activity-on-arc network for this project. **(2)**

(b) Draw the activity-on-arc network for the project. Label events with circled numbers, activities with their letter and duration, and show the dummy activity as a dashed arrow. **(5)**

(Total for Question 3 is 7 marks)

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4 A bakery makes x batches of rolls and y batches of loaves each day. The constraints are $x + 4y \leq 28$, $3x + 3y \leq 39$, $x \geq 0$ and $y \geq 0$. The profit, in hundreds of pounds, is $P = 3x + y$, and the profit is to be maximised.

(a) Draw the feasible region and find the coordinates of each of its vertices. **(3)**

(b) Evaluate P at each vertex and hence find the maximum profit and the values of x and y which give it. **(3)**

(Total for Question 4 is 6 marks)

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5 A project consists of activities A, B, C, D, E, F, G, H, I. Each activity is listed with its duration in days and the activities that must be completed before it can start: A (duration 7, after none); B (duration 9, after A); C (duration 5, after B); D (duration 4, after A, B); E (duration 8, after B); F (duration 4, after A); G (duration 6, after E); H (duration 2, after F); I (duration 5, after F).

(a) Carry out a forward pass to find the earliest start time of each activity, and state the minimum completion time of the project. **(3)**

(b) Carry out a backward pass and hence find the critical activities. **(3)**

(c) Find the total float of activity C. **(2)**

(d) Find a lower bound for the number of workers needed to complete the project in the minimum time, if each activity needs one worker. **(2)**

(Total for Question 5 is 10 marks)

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6 A factory makes x units of product P and y units of product Q each week. The constraints are $3x + y \leq 32$, $3x + 3y \leq 48$, $x \geq 0$ and $y \geq 0$. The profit, in hundreds of pounds, is $P = 5x + 2y$, and the profit is to be maximised.

(a) Draw the feasible region and find the coordinates of each of its vertices. **(3)**

(b) Evaluate P at each vertex and hence find the maximum profit and the values of x and y which give it. **(3)**

(Total for Question 6 is 6 marks)

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7 A project consists of activities A, B, C, D, E, F, G, H. Each activity is listed with its duration in days and the activities that must be completed before it can start: A (duration 5, after none); B (duration 5, after A); C (duration 6, after B); D (duration 8, after C); E (duration 4, after D); F (duration 4, after A, D); G (duration 6, after D, E); H (duration 8, after E).

(a) Carry out a forward pass to find the earliest start time of each activity, and state the minimum completion time of the project. **(3)**

(b) Carry out a backward pass and hence find the critical activities. **(3)**

(c) Find the total float of activity F. **(2)**

(d) Find a lower bound for the number of workers needed to complete the project in the minimum time, if each activity needs one worker. **(2)**

(Total for Question 7 is 10 marks)

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Mark scheme · 4.4 Decision Maths: Critical Path Analysis and Linear Programming: 1-hour Pack

Question 1

(a) earliest start = latest earliest finish of the immediate predecessors **M1**

(a) earliest starts: A 0, B 9, C 11, D 9, E 16, F 16, G 24, H 9 **A1**

(a) minimum completion time = 31 days **A1**

(a) Answer: 31 days

(b) latest finish = smallest latest start of the successors **M1**

(b) latest finishes: A 9, B 24, C 31, D 16, E 24, F 31, G 31, H 31 **A1**

(b) critical activities (zero float): A, D, E, G **A1**

(b) Answer: A, D, E, G

(c) total float = latest finish - earliest start - duration **M1**

(c) 24 - 9 - 2 = 13 days **A1**

(c) Answer: 13 days

(d) total duration of all activities = 41 **M1**

(d) 41/31 = 1.32, so at least 2 workers **A1**

(d) Answer: 2

Question 2

(a) plots both constraint lines and shades the unwanted region **M1**

(a) finds the intercepts with the axes and the intersection of the two lines **M1**

(a) vertices: (0, 0), (0, 8), (4, 7), (25, 0) **A1**

(a) Answer: (0, 0), (0, 8), (4, 7), (25, 0)

(b) evaluates P at each vertex **M1**

(b) values: 0, 16, 34, 125 **A1**

(b) maximum P = 125 at (25, 0) **A1**

(b) Answer: P = 125 at (25, 0)

Question 3

(a) recognise that D depends on both A and B, whereas C depends on A alone, so A cannot simply end at the same event as B without wrongly forcing C to also wait for B (oe) **M1**

(a) state that a dummy activity (zero duration) is used to carry the completion of A through to the event that D leaves from, without altering the earliest start time of C (oe) **A1**

(a) Answer: One dummy activity is needed, to feed A's completion into D's event without also delaying C.

(b) single start event (event 1), with A and B both shown leaving it **B1**

(b) C shown leaving the event reached only by A (event 2), with duration 3 **B1**

(b) dummy activity (dashed, 0 duration) drawn from the event reached by A (event 2) to the event reached by B (event 3) **B1**

(b) D shown leaving event 3 (reached by both B directly and A via the dummy), with duration 2 **B1**

(b) single end event reached by both C and D, network fully connected with no redundant or crossing dummies **B1**

(b) Answer: Network: 1 -> 2 (A, 2 days); 1 -> 3 (B, 1 day); 2 -> 3 (dummy, 0 days); 2 -> 4 (C, 3 days); 3 -> 4 (D, 2 days).

Question 4

(a) plots both constraint lines and shades the unwanted region **M1**

(a) finds the intercepts with the axes and the intersection of the two lines **M1**

(a) vertices: (0, 0), (0, 7), (8, 5), (13, 0) **A1**

(a) Answer: (0, 0), (0, 7), (8, 5), (13, 0)

(b) evaluates P at each vertex **M1**

(b) values: 0, 7, 29, 39 **A1**

(b) maximum P = 39 at (13, 0) **A1**

(b) Answer: P = 39 at (13, 0)

Question 5

(a) earliest start = latest earliest finish of the immediate predecessors **M1**

(a) earliest starts: A 0, B 7, C 16, D 16, E 16, F 7, G 24, H 11, I 11 **A1**

(a) minimum completion time = 30 days **A1**

(a) Answer: 30 days

(b) latest finish = smallest latest start of the successors **M1**

(b) latest finishes: A 7, B 16, C 30, D 30, E 24, F 25, G 30, H 30, I 30 **A1**

(b) critical activities (zero float): A, B, E, G **A1**

(b) Answer: A, B, E, G

(c) total float = latest finish - earliest start - duration **M1**

(c) 30 - 16 - 5 = 9 days **A1**

(c) Answer: 9 days

(d) total duration of all activities = 50 **M1**

(d) 50/30 = 1.67, so at least 2 workers **A1**

(d) Answer: 2

Question 6

(a) plots both constraint lines and shades the unwanted region **M1**

(a) finds the intercepts with the axes and the intersection of the two lines **M1**

(a) vertices: (0, 0), (0, 16), (8, 8), (32/3, 0) **A1**

(a) Answer: (0, 0), (0, 16), (8, 8), (32/3, 0)

(b) evaluates P at each vertex **M1**

(b) values: 0, 32, 56, 160/3 **A1**

(b) maximum P = 56 at (8, 8) **A1**

(b) Answer: P = 56 at (8, 8)

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Question 7

(a) earliest start = latest earliest finish of the immediate predecessors **M1**

(a) earliest starts: A 0, B 5, C 10, D 16, E 24, F 24, G 28, H 28 **A1**

(a) minimum completion time = 36 days **A1**

(a) Answer: 36 days

(b) latest finish = smallest latest start of the successors **M1**

(b) latest finishes: A 5, B 10, C 16, D 24, E 28, F 36, G 36, H 36 **A1**

(b) critical activities (zero float): A, B, C, D, E, H **A1**

(b) Answer: A, B, C, D, E, H

(c) total float = latest finish - earliest start - duration **M1**

(c) 36 - 24 - 4 = 8 days **A1**

(c) Answer: 8 days

(d) total duration of all activities = 46 **M1**

(d) 46/36 = 1.28, so at least 2 workers **A1**

(d) Answer: 2

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