3.1 Further Mechanics: Momentum, Impulse and Collisions

Total Marks

AQA 7367 · Calculator allowed · about 230 minutes · Write-in edition

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 During a hockey training drill, Priya strikes a hockey ball of mass 0.16 kg that is travelling towards her at 12 m/s. The impact reverses the ball's direction of travel and sends it away from her at 18 m/s. The stick is in contact with the ball for 0.025 seconds.

(a) Calculate the magnitude of the impulse exerted on the ball by the stick. **(3)**

(b) Calculate the magnitude of the average force exerted on the ball by the stick during the impact. **(2)**

(Total for Question 1 is 5 marks)

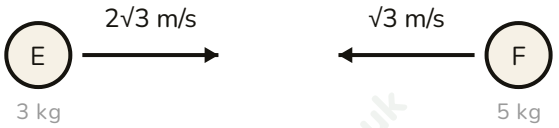
2 This question tests recall of the key definitions used in collision problems.

(a) State Newton's experimental law of restitution for the direct impact of two smooth spheres, defining any symbols used. **(2)**

(b) State the range of possible values of e, and describe the physical significance of the values e = 0 and e = 1. **(2)**

(Total for Question 2 is 4 marks)

3 Two smooth spheres, E and F, of masses 3 kg and 5 kg respectively, move towards each other on a smooth horizontal plane and collide directly. Before the collision, E has speed $2\sqrt{3}$ m/s and F has speed $\sqrt{3}$ m/s, moving in opposite directions. The coefficient of restitution between E and F is e = 1/3.



(a) Taking E's initial direction of motion as positive, find the exact velocities of E and F immediately after the collision, giving your answers in the form k $\sqrt{3}$, where k is a rational number. **(6)**

(b) Find, in exact form, the loss in kinetic energy due to the collision. **(2)**

(Total for Question 3 is 8 marks)

4 Three smooth spheres, X, Y and Z, of masses 1 kg, 2 kg and 3 kg respectively, lie at rest in a straight line on a smooth horizontal table, with Y between X and Z. Sphere X is projected with speed 10 m/s towards Y, which is initially at rest. The coefficient of restitution between X and Y is 0.5.



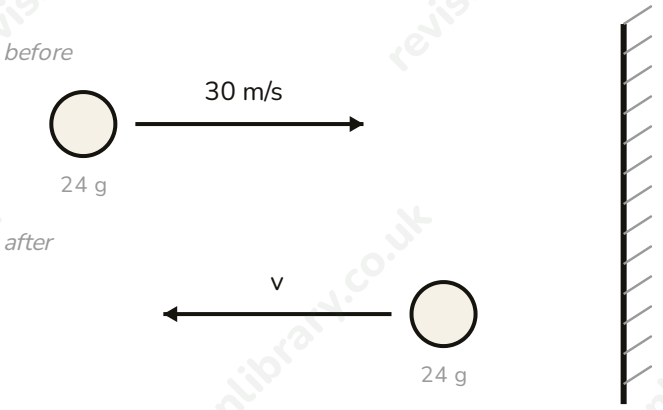
(a) Find the velocities of X and Y immediately after their collision. **(5)**

(b) Sphere Y then goes on to collide directly with sphere Z, which is also initially at rest. The coefficient of restitution between Y and Z is 0.8. Find the velocities of Y and Z immediately after this second collision. **(5)**

(c) Explain why sphere Y must go on to collide with sphere X a second time. **(3)**

(Total for Question 4 is 13 marks)

5 A squash ball of mass 24 grams is served horizontally at a smooth vertical wall with speed 30 m/s. It rebounds and returns along the same horizontal line. The coefficient of restitution between the ball and the wall is e = 0.75.



(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Calculate the impulse exerted by the wall on the ball, stating an appropriate direction. **(3)**

(c) Calculate the percentage of the ball's kinetic energy that is lost in the impact, giving your answer to 3 significant figures. **(2)**

(Total for Question 5 is 7 marks)

6 A small ball of mass 1.5 kg hits a smooth fixed vertical wall at right angles with speed 6 m/s. The coefficient of restitution between the ball and the wall is 3/4.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

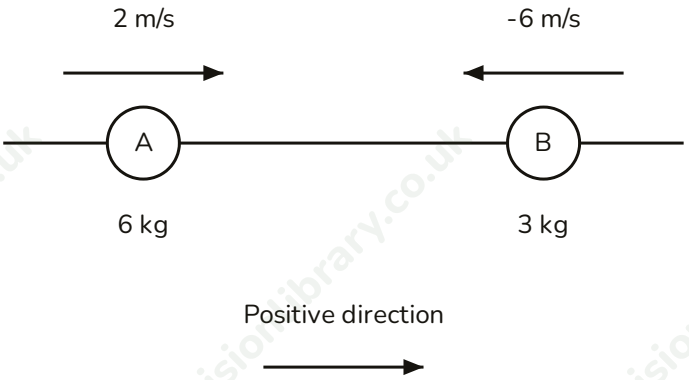
(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 1 m/s. **(4)**

(Total for Question 6 is 10 marks)

7 Two smooth spheres A and B, of masses 6 kg and 3 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 2 m/s and B has velocity -6 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 1/4.

Diagram NOT accurately drawn



(a) Find the velocities of A and B immediately after the collision. **(4)**

(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 7 is 9 marks)

8 A small ball of mass 0.2 kg hits a smooth fixed vertical wall at right angles with speed 9 m/s. The coefficient of restitution between the ball and the wall is 1/2.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 1 m/s. **(4)**

(Total for Question 8 is 10 marks)

9 A small ball of mass 0.2 kg hits a smooth fixed vertical wall at right angles with speed 20 m/s. The coefficient of restitution between the ball and the wall is 1/3.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

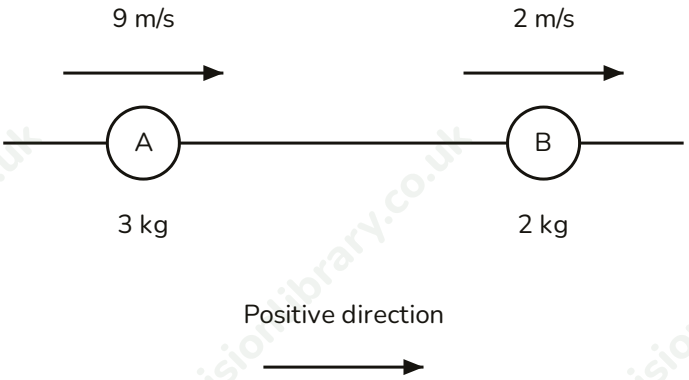
(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 2 m/s. **(4)**

(Total for Question 9 is 10 marks)

10 Two smooth spheres A and B, of masses 3 kg and 2 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 9 m/s and B has velocity 2 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 1/4.

Diagram NOT accurately drawn



(a) Find the velocities of A and B immediately after the collision. **(4)**

(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 10 is 9 marks)

11 A particle P of mass 3 kg is moving with speed 7 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 7 kg, which is moving along the same line with velocity 4 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.25 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 11 is 10 marks)

12 A particle P of mass 8 kg is moving with speed 2 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 1 kg, which is moving along the same line with velocity 1 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.5 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 12 is 10 marks)

13 A particle P of mass 5 kg is moving with speed 10 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 4 kg, which is moving along the same line with velocity 2 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

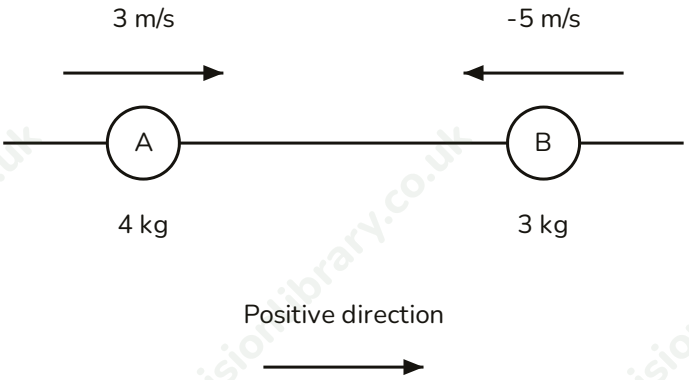
(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.1 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 13 is 10 marks)

14 Two smooth spheres A and B, of masses 4 kg and 3 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 3 m/s and B has velocity -5 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 2/3.

Diagram NOT accurately drawn



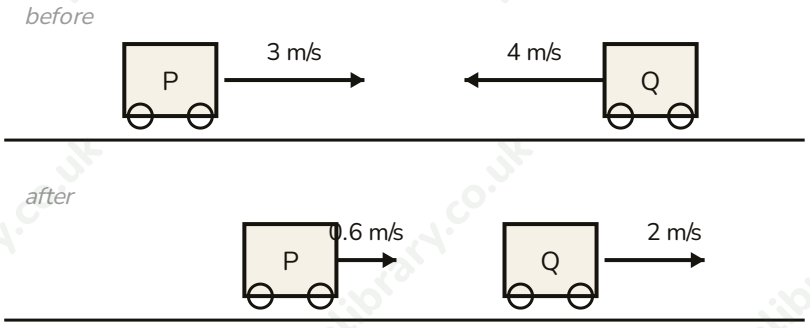
(a) Find the velocities of A and B immediately after the collision. **(4)**

(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 14 is 9 marks)

15 Two trolleys, P of mass 5 kg and Q of mass 2 kg, are moving towards each other on a smooth horizontal track and collide directly. Immediately before the collision, P has speed 3 m/s and Q has speed 4 m/s, and they are moving in opposite directions. Immediately after the collision, P has velocity 0.6 m/s and Q has velocity 2 m/s, both now in the direction of P's original motion.



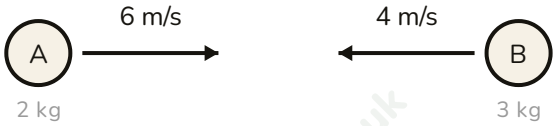
(a) By calculating the total momentum immediately before and immediately after the collision, show that momentum is conserved in this collision. **(3)**

(b) Calculate the coefficient of restitution between P and Q for this collision. **(3)**

(c) State, with a reason, whether this collision could be perfectly elastic. **(1)**

(Total for Question 15 is 7 marks)

16 Two smooth spheres, A and B, of mass 2 kg and 3 kg respectively, are free to move along the same straight line on a smooth horizontal table. Sphere A is projected towards B with speed 6 m/s, and at the same instant B is projected towards A with speed 4 m/s, so that the spheres collide directly. The coefficient of restitution between A and B is 0.5.



(a) Taking the direction of A's initial motion as positive, write down an equation for conservation of momentum for this collision. **(2)**

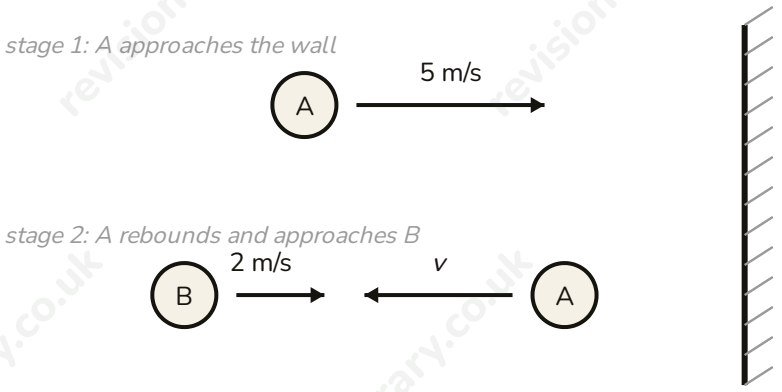
(b) Write down Newton's law of restitution (the restitution equation) for this collision. **(2)**

(c) Hence find the velocities of A and B immediately after the collision, stating clearly the direction in which each sphere moves. **(5)**

(d) Show that the collision results in a loss of kinetic energy, and find this loss. **(3)**

(Total for Question 16 is 12 marks)

17 A smooth sphere A of mass 4 kg is moving with speed 5 m/s directly towards a smooth vertical wall. The coefficient of restitution between A and the wall is 0.4.



(a) Find the speed of A immediately after it rebounds from the wall. **(2)**

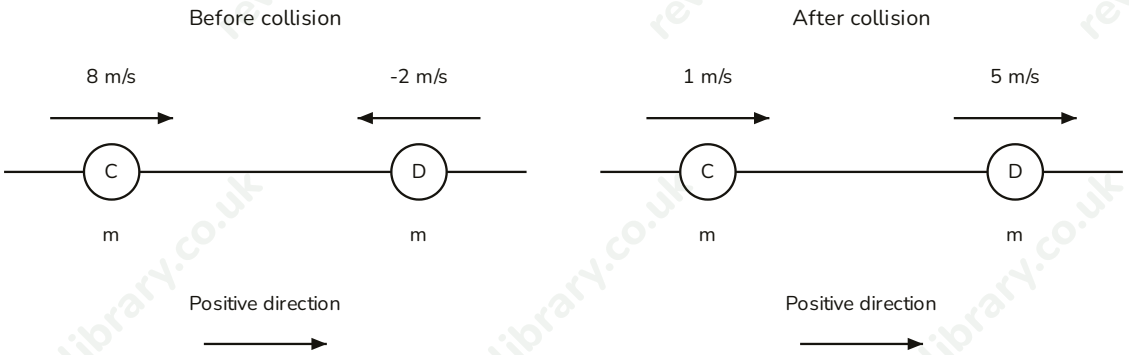
(b) After rebounding, A moves away from the wall and collides directly with a second smooth sphere, B, of mass 6 kg, which is moving towards the wall (that is, towards A) with speed 2 m/s. The coefficient of restitution between A and B is 0.75. Taking the direction away from the wall as positive, find the velocities of A and B immediately after this second collision. **(6)**

(c) Determine whether A and B collide again, giving a full reason for your answer. **(3)**

(Total for Question 17 is 11 marks)

18 Two small spheres, C and D, of equal mass m, move towards each other along the same straight line and collide directly. Immediately before the collision, C has velocity 8 m/s and D has velocity -2 m/s. Immediately after the collision, C has velocity 1 m/s and D has velocity 5 m/s.

Diagram NOT accurately drawn

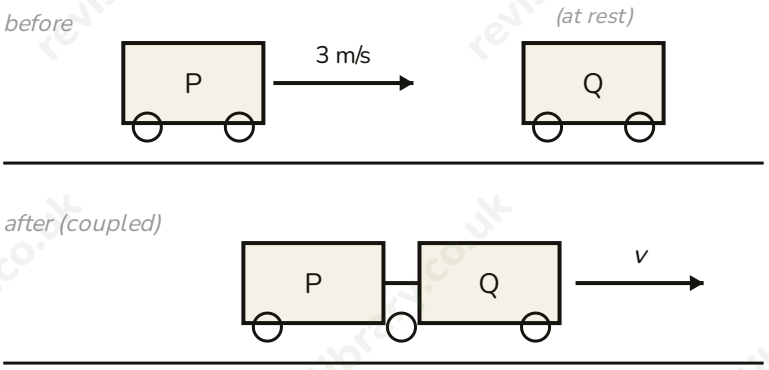


(a) Verify that this is consistent with conservation of momentum, and find the coefficient of restitution e for this collision. **(4)**

(b) Given that m = 0.8 kg, calculate the loss in kinetic energy due to the collision. **(2)**

(Total for Question 18 is 6 marks)

19 Two railway trucks, P of mass 2400 kg travelling at 3 m/s, and Q of mass 3600 kg initially at rest, collide and couple together so that they move with a common velocity immediately after the collision (that is, e = 0 for this impact).



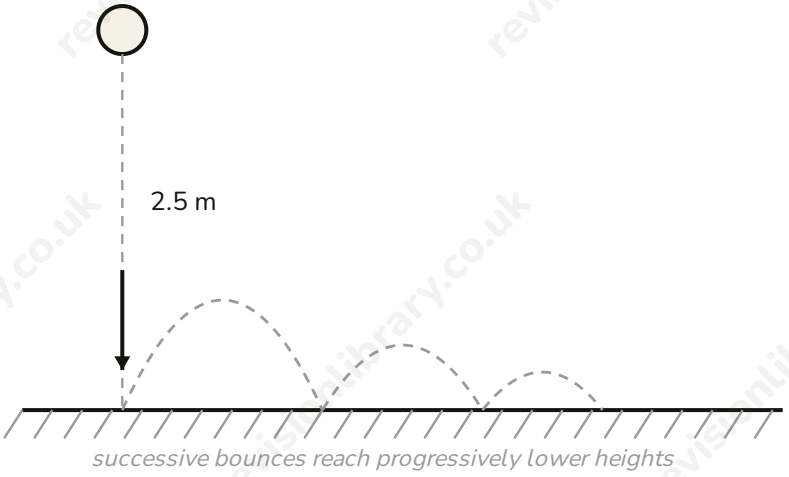
(a) Find the common velocity of the trucks immediately after the collision. **(3)**

(b) Calculate the loss in kinetic energy due to the collision. **(3)**

(c) Let m1 be the mass of a moving particle that collides directly with, and coalesces with, a second particle of mass m2 which is initially at rest (e = 0). Show that the fraction of the initial kinetic energy lost in the collision is m2 / (m1 + m2), and verify this general result using your answers to parts (a) and (b). **(3)**

(Total for Question 19 is 9 marks)

20 A ball is dropped from rest from a height of 2.5 m onto smooth, horizontal ground. The coefficient of restitution between the ball and the ground is $e = 0.6$. Air resistance may be ignored, and $g = 9.8\ m/s^2$ throughout this question.



(a) Find the speed of the ball immediately before it first strikes the ground. **(2)**

(b) Find the height to which the ball rebounds after the first bounce. **(3)**

(c) Show that the height reached after the nth bounce is $h_n = 2.5\ e^{2n}$, and hence find the height reached after the third bounce, giving your answer to 3 significant figures. **(4)**

(Total for Question 20 is 9 marks)

Mark scheme · 3.1 Further Mechanics: Momentum, Impulse and Collisions

Question 1

(a) uses impulse = m(v - u), taking the ball's initial direction of travel as positive so u = 12 and v = -18 (oe) **M1**

(a) correct unsimplified expression, e.g. 0.16 x (-18 - 12) **A1**

(a) magnitude of impulse = 4.8 Ns, cao **A1**

(a) Answer: 4.8 Ns

(b) uses force = impulse / time, F = 4.8 / 0.025 (ft on their impulse from part (a)) **M1**

(b) F = 192 N, cao **A1**

(b) Answer: 192 N

Question 2

(a) correct statement, e.g. v2 - v1 = -e(u2 - u1) (oe), or 'speed of separation = e x speed of approach' **B1**

(a) e correctly defined as the coefficient of restitution: a constant for the pair of colliding surfaces, independent of the masses and speeds involved **B1**

(a) Answer: v2 - v1 = -e(u2 - u1), where u1, u2 are the velocities before impact, v1, v2 the velocities after impact, and e is the coefficient of restitution between the two surfaces

(b) $0 \le e \le 1$ **B1**

(b) e = 0 describes a perfectly inelastic collision (the spheres coalesce and move with a common velocity, the maximum possible kinetic energy is lost); e = 1 describes a perfectly elastic collision (no kinetic energy is lost) **B1**

(b) Answer: $0 \le e \le 1$; e = 0 is perfectly inelastic (spheres coalesce), e = 1 is perfectly elastic (no KE lost)

Question 3

(a) momentum equation: $3(2\sqrt{3}) + 5(-\sqrt{3}) = 3v_E + 5v_F$ **M1**

(a) simplified to $3v_E + 5v_F = \sqrt{3}$ **A1**

(a) restitution equation: $v_F - v_E = -(1/3)(-\sqrt{3} - 2\sqrt{3})$ **M1**

(a) simplified to $v_F - v_E = \sqrt{3}$ **A1**

(a) solves the two equations simultaneously (e.g. substitutes $v_F = v_E + \sqrt{3}$) **dM1**

(a) $v_E = -(1/2)\sqrt{3}$ m/s and $v_F = (1/2)\sqrt{3}$ m/s, both exact and correct (oe equivalent surd forms), cao **A1**

(a) Answer: $v_E = -(1/2)\sqrt{3}$ m/s (i.e. $(\sqrt{3}/2)$ m/s reversed); $v_F = (1/2)\sqrt{3}$ m/s

(b) KE before = $0.5(3)(2\sqrt{3})^2 + 0.5(5)(\sqrt{3})^2$; KE after = $0.5(3)(\sqrt{3}/2)^2 + 0.5(5)(\sqrt{3}/2)^2$, using their exact velocities from (a) (ft) **M1**

(b) loss = 25.5 - 3 = 22.5 J, cao (exact) **A1**

(b) Answer: 22.5 J

Question 4

(a) momentum equation: $1(10) + 2(0) = 1v_X + 2v_Y$ **M1**

(a) simplified to $v_X + 2v_Y = 10$ **A1**

(a) restitution equation: $v_Y - v_X = -0.5(0 - 10)$ **M1**

(a) simplified to $v_Y - v_X = 5$ **A1**

(a) solves to $v_X = 0$ and $v_Y = 5$ m/s, both correct, cao **A1**

(a) Answer: $v_X = 0$ (X is brought to rest); $v_Y = 5$ m/s (in X's original direction)

(b) momentum equation using Y's velocity from part (a): $2(5) + 3(0) = 2v_Y' + 3v_Z$ (ft) **M1**

(b) simplified to $2v_Y' + 3v_Z = 10$ **A1**

(b) restitution equation: $v_Z - v_Y' = -0.8(0 - 5)$ **M1**

(b) simplified to $v_Z - v_Y' = 4$ **A1**

(b) solves to $v_Y' = -0.4$ m/s and $v_Z = 3.6$ m/s, both correct, cao **A1**

(b) Answer: $v_Y' = -0.4$ m/s (Y rebounds, moving back towards X); $v_Z = 3.6$ m/s

(c) states $v_X = 0$ from part (a), so with no external horizontal force acting on it, X remains at rest at its post-collision position indefinitely (unless struck again) **B1**

(c) states $v_Y' = -0.4$ m/s from part (b), i.e. Y is moving back in the direction of X after its collision with Z **B1**

(c) concludes that since Y moves directly towards the stationary X along the same line, with nothing between them, a second X-Y collision is inevitable **B1**

(c) Answer: Yes: X remains at rest ($v_X = 0$) while Y rebounds off Z at 0.4 m/s back towards X, so Y must eventually collide with X again

Question 5

(a) speed after = e x speed before = 0.75 x 30 **M1**

(a) 22.5 m/s, cao **A1**

(a) Answer: 22.5 m/s

(b) impulse = m(v-u), taking the ball's original direction as positive so u=30, v=-22.5 (ft from part (a)) **M1**

(b) 0.024 x (-22.5-30) = -1.26, magnitude 1.26 Ns **A1**

(b) states the direction correctly: away from the wall (opposite to the ball's original direction of travel) **A1**

(b) Answer: 1.26 Ns, directed away from the wall

(c) KE before = $0.5(0.024)(30^2)$; KE after = $0.5(0.024)(22.5^2)$ (or uses the shortcut fraction lost = $1 - e^2$) **M1**

(c) 43.8% (awrt 3 s.f.), cao **A1**

(c) Answer: 43.8% (3 s.f.)

Question 6

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 3/4 x 6 = 4.500 m/s **A1**

(a) Answer: 4.500 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 1.5(6 + 4.500) = 15.8 N s **A1**

(b) Answer: 15.8 N s

(c) KE before - KE after **M1**

(c) $(1/2)(1.5)(6^2)(1 - 9/16) = 11.8$ J **A1**

(c) Answer: 11.8 J

(d) forms $6 \times (3/4)^n < 1$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) $n > 6.228$ **A1**

(d) $n = 7$ **A1**

(d) Answer: 7

Question 7

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) **M1**

(a) v1 = -4/3 m/s **A1**

(a) v2 = 2/3 m/s **A1**

(a) Answer: v_A = -4/3 m/s, v_B = 2/3 m/s

(b) impulse = change in momentum of B **M1**

(b) 3(2/3 - (-6)) = 20 N s **A1**

(b) Answer: 20 N s

(c) KE before - KE after **M1**

(c) before 66 J, after 6 J **A1**

(c) loss = 60 J **A1**

(c) Answer: 60 J

Question 8

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 1/2 x 9 = 4.500 m/s **A1**

(a) Answer: 4.500 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.2(9 + 4.500) = 2.70 N s **A1**

(b) Answer: 2.70 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.2)(9^2)(1 - 1/4) = 6.075 = 6.08$ J **A1**

(c) Answer: 6.08 J

(d) forms $9 \times (1/2)^n < 1$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) $n > 3.170$ **A1**

(d) $n = 4$ **A1**

(d) Answer: 4

Question 9

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 1/3 x 20 = 6.667 m/s **A1**

(a) Answer: 6.667 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.2(20 + 6.667) = 5.33 N s **A1**

(b) Answer: 5.33 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.2)(20^2)(1 - 1/9) = 35.6$ J **A1**

(c) Answer: 35.6 J

(d) forms $20 \times (1/3)^n < 2$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) $n > 2.096$ **A1**

(d) $n = 3$ **A1**

(d) Answer: 3

Question 10

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) **M1**

(a) v1 = 11/2 m/s **A1**

(a) v2 = 29/4 m/s **A1**

(a) Answer: v_A = 11/2 m/s, v_B = 29/4 m/s

(b) impulse = change in momentum of B **M1**

(b) 2(29/4 - (2)) = 21/2 N s **A1**

(b) Answer: 21/2 N s

(c) KE before - KE after **M1**

(c) before 251/2 J, after 1567/16 J **A1**

(c) loss = 441/16 J **A1**

(c) Answer: 441/16 J

Question 11

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 21 + (28) = 10v **A1**

(a) v = 49/10 m/s **A1**

(a) Answer: 49/10 m/s

(b) change of momentum of P **M1**

(b) 3(49/10 - 7) = -63/10 N s **A1**

(b) Answer: 63/10 N s

(c) KE before - KE after **M1**

(c) 259/2 - 2401/20 **A1**

(c) 189/20 J **A1**

(c) Answer: 189/20 J

(d) force = impulse/time **M1**

(d) 25.2 N **A1**

(d) Answer: 25.2 N

Question 12

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v — **M1**

(a) 16 + (1) = 9v — **A1**

(a) v = 17/9 m/s — **A1**

(a) Answer: 17/9 m/s

(b) change of momentum of P — **M1**

(b) 8(17/9 - 2) = -8/9 N s — **A1**

(b) Answer: 8/9 N s

(c) KE before - KE after — **M1**

(c) 33/2 - 289/18 — **A1**

(c) 4/9 J — **A1**

(c) Answer: 4/9 J

(d) force = impulse/time — **M1**

(d) 1.78 N — **A1**

(d) Answer: 1.78 N

Question 13

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v — **M1**

(a) 50 + (8) = 9v — **A1**

(a) v = 58/9 m/s — **A1**

(a) Answer: 58/9 m/s

(b) change of momentum of P — **M1**

(b) 5(58/9 - 10) = -160/9 N s — **A1**

(b) Answer: 160/9 N s

(c) KE before - KE after — **M1**

(c) 258 - 1682/9 — **A1**

(c) 640/9 J — **A1**

(c) Answer: 640/9 J

(d) force = impulse/time — **M1**

(d) 178 N — **A1**

(d) Answer: 178 N

Question 14

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 — **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) — **M1**

(a) v1 = -19/7 m/s — **A1**

(a) v2 = 55/21 m/s — **A1**

(a) Answer: v_A = -19/7 m/s, v_B = 55/21 m/s

(b) impulse = change in momentum of B — **M1**

(b) 3(55/21 - (-5)) = 160/7 N s — **A1**

(b) Answer: 160/7 N s

(c) KE before - KE after — **M1**

(c) before 111/2 J, after 1051/42 J — **A1**

(c) loss = 640/21 J — **A1**

(c) Answer: 640/21 J

Question 15

(a) momentum before, taking P's original direction as positive: 5(3) + 2(-4) — **M1**

(a) momentum before = 7 kg m/s — **A1**

(a) momentum after = 5(0.6) + 2(2) = 7 kg m/s, equal to the value before, so momentum is conserved (cso) — **B1**

(a) Answer: Momentum before = momentum after = 7 kg m/s, so momentum is conserved

(b) uses Newton's law of restitution, e = -(v_Q - v_P) / (u_Q - u_P) — **M1**

(b) correct unsimplified substitution, e = -(2 - 0.6) / (-4 - 3) — **A1**

(b) e = 0.2, cao — **A1**

(b) Answer: e = 0.2

(c) no, since e = 0.2 is not equal to 1 (a perfectly elastic collision requires e = 1, oe stating kinetic energy is not conserved) — **B1**

(c) Answer: No, since e = 0.2 does not equal 1

Question 16

(a) 2(6) + 3(-4) = 2v_A + 3v_B (oe, correct masses and signed velocities substituted) — **M1**

(a) simplified to 2v_A + 3v_B = 0 — **A1**

(a) Answer: 2v_A + 3v_B = 0

(b) v_B - v_A = -e(u_B - u_A) = -0.5(-4 - 6) (oe) — **M1**

(b) simplified to v_B - v_A = 5 — **A1**

(b) Answer: v_B - v_A = 5

(c) rearranges one equation, e.g. v_A = -1.5v_B from part (a) — **M1**

(c) substitutes into the equation from part (b) to form one equation in one unknown, v_B - (-1.5v_B) = 5 — **dM1**

(c) v_B = 2 m/s, cao — **A1**

(c) v_A = -3 m/s, cao (ft from their v_B if the method is correct) — **A1**

(c) correctly interprets the signs: A rebounds, moving at 3 m/s in the direction opposite to its initial motion; B continues moving at 2 m/s in A's original direction — **B1**

(c) Answer: v_A = -3 m/s (i.e. 3 m/s reversed); v_B = 2 m/s (continuing in A's original direction)

(d) KE before = 0.5(2)(6^2) + 0.5(3)(4^2) — **M1**

(d) KE after = 0.5(2)(3^2) + 0.5(3)(2^2), using their velocities from part (c) (ft) — **M1**

(d) loss = 60 - 15 = 45.0 J, cao — **A1**

(d) Answer: Kinetic energy loss = 45.0 J

Question 17

(a) speed after = e x speed before = 0.4 x 5 **M1**

(a) 2 m/s (moving away from the wall), cao **A1**

(a) Answer: 2 m/s, away from the wall

(b) momentum equation: $4(2) + 6(-2) = 4v_A + 6v_B$ **M1**

(b) simplified to $2v_A + 3v_B = -2$ (oe, e.g. $4v_A+6v_B=-4$) **A1**

(b) restitution equation: $v_B - v_A = -0.75(-2 - 2)$ **M1**

(b) simplified to $v_B - v_A = 3$ **A1**

(b) solves the two equations simultaneously **dM1**

(b) $v_A = -2.2$ m/s and $v_B = 0.8$ m/s, both correct, cao **A1**

(b) Answer: $v_A = -2.2$ m/s (2.2 m/s back towards the wall); $v_B = 0.8$ m/s (away from the wall)

(c) recognises A is now moving back towards the wall at 2.2 m/s while B moves away at a constant 0.8 m/s, so A must strike the wall again before it could catch B **B1**

(c) finds A's speed after rebounding from the wall a second time, 0.4 x 2.2 **M1**

(c) 0.88 m/s, which exceeds B's constant speed of 0.8 m/s, so A gains on B and a second A-B collision must occur **A1**

(c) Answer: Yes, A and B collide again: A rebounds from the wall a second time at 0.88 m/s, faster than B's steady 0.8 m/s, so A catches up with B

Question 18

(a) momentum before = m(8) + m(-2) = 6m; momentum after = m(1) + m(5) = 6m, equal for any m, so conserved **M1**

(a) correctly states both totals equal 6m (oe) **A1**

(a) uses $e = -(v_D - v_C) / (u_D - u_C) = -(5 - 1) / (-2 - 8)$ **M1**

(a) e = 0.4, cao **A1**

(a) Answer: Momentum before = after = 6m kg m/s (conserved); e = 0.4

(b) KE before = $0.5(0.8)(8^2) + 0.5(0.8)(2^2)$; KE after = $0.5(0.8)(1^2) + 0.5(0.8)(5^2)$ **M1**

(b) loss = 27.2 - 10.4 = 16.8 J, cao **A1**

(b) Answer: 16.8 J

Question 19

(a) momentum equation: $2400(3) + 3600(0) = (2400+3600)v$ **M1**

(a) $7200 = 6000v$ (oe) **A1**

(a) v = 1.2 m/s, cao **A1**

(a) Answer: 1.2 m/s

(b) KE before = $0.5(2400)(3^2)$ [Q at rest contributes 0] **M1**

(b) KE after = $0.5(6000)(1.2^2)$, using their v from part (a) (ft) **A1**

(b) loss = 10800 - 4320 = 6480 J, cao **A1**

(b) Answer: 6480 J

(c) uses momentum conservation to write the common velocity as v = m1 u / (m1 + m2) **M1**

(c) forms KE before = $0.5\ m1\ u^2$ and KE after = $0.5(m1+m2)v^2$, and simplifies the loss to $0.5\ m1\ u^2$ x [m2/(m1+m2)] **M1**

(c) divides by the initial KE to obtain fraction lost = m2/(m1+m2) (cso), and verifies numerically: m2/(m1+m2) = 3600/6000 = 0.6, matching 6480/10800 = 0.6 from parts (a) and (b) **A1**

(c) Answer: Fraction lost = m2/(m1+m2); numerically 3600/6000 = 0.6, matching 6480/10800 = 0.6

Question 20

(a) uses $v^2 = u^2 + 2gh$ with u = 0, h = 2.5, g = 9.8 **M1**

(a) v = 7 m/s, cao (exact) **A1**

(a) Answer: 7 m/s

(b) speed just after first bounce = e x 7 = 0.6 x 7 = 4.2 m/s (ft from part (a)) **M1**

(b) uses $0 = (4.2)^2 - 2gh$ to find the rebound height **M1**

(b) h = 0.9 m, cao **A1**

(b) Answer: 0.9 m

(c) recognises that since rebound speed = e x (impact speed) and height is proportional to $(\text{speed})^2$, each successive rebound height is e^2 times the previous one **M1**

(c) correctly derives $h_n = 2.5\ e^{2n}$ (cso), noting $h_1 = 2.5\ e^2 = 0.9$ agrees with part (b) **A1**

(c) substitutes n = 3: $h_3 = 2.5 \times (0.6)^6$ **M1**

(c) $h_3 = 0.117$ m (awrt 3 s.f.), cao **A1**

(c) Answer: $h_n = 2.5\ e^{2n}$; h_3 = awrt 0.117 m