4.4 Decision Maths: Critical Path Analysis and Linear Programming: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question concerns the vocabulary used when analysing an activity network for a project.

(a) State what is meant by a critical activity of a project network. **(1)**

(b) State what is meant by the total float of an activity. **(1)**

(c) State what is meant by the free float of an activity. **(1)**

(d) State why a dummy activity may be needed when drawing an activity-on-arc network. **(1)**

(Total for Question 1 is 4 marks)

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2 Amina runs a small business making two types of scented candle: 'Meadow' candles and 'Harbour' candles, each week. Let m be the number of Meadow candles made per week, and h be the number of Harbour candles made per week.

Each Meadow candle uses 200 g of wax and takes 15 minutes of hand-pouring time. Each Harbour candle uses 350 g of wax and takes 10 minutes of hand-pouring time.

Each week, Amina has 21 kg of wax and 20 hours of hand-pouring time available.

Market research shows Amina should make at least twice as many Meadow candles as Harbour candles.

Amina must make at least 10 Harbour candles each week to fulfil a standing order.

The profit is £3.50 per Meadow candle and £4.20 per Harbour candle.

(a) Define the decision variables m and h. **(1)**

(b) Write down the objective function, stating whether it should be maximised or minimised. **(1)**

(c) Write down all the constraints for this linear programming problem, other than the restriction found in part (d). **(4)**

(d) State any further restriction that applies to m and h, and explain why it is necessary. **(1)**

(Total for Question 2 is 7 marks)

3 Discuss the similarities and differences between the objective line (ruler) method and the vertex method for solving a two-variable linear programming problem graphically. Your answer should refer to how each method locates the optimal solution, how each method deals with a problem that has more than one optimal solution, and which method is generally more efficient when the feasible region has many vertices.

(Total for Question 3 is 7 marks)

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4 A project consists of activities A, B, C, D, E, F, G, H, I. Each activity is listed with its duration in days and the activities that must be completed before it can start: A (duration 4, after none); B (duration 5, after A); C (duration 2, after A); D (duration 2, after A); E (duration 4, after C); F (duration 4, after D); G (duration 9, after C); H (duration 8, after B); I (duration 3, after H).

(a) Carry out a forward pass to find the earliest start time of each activity, and state the minimum completion time of the project. **(3)**

(b) Carry out a backward pass and hence find the critical activities. **(3)**

(c) Find the total float of activity C. **(2)**

(d) Find a lower bound for the number of workers needed to complete the project in the minimum time, if each activity needs one worker. **(2)**

(Total for Question 4 is 10 marks)

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5 A factory makes x units of product P and y units of product Q each week. The constraints are $x + 3y \leq 16$, $4x + 3y \leq 37$, $x \geq 0$ and $y \geq 0$. The profit, in hundreds of pounds, is $P = 2x + 3y$, and the profit is to be maximised.

(a) Draw the feasible region and find the coordinates of each of its vertices. **(3)**

(b) Evaluate P at each vertex and hence find the maximum profit and the values of x and y which give it. **(3)**

(Total for Question 5 is 6 marks)

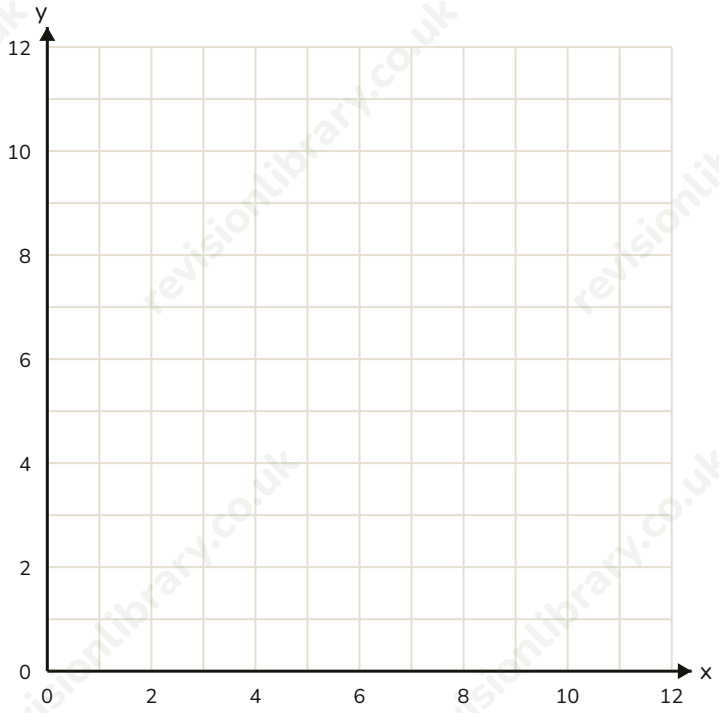
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6 Bexley Outdoor Furniture makes garden benches (x) and garden tables (y) each week. Each bench needs 2 units of timber and 1 hour of labour; each table needs 1 unit of timber and 3 hours of labour. Each week, 20 units of timber and 30 hours of labour are available. The profit is £40 per bench and £60 per table. Bexley wants to maximise weekly profit $P = 40x + 60y$, subject to the constraints:

$2x + y \leq 20$ (timber)

$x + 3y \leq 30$ (labour)

$x \geq 0, y \geq 0$



(a) On the grid provided, draw the lines $2x + y = 20$ and $x + 3y = 30$, and shade the feasible region satisfying all the constraints. **(4)**

(b) Find the coordinates of each vertex of the feasible region. **(5)**

(c) Use the vertex method to determine the number of benches and tables Bexley should make each week to maximise weekly profit, and state the maximum weekly profit. **(3)**

(Total for Question 6 is 12 marks)

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Mark scheme · 4.4 Decision Maths: Critical Path Analysis and Linear Programming: Test Pack

Question 1

(a) an activity with zero total float, so any delay to it delays the completion of the whole project (oe) **B1**

(a) Answer: An activity with zero total float; any delay to it delays the whole project.

(b) the amount of time an activity may be delayed (or extended) without affecting the overall completion time of the project, assuming all other activities occur as originally planned (oe) **B1**

(b) Answer: The amount an activity can be delayed without delaying the whole project.

(c) the amount of time an activity may be delayed without affecting the earliest start time of any following activity (oe) **B1**

(c) Answer: The amount an activity can be delayed without delaying the earliest start of any following activity.

(d) to show a logical dependency between activities that would not otherwise be represented correctly by the network (for example when one activity depends on more than one predecessor, but another activity leaving the same event does not) (oe); accept: to avoid two activities sharing the same start and end events **B1**

(d) Answer: To represent a logical dependency correctly (or to avoid two activities sharing the same pair of events) when this cannot otherwise be shown.

Question 2

(a) m = number of Meadow candles made per week; h = number of Harbour candles made per week **B1**

(a) Answer: m = number of Meadow candles made per week; h = number of Harbour candles made per week.

(b) maximise P = 3.5m + 4.2h oe **B1**

(b) Answer: Maximise P = 3.5m + 4.2h.

(c) 200m + 350h ≤ 21000 oe (e.g. 4m + 7h ≤ 420) [wax] **B1**

(c) 15m + 10h ≤ 1200 oe (e.g. 3m + 2h ≤ 240) [hand-pouring time] **B1**

(c) m ≥ 2h [ratio] **B1**

(c) h ≥ 10 [standing order] **B1**

(c) Answer: 200m + 350h ≤ 21000; 15m + 10h ≤ 1200; m ≥ 2h; h ≥ 10.

(d) m, h ≥ 0 and both integers, since it is not possible to make a negative or fractional number of candles (oe) **B1**

(d) Answer: m, h must be non-negative integers, since candles cannot be made in negative or fractional quantities.

Question 3

Level 0 No relevant comments made, or the response fails to address the question. **0**

Level 1 A basic description of at most one method is given, with little or no valid comparison between the two. **1-2**

Level 2 Both methods are described with reasonable accuracy, and at least one valid comparison point is made (for example relating to efficiency or multiple optima), but the discussion is not fully developed or contains minor inaccuracies. **3-5**

Level 3 Both methods are described accurately and at least three distinct, well-explained comparison points are made (for example locating the optimum, efficiency for many vertices, and recognising multiple optimal solutions), with correct terminology used throughout. **6-7**

Indicative content:

- Both methods require the feasible region to be drawn accurately first, and both are guaranteed to find the same optimal solution, since the optimum of a linear objective function over a polygonal feasible region always occurs at a vertex (or along a whole edge, in the case of multiple optima).
- The vertex method finds the coordinates of every vertex of the feasible region, then evaluates the objective function at each one directly, selecting the vertex giving the best (maximum or minimum) value.
- The objective line method instead draws one sample line of the objective function (for a convenient value), then slides a ruler parallel to this line across the feasible region, in the direction that improves the objective, identifying the last vertex the ruler touches before leaving the feasible region.
- The vertex method can become slow for a feasible region with many vertices, since every vertex's coordinates must be found (often by solving simultaneous equations) and the objective evaluated at each one; the objective line method avoids this, since the ruler can be moved directly to the optimal vertex without evaluating the objective everywhere else.
- The objective line method requires care in drawing the sample objective line accurately and in judging its gradient correctly relative to the constraint lines; a poorly drawn line can lead to the wrong vertex being identified, whereas the vertex method's evaluations are more mechanical and less prone to this type of error.
- If the objective line is parallel to one of the edges of the feasible region, there are infinitely many optimal solutions (every point on that edge, not just a single vertex); this is easy to spot with the objective line method, since the ruler lies flat along the whole edge at the optimal position, whereas with the vertex method it only becomes apparent when two adjacent vertices are found to give exactly equal objective values.
- In practice, the vertex method is often preferred for a feasible region with few vertices (since it is systematic and easy to check), while the objective line method is generally more efficient, and more revealing about the structure of the optimal solution, when the feasible region has many vertices.

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Question 4

(a) earliest start = latest earliest finish of the immediate predecessors **M1**

(a) earliest starts: A 0, B 4, C 4, D 4, E 6, F 6, G 6, H 9, I 17 **A1**

(a) minimum completion time = 20 days **A1**

(a) Answer: 20 days

(b) latest finish = smallest latest start of the successors **M1**

(b) latest finishes: A 4, B 9, C 11, D 16, E 20, F 20, G 20, H 17, I 20 **A1**

(b) critical activities (zero float): A, B, H, I **A1**

(b) Answer: A, B, H, I

(c) total float = latest finish - earliest start - duration **M1**

(c) 11 - 4 - 2 = 5 days **A1**

(c) Answer: 5 days

(d) total duration of all activities = 41 **M1**

(d) 41/20 = 2.05, so at least 3 workers **A1**

(d) Answer: 3

Question 5

(a) plots both constraint lines and shades the unwanted region **M1**

(a) finds the intercepts with the axes and the intersection of the two lines **M1**

(a) vertices: (0, 0), (0, 16/3), (7, 3), (37/4, 0) **A1**

(a) Answer: (0, 0), (0, 16/3), (7, 3), (37/4, 0)

(b) evaluates P at each vertex **M1**

(b) values: 0, 16, 23, 37/2 **A1**

(b) maximum P = 23 at (7, 3) **A1**

(b) Answer: P = 23 at (7, 3)

Question 6

(a) line $2x + y = 20$ correctly drawn through (0,20) and (10,0) **B1**

(a) line $x + 3y = 30$ correctly drawn through (0,10) and (15,5) or (30,0) **B1**

(a) correct feasible region identified, bounded by the two lines and both axes **B1**

(a) region unambiguously indicated (shaded or labelled R), consistent with $x \geq 0$, $y \geq 0$ **B1**

(a) Answer: Feasible region bounded by (0,0), (10,0), (6,8), (0,10).

(b) (0,0) **B1**

(b) (10,0) **B1**

(b) (0,10) **B1**

(b) solving $2x + y = 20$ and $x + 3y = 30$ simultaneously **M1**

(b) (6,8) cao **A1**

(b) Answer: (0,0), (10,0), (6,8), (0,10).

(c) evaluate $P = 40x + 60y$ at each vertex (at least two shown) **M1**

(c) (0,0): P=0; (10,0): P=400; (6,8): P=720; (0,10): P=600, with (6,8) correctly identified as giving the greatest value **A1**

(c) conclusion: 6 benches and 8 tables, maximum profit £720 cao **A1**

(c) Answer: 6 benches and 8 tables; maximum weekly profit £720.

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