3.2 Further Mechanics: Work, Energy and Power: 1-hour Pack

Total Marks

AQA 7367 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Formulae and modelling assumptions used in this worksheet

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Unless a question states otherwise, take g = 9.8 m s^-2 and model each object as a particle. Strings and ropes are light and inextensible, and any pulleys or engines are treated as smooth or ideal unless stated. Key results used across this worksheet: work done by a constant force F acting through a displacement s at angle theta to the direction of motion is W = F*s*cos(theta); kinetic energy is KE = (1/2)*m*v^2; gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle equals the change in its kinetic energy; power is the rate of doing work, so for a force F moving its point of application at speed v in the direction of F, P = F*v, and the work done over a time interval is W = integral of P with respect to t; for a force that varies with displacement, F(x), the work done as the point of application moves from x = a to x = b is W = integral from a to b of F(x) dx.

1 A metal ball of mass 0.5 kg is released from rest at a height of 2 m above the surface of soft sand. It penetrates a distance of 0.08 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take g = 9.8 m/s^2.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 1 is 5 marks)

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2 A car of mass 800 kg has an engine working at a constant rate of 20 kW. The resistance to motion is a constant 400 N whatever the speed. Take $g = 9.8\ m/s^2$.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 15 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/25$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 2 is 9 marks)

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3 A metal ball of mass 1 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.2 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 3 is 5 marks)

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4 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 3/5$. The length of the plane is 10 m and the coefficient of friction between the particle and the plane is 0.25. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8\ m/s^2$.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 4 is 9 marks)

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5 A metal ball of mass 4 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.05 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

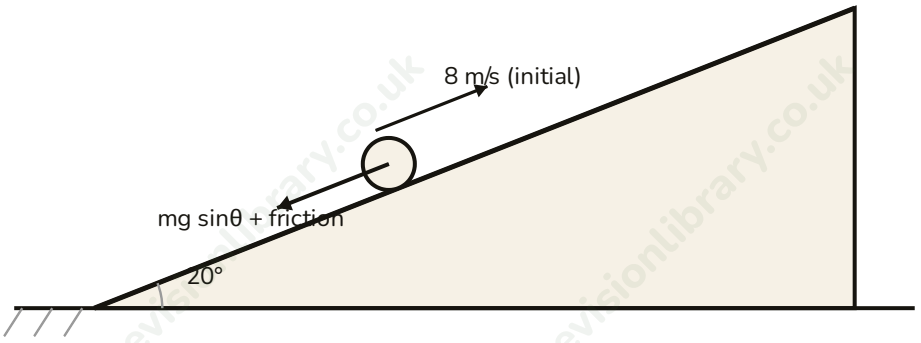
(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 5 is 5 marks)

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6 A particle of mass 0.5 kg is projected up a line of greatest slope of a rough plane inclined at 20 degrees to the horizontal, with initial speed 8 m/s. The coefficient of friction between the particle and the plane is 0.25.



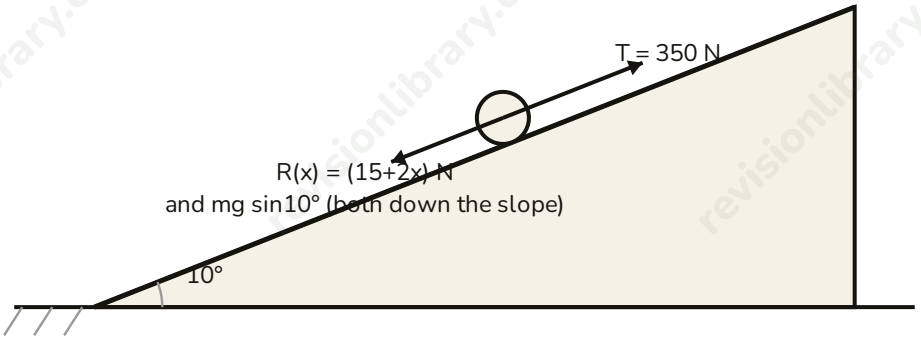
(a) Use the work-energy principle to find the distance the particle travels up the plane before it comes to instantaneous rest. **(5)**

(b) Determine, with justification, whether the particle subsequently slides back down the plane. **(3)**

(Total for Question 6 is 8 marks)

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7 A sledge of mass 40 kg is pulled up a straight slope inclined at 10 degrees to the horizontal by a rope parallel to the slope, in which the tension is a constant 350 N. Due to the changing surface of the slope, the resistance to motion (excluding gravity) when the sledge is a distance x metres up the slope ($0 \leq x \leq 12$) is modelled by $R = (15 + 2x)$ newtons. The sledge starts from rest at the bottom of the slope.



sledge starts from rest at the bottom of the slope

(a) Find the work done against the variable resistance as the sledge moves from $x = 0$ to $x = 12$. **(3)**

(b) Using the work-energy principle, find the speed of the sledge when $x = 12$. **(6)**

(Total for Question 7 is 9 marks)

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Mark scheme · 3.2 Further Mechanics: Work, Energy and Power: 1-hour Pack

Question 1

(a) conservation of energy (or $v^2 = 2gh$)	**M1**
(a) $v = \sqrt{2 \times 9.8 \times 2} = 6.26$ m/s	**A1**
(a) Answer: 6.26 m/s	
(b) loss of GPE over the whole descent = work done against the resistance	**M1**
(b) $0.5(9.8)(2 + 0.08) = R \times 0.08$	**A1**
(b) R = 127 N	**A1**
(b) Answer: 127 N	

Question 2

(a) at maximum speed the driving force equals the resistance: F = R	**M1**
(a) uses P = Fv	**M1**
(a) v = 20000/400 = 50.0 m/s	**A1**
(a) Answer: 50.0 m/s	
(b) driving force = 20000/15	**M1**
(b) applies F = ma: driving force - resistance = ma	**M1**
(b) $a = 1.2$ m/s^2	**A1**
(b) Answer: 1.2 m/s^2	
(c) resolves along the slope: F = R + mg sin α	**M1**
(c) uses P = Fv	**M1**
(c) v = 20000/(400 + 800(9.8)/25) = 28.0 m/s	**A1**
(c) Answer: 28.0 m/s	

Question 3

(a) conservation of energy (or $v^2 = 2gh$)	**M1**
(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s	**A1**
(a) Answer: 5.42 m/s	
(b) loss of GPE over the whole descent = work done against the resistance	**M1**
(b) $1(9.8)(1.5 + 0.2) = R \times 0.2$	**A1**
(b) R = 83.3 N	**A1**
(b) Answer: 83.3 N	

Question 4

(a) friction = μ R = μ mg cos α	**M1**
(a) work = friction x distance	**M1**
(a) 39.2 J	**A1**
(a) Answer: 39.2 J	
(b) KE gained = loss in GPE - work done against friction	**M1**
(b) $(1/2)2v^2 = 2(9.8)(10)(3/5) - 39.200$	**A1**
(b) v = 8.85 m/s	**A1**
(b) Answer: 8.85 m/s	
(c) work done against friction = initial KE	**M1**
(c) $0.3 \times 2 \times 9.8 \times s = (1/2)2(8.854)^2$	**M1**
(c) s = 13.3 m	**A1**
(c) Answer: 13.3 m	

Question 5

(a) conservation of energy (or $v^2 = 2gh$)	**M1**
(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s	**A1**
(a) Answer: 5.42 m/s	
(b) loss of GPE over the whole descent = work done against the resistance	**M1**
(b) $4(9.8)(1.5 + 0.05) = R \times 0.05$	**A1**
(b) R = 1220 N	**A1**
(b) Answer: 1220 N	

Question 6

(a) initial KE = $(1/2)*0.5*8^2 = 16$ (J)	**M1**
(a) identifies the resisting force acting down the slope as m*g*sin(20 degrees) + μ*m*g*cos(20 degrees)	**M1**
(a) resisting force = 2.83 N, awrt	**A1**
(a) equates initial KE to (resisting force)*d and rearranges for d, ft	**M1**
(a) d = 5.66 m, awrt, cao	**A1**
(a) Answer: d = awrt 5.66 m	
(b) compares tan(20 degrees) with the coefficient of friction μ = 0.25 (or equivalently compares m*g*sin(20 degrees) with μ*m*g*cos(20 degrees))	**M1**
(b) tan(20 degrees) = 0.364, awrt	**A1**
(b) since tan(20 degrees) > μ (0.364 > 0.25), the component of gravity down the slope exceeds the maximum available friction force, so the particle does slide back down, oe	**A1**
(b) Answer: Yes, the particle slides back down, since tan(20 degrees) = awrt 0.364 > μ = 0.25.	

Question 7

(a) sets up work against resistance = integral from 0 to 12 of (15 + 2x) dx	**M1**
(a) integrates correctly to $15x + x^2$, and substitutes limits	**M1**
(a) 324 J, cao	**A1**
(a) Answer: Work against resistance = 324 J	
(b) work done by the tension = 350*12 = 4200 (J)	**M1**
(b) work done against gravity = m*g*sin(10 degrees)*12	**M1**
(b) work done against gravity = 817 J, awrt	**A1**
(b) work done against resistance = 324 J, ft (a)	**B1**
(b) applies the work-energy principle: $(1/2)*40*v^2 - 0 =$ 4200 - (their gravity work) - (their resistance work), ft	**M1**
(b) v = 12.4 m/s, awrt, cao	**A1**
(b) Answer: v = awrt 12.4 m/s	

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