4.5 Decision: Algorithms and Graph Theory Depth: 30-minute Pack

Total Marks

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Kwame runs a small courier company. One morning he must pack parcels of the following weights, in kg, into delivery bins that can each hold a maximum of 30 kg:
13, 8, 19, 11, 6, 14, 9, 17

(a) Calculate a lower bound for the number of bins needed to pack all eight parcels. **(2)**

(b) Apply the first-fit algorithm to the parcels, in the order given, packing them into bins of capacity 30 kg. State the contents of each bin, and the number of bins used. **(3)**

(c) Sort the parcel weights into decreasing order and apply first-fit decreasing to the same set of parcels, packing into bins of capacity 30 kg. State the contents of each bin, and the number of bins used. **(3)**

(Total for Question 1 is 8 marks)

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2 Aaliyah is organising a charity obstacle-course event with five checkpoints, P, Q, R, S, T, linked by permanently open trail paths. The distances between checkpoints connected by a trail, in metres, are: PQ = 120, PR = 200, PS = 340, QR = 90, QT = 260, RS = 150, RT = 210, ST = 180 (there is no direct trail between any other pair of checkpoints).

(a) The organisers need to run a single length of event cable connecting all five checkpoints, using the least total length of cable. Use Kruskal's algorithm to find this minimum length, showing the edges you accept and at least one edge you reject, with a reason. **(4)**

(b) A safety marshal, Owen, must travel from the checkpoint at P to the checkpoint at T, using the original trail network (not necessarily the minimum cable route found in part (a)). Use Dijkstra's algorithm, starting at P, to find the length of the shortest route from P to T. Show the order in which each checkpoint receives its final label, and state the shortest route. **(5)**

(c) Crates of bottled water, of mass 14, 22, 9, 18, 27, 11 and 16 kg, must be loaded into vans, each with a capacity of 45 kg. Calculate a lower bound for the number of vans required, then apply first-fit decreasing to pack the crates. State the contents of each van, and state whether the lower bound has been achieved. **(4)**

(Total for Question 2 is 13 marks)

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3 A network has vertices A, B, C, D, E, F, G and the following weighted arcs: AD = 18, AE = 25, BC = 33, BD = 15, BF = 32, CG = 5, DE = 6, DF = 19, EF = 24, EG = 28.

(a) Use Kruskal's algorithm to find a minimum spanning tree. List the arcs in the order in which you add them to the tree. **(3)**

(b) State the total weight of your minimum spanning tree. **(1)**

(c) Use Prim's algorithm starting at vertex A to find the minimum spanning tree, listing the arcs in the order added. **(3)**

(Total for Question 3 is 7 marks)

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Mark scheme · 4.5 Decision: Algorithms and Graph Theory Depth: 30-minute Pack

Question 1

(a) total weight = 13+8+19+11+6+14+9+17 = 97 kg **M1**

(a) lower bound = ceiling(97/30) = 4 cao **A1**

(a) Answer: Lower bound = 4 bins.

(b) correct method: each parcel placed in the first bin (in order) with enough remaining space, opening a new bin only when none of the existing bins can fit it **M1**

(b) correct bin contents: Bin 1 {13, 8, 6}, Bin 2 {19, 11}, Bin 3 {14, 9}, Bin 4 {17} **A1**

(b) 4 bins used cao **B1**

(b) Answer: Bin 1: 13, 8, 6 (27 kg). Bin 2: 19, 11 (30 kg). Bin 3: 14, 9 (23 kg). Bin 4: 17 (17 kg). 4 bins used.

(c) correct decreasing order: 19, 17, 14, 13, 11, 9, 8, 6 **M1**

(c) correct bin contents: Bin 1 {19, 11}, Bin 2 {17, 13}, Bin 3 {14, 9, 6}, Bin 4 {8} **A1**

(c) 4 bins used cao, matching the lower bound found in part (a), so this packing is optimal **B1**

(c) Answer: Bin 1: 19, 11 (30 kg). Bin 2: 17, 13 (30 kg). Bin 3: 14, 9, 6 (29 kg). Bin 4: 8 (8 kg). 4 bins used, matching the lower bound.

Question 2

(a) attempts edges in increasing order of distance: QR(90), PQ(120), RS(150), ST(180), PR(200), ... **M1**

(a) correctly accepts QR(90), PQ(120), RS(150), ST(180), connecting all five checkpoints with 4 edges and no cycle **A1**

(a) total minimum length = 90+120+150+180 = 540 (m) cao **A1**

(a) correctly rejects an edge that would form a cycle, e.g. PR(200), since P and R are already connected via P-Q-R using the edges already accepted (oe) **B1**

(a) Answer: Minimum cable length = 540 m, using edges QR, PQ, RS, ST; PR (200) is rejected as it would form a cycle.

(b) P permanently labelled 0; initial working values Q = 120 (via P), R = 200 (via P), S = 340 (via P) **M1**

(b) Q permanently labelled second, value 120 **A1**

(b) R permanently labelled third, value 200 (via P); notes Q's route to R, 120+90=210, is not an improvement **A1**

(b) S permanently labelled fourth, value 340 (via P); notes R's routes to S (200+150=350) and T (200+210=410) are not improvements over the best values already known **A1**

(b) T permanently labelled fifth (last), value 380 (via Q, since 120+260=380); route P-Q-T stated **A1**

(b) Answer: Order of permanent labelling: P(0), Q(120), R(200), S(340), T(380). Shortest route P-Q-T, length 380 m.

(c) total mass = 14+22+9+18+27+11+16 = 117 kg, lower bound = ceiling(117/45) = 3 **B1**

(c) correct decreasing order: 27, 22, 18, 16, 14, 11, 9 **M1**

(c) correct van contents: Van 1 {27, 18}, Van 2 {22, 16}, Van 3 {14, 11, 9} **A1**

(c) 3 vans used, matching the lower bound, so this packing is optimal cao **B1**

(c) Answer: Lower bound = 3 vans. Van 1: 27, 18 (45 kg). Van 2: 22, 16 (38 kg). Van 3: 14, 11, 9 (34 kg). 3 vans used, matching the lower bound.

Question 3

(a) sorts the arcs in increasing order of weight and selects the smallest **M1**

(a) adds each next smallest arc that does not form a cycle **M1**

(a) arcs selected: CG (5), DE (6), BD (15), AD (18), DF (19), EG (28) **A1**

(a) Answer: CG (5), DE (6), BD (15), AD (18), DF (19), EG (28)

(b) 91 **B1**

(b) Answer: 91

(c) starts at A and adds the shortest arc joining a tree vertex to a non-tree vertex each time **M1**

(c) first three arcs correct **A1**

(c) arcs added: AD (18), DE (6), BD (15), DF (19), EG (28), CG (5) **A1**

(c) Answer: AD (18), DE (6), BD (15), DF (19), EG (28), CG (5)

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