4.5 Decision: Algorithms and Graph Theory Depth: 1-hour Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AC = 8, AG = 18, BC = 7, BD = 27, BE = 26, BF = 11, BG = 39, CE = 24, DG = 37.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 1 is 7 marks)

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2 The list 2, 30, 52, 50, 8, 20, 59, 41, 31 is to be sorted into ascending order using a bubble sort. In each pass, adjacent items are compared from the left and swapped if they are in the wrong order.

(a) Show the list after the first pass and state the number of swaps made in the first pass. **(3)**

(b) Show the list after the second pass. **(2)**

(c) State the number of passes needed to sort the list completely, not counting a final pass in which no swaps are made, and the total number of swaps. **(2)**

(Total for Question 2 is 7 marks)

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3 The items with the following sizes are to be packed into bins of capacity 50: 18, 5, 40, 28, 31, 8, 26, 20, 35, 13.

(a) Find a lower bound for the number of bins needed. **(2)**

(b) Use the first-fit algorithm, taking the items in the order given, to pack the items into bins. State the contents of each bin. **(3)**

(c) Use the first-fit decreasing algorithm to pack the items. State the number of bins used and the contents of each bin. **(3)**

(Total for Question 3 is 8 marks)

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4 The list 53, 14, 25, 2, 11, 1, 33, 23, 39 is to be sorted into ascending order using a bubble sort. In each pass, adjacent items are compared from the left and swapped if they are in the wrong order.

(a) Show the list after the first pass and state the number of swaps made in the first pass. **(3)**

(b) Show the list after the second pass. **(2)**

(c) State the number of passes needed to sort the list completely, not counting a final pass in which no swaps are made, and the total number of swaps. **(2)**

(Total for Question 4 is 7 marks)

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5 The graph G has vertices A, B, C, D, E, F and edges AB, AC, AD, AE, BC, BE, BF, CD, CE, CF, EF.

(a) Find the degree of each vertex, and verify the handshaking lemma for G. **(3)**

(b) Classify G as Eulerian, semi-Eulerian or neither, giving a reason. **(2)**

(c) The complete graph K_9 has 9 vertices, each joined to every other. Find its number of edges, and state whether it is Eulerian. **(3)**

(Total for Question 5 is 8 marks)

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6 The items with the following sizes are to be packed into bins of capacity 20: 7, 12, 7, 5, 8, 8, 3, 5, 11, 7.

(a) Find a lower bound for the number of bins needed. **(2)**

(b) Use the first-fit algorithm, taking the items in the order given, to pack the items into bins. State the contents of each bin. **(3)**

(c) Use the first-fit decreasing algorithm to pack the items. State the number of bins used and the contents of each bin. **(3)**

(Total for Question 6 is 8 marks)

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7 A water company is planning new pipeline connections between seven reservoirs, A to G. The table below gives the cost, in £1000s, of laying a direct pipeline between each pair of reservoirs that could feasibly be connected:

AB = 7, AC = 9, BC = 5, BD = 8, CD = 6, DE = 4, EF = 3, EG = 9, FG = 6, CE = 10, BE = 11, DF = 12

(a) Use Kruskal's algorithm to find a minimum spanning tree for this network. List the edges in increasing order of cost, stating whether each is accepted or rejected as you reach it, until every reservoir is connected. **(5)**

(b) State the total cost of the minimum spanning tree found in part (a). **(1)**

(c) The next edge Kruskal's algorithm would consider is BD, of cost 8. Explain why this edge cannot be selected. **(1)**

(Total for Question 7 is 7 marks)

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Mark scheme · 4.5 Decision: Algorithms and Graph Theory Depth: 1-hour Pack

Question 1

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: BC (7), AC (8), BF (11), AG (18), CE (24), BD (27) **A1**

(a) Answer: BC (7), AC (8), BF (11), AG (18), CE (24), BD (27)

(b) 95 **B1**

(b) Answer: 95

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AC (8), BC (7), BF (11), AG (18), CE (24), BD (27) **A1**

(c) Answer: AC (8), BC (7), BF (11), AG (18), CE (24), BD (27)

Question 2

(a) compares adjacent pairs from the left, swapping when the left item is larger **M1**

(a) list after pass 1: 2, 30, 50, 8, 20, 52, 41, 31, 59 **A1**

(a) 5 swaps **B1**

(a) Answer: 2, 30, 50, 8, 20, 52, 41, 31, 59; 5 swaps

(b) repeats the process, ignoring the final item (already in place) **M1**

(b) 2, 30, 8, 20, 50, 41, 31, 52, 59 **A1**

(b) Answer: 2, 30, 8, 20, 50, 41, 31, 52, 59

(c) 4 passes **B1**

(c) 14 swaps in total **B1**

(c) Answer: 4 passes, 14 swaps

Question 3

(a) total size = 224 **M1**

(a) 224/50 = 4.48, so at least 5 bins **A1**

(a) Answer: 5

(b) places each item in the first bin with enough room, in the order given **M1**

(b) first bins correct **A1**

(b) bins: (18, 5, 8, 13); (40); (28, 20); (31); (26); (35) **A1**

(b) Answer: (18, 5, 8, 13); (40); (28, 20); (31); (26); (35)

(c) sorts the items into descending order **M1**

(c) applies first-fit to the sorted list **M1**

(c) 5 bins: (40, 8); (35, 13); (31, 18); (28, 20); (26, 5) **A1**

(c) Answer: 5 bins: (40, 8); (35, 13); (31, 18); (28, 20); (26, 5)

Question 4

(a) compares adjacent pairs from the left, swapping when the left item is larger **M1**

(a) list after pass 1: 14, 25, 2, 11, 1, 33, 23, 39, 53 **A1**

(a) 8 swaps **B1**

(a) Answer: 14, 25, 2, 11, 1, 33, 23, 39, 53; 8 swaps

(b) repeats the process, ignoring the final item (already in place) **M1**

(b) 14, 2, 11, 1, 25, 23, 33, 39, 53 **A1**

(b) Answer: 14, 2, 11, 1, 25, 23, 33, 39, 53

(c) 5 passes **B1**

(c) 18 swaps in total **B1**

(c) Answer: 5 passes, 18 swaps

Question 5

(a) counts the edges at each vertex **M1**

(a) degrees: A 4, B 4, C 5, D 2, E 4, F 3 **A1**

(a) sum of degrees = 22 = 2 x 11 edges **A1**

(a) Answer: A 4, B 4, C 5, D 2, E 4, F 3; sum 22

(b) counts the vertices of odd degree: C, F **M1**

(b) semi-Eulerian (has an Eulerian trail but no cycle) **A1**

(b) Answer: semi-Eulerian (has an Eulerian trail but no cycle)

(c) 9(9 - 1)/2 **M1**

(c) 36 edges **A1**

(c) each vertex has degree 8, which is even, so K_9 is Eulerian **B1**

(c) Answer: 36 edges; Eulerian

Question 6

(a) total size = 73 **M1**

(a) 73/20 = 3.65, so at least 4 bins **A1**

(a) Answer: 4

(b) places each item in the first bin with enough room, in the order given **M1**

(b) first bins correct **A1**

(b) bins: (7, 12); (7, 5, 8); (8, 3, 5); (11, 7) **A1**

(b) Answer: (7, 12); (7, 5, 8); (8, 3, 5); (11, 7)

(c) sorts the items into descending order **M1**

(c) applies first-fit to the sorted list **M1**

(c) 4 bins: (12, 8); (11, 8); (7, 7, 5); (7, 5, 3) **A1**

(c) Answer: 4 bins: (12, 8); (11, 8); (7, 7, 5); (7, 5, 3)

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Question 7

(a) attempts edges in increasing order of cost: EF(3), DE(4), BC(5), CD(6), FG(6), AB(7), ... **M1**

(a) accepts EF(3), DE(4), BC(5), each correctly joining a new component with no cycle **A1**

(a) accepts CD(6) and FG(6) (either order, since tied), each correctly joining a new component **A1**

(a) accepts AB(7), correctly completing the spanning tree **A1**

(a) correctly states the tree is complete once 6 edges have been chosen, since a spanning tree of 7 reservoirs requires exactly 6 edges **B1**

(a) Answer: Accepted, in order: EF(3), DE(4), BC(5), CD(6), FG(6), AB(7). Tree complete with 6 edges.

(b) 31 (i.e. £31,000) cao **B1**

(b) Answer: 31 (£31,000).

(c) B and D are already connected via the edges already chosen (the path B-C-D), so adding BD would form a cycle, which Kruskal's algorithm must avoid (oe) **B1**

(c) Answer: B and D are already connected (via B-C-D using edges already selected), so BD would create a cycle and must be rejected.

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