3.2 Further Mechanics: Work, Energy and Power

AQA 7367 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Formulae and modelling assumptions used in this worksheet

Revision Library

Unless a question states otherwise, take g = 9.8 m s^-2 and model each object as a particle. Strings and ropes are light and inextensible, and any pulleys or engines are treated as smooth or ideal unless stated. Key results used across this worksheet: work done by a constant force F acting through a displacement s at angle theta to the direction of motion is W = F*s*cos(theta); kinetic energy is KE = (1/2)*m*v^2; gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle equals the change in its kinetic energy; power is the rate of doing work, so for a force F moving its point of application at speed v in the direction of F, P = F*v, and the work done over a time interval is W = integral of P with respect to t; for a force that varies with displacement, F(x), the work done as the point of application moves from x = a to x = b is W = integral from a to b of F(x) dx.

1 This question concerns the work-energy principle for a particle moving under a system of forces.

(a) State the work-energy principle. **(1)**

(b) A particle of mass 2 kg is initially moving at 3 m/s. A resultant force acting on the particle does 60 J of work on it as it moves. Find the final speed of the particle. **(3)**

(Total for Question 1 is 4 marks)

2 A resultant force of constant magnitude 30 N acts on a body of mass 5 kg, in the direction of its motion, while the body is initially moving at 2 m/s. The force continues to act as the body moves a further distance of 10 m in a straight line.

(a) Find the speed of the body after it has moved the 10 m distance. **(4)**

(Total for Question 2 is 4 marks)

---

3 A metal ball of mass 4 kg is released from rest at a height of 3 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8$ m/s^2.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 3 is 5 marks)

4 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 8 m and the coefficient of friction between the particle and the plane is 0.15. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.2. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 4 is 9 marks)

5 A metal ball of mass 0.5 kg is released from rest at a height of 2 m above the surface of soft sand. It penetrates a distance of 0.08 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 5 is 5 marks)

6 A car of mass 800 kg has an engine working at a constant rate of 20 kW. The resistance to motion is a constant 400 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 15 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/25$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 6 is 9 marks)

7 A metal ball of mass 1 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.2 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 7 is 5 marks)

8 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 3/5$. The length of the plane is 10 m and the coefficient of friction between the particle and the plane is 0.25. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 8 is 9 marks)

9 A metal ball of mass 4 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.05 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 9 is 5 marks)

10 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 6 m and the coefficient of friction between the particle and the plane is 0.1. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 10 is 9 marks)

11 A metal ball of mass 4 kg is released from rest at a height of 2.5 m above the surface of soft sand. It penetrates a distance of 0.2 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 11 is 5 marks)

12 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 12 m and the coefficient of friction between the particle and the plane is 0.25. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 12 is 9 marks)

13 A metal ball of mass 1 kg is released from rest at a height of 3 m above the surface of soft sand. It penetrates a distance of 0.2 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 13 is 5 marks)

14 A metal ball of mass 2 kg is released from rest at a height of 3 m above the surface of soft sand. It penetrates a distance of 0.08 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 14 is 5 marks)

15 A car of mass 1500 kg has an engine working at a constant rate of 20 kW. The resistance to motion is a constant 500 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 10 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/20$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 15 is 9 marks)

16 A car of mass 800 kg has an engine working at a constant rate of 60 kW. The resistance to motion is a constant 400 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 19 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/25$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 16 is 9 marks)

17 A particle of mass 5 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 4/5$. The length of the plane is 6 m and the coefficient of friction between the particle and the plane is 0.15. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.4. Take $g = 9.8$ m/s^2.

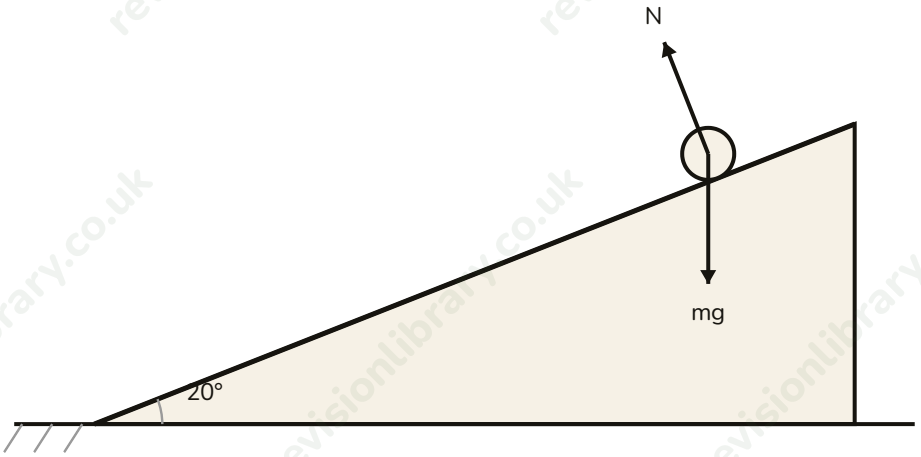
(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 17 is 9 marks)

18 A small block of mass 1.5 kg is released from rest at the top of a smooth incline of length 6 m, inclined at 20 degrees to the horizontal, and slides down to the bottom of the incline.



released from rest at the top; incline length 6 m

(a) Use energy methods to find the speed of the block at the bottom of the incline. **(3)**

(b) Show, by resolving forces along the incline and using a suvat equation, that the same speed at the bottom is obtained as in part (a). **(3)**

(Total for Question 18 is 6 marks)

19 A particle of mass 5 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 12 m and the coefficient of friction between the particle and the plane is 0.1. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.2. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 19 is 9 marks)

20 A car of mass 1500 kg has an engine working at a constant rate of 20 kW. The resistance to motion is a constant 400 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 10 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/40$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 20 is 9 marks)

21 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 6 m and the coefficient of friction between the particle and the plane is 0.15. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

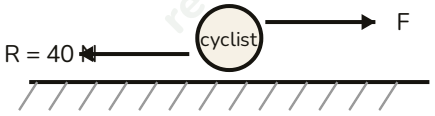
(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

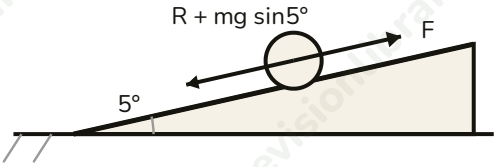
(Total for Question 21 is 9 marks)

22 A cyclist, Owen, and his cycle have a combined mass of 80 kg. Cycling along a horizontal road at a constant speed of 6 m/s, Owen experiences a constant resistance to motion of 40 N.

(a) horizontal road



(b) incline at 5°



(a) Calculate the power that Owen develops while cycling at this constant speed on the horizontal road. **(2)**

(b) Owen then rides up a hill inclined at 5 degrees to the horizontal, continuing to develop the same constant power as in part (a) and experiencing the same resistance of 40 N. Find the new constant speed at which Owen travels up the hill. **(5)**

(Total for Question 22 is 7 marks)

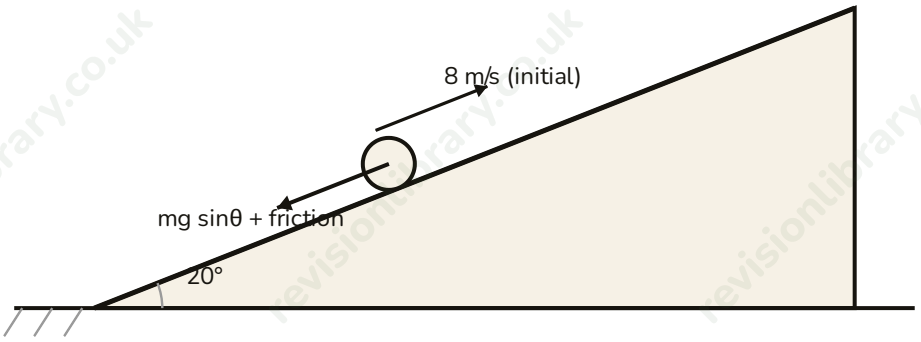
23 A go-kart of mass 120 kg moves from rest along a smooth horizontal track. At time t seconds after starting ($0 \leq t \leq 8$), the power developed by its engine is $P = (300 + 40t)$ watts.

(a) Find the total work done by the engine during the first 8 seconds of motion. **(3)**

(b) Hence find the speed of the go-kart at $t = 8$ seconds. **(3)**

(Total for Question 23 is 6 marks)

24 A particle of mass 0.5 kg is projected up a line of greatest slope of a rough plane inclined at 20 degrees to the horizontal, with initial speed 8 m/s. The coefficient of friction between the particle and the plane is 0.25.

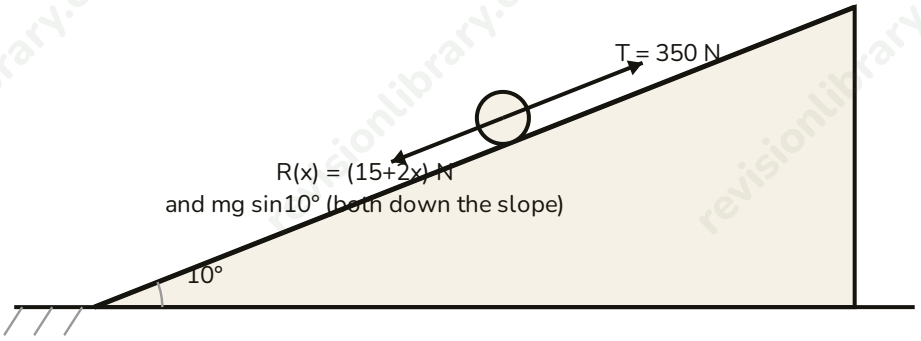


(a) Use the work-energy principle to find the distance the particle travels up the plane before it comes to instantaneous rest. **(5)**

(b) Determine, with justification, whether the particle subsequently slides back down the plane. **(3)**

(Total for Question 24 is 8 marks)

25 A sledge of mass 40 kg is pulled up a straight slope inclined at 10 degrees to the horizontal by a rope parallel to the slope, in which the tension is a constant 350 N. Due to the changing surface of the slope, the resistance to motion (excluding gravity) when the sledge is a distance x metres up the slope ($0 \leq x \leq 12$) is modelled by $R = (15 + 2x)$ newtons. The sledge starts from rest at the bottom of the slope.



sledge starts from rest at the bottom of the slope

(a) Find the work done against the variable resistance as the sledge moves from $x = 0$ to $x = 12$. **(3)**

(b) Using the work-energy principle, find the speed of the sledge when $x = 12$. **(6)**

(Total for Question 25 is 9 marks)

Mark scheme · 3.2 Further Mechanics: Work, Energy and Power

Question 1

(a) the total work done by the resultant of all the forces acting on a particle equals the change in its kinetic energy, oe **B1**

(a) Answer: The total work done by the resultant force acting on a particle equals the change in its kinetic energy.

(b) final KE = initial KE + work done = $(1/2)*2*3^2 + 60$ **M1**

(b) rearranges to $v^2 = 2*(\text{final KE})/2$, i.e. $v^2 = \text{final KE}$ **M1**

(b) v = 8.31 m/s, awrt, cao **A1**

(b) Answer: v = awrt 8.31 m/s

Question 2

(a) work done by resultant force = F*s = 30*10 = 300 (J) **M1**

(a) applies the work-energy principle: $(1/2)*5*v^2 - (1/2)*5*2^2 = 300$ **M1**

(a) $v^2 = 124$, oe **A1**

(a) v = 11.1 m/s, awrt, cao **A1**

(a) Answer: v = awrt 11.1 m/s

Question 3

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 3} = 7.67$ m/s **A1**

(a) Answer: 7.67 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) 4(9.8)(3 + 0.1) = R x 0.1 **A1**

(b) R = 1220 N **A1**

(b) Answer: 1220 N

Question 4

(a) friction = μ R = μ mg cos α **M1**

(a) work = friction x distance **M1**

(a) 9.05 J **A1**

(a) Answer: 9.05 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)2v^2 = 2(9.8)(8)(12/13) - 9.046$ **A1**

(b) v = 11.6 m/s **A1**

(b) Answer: 11.6 m/s

(c) work done against friction = initial KE **M1**

(c) $0.2 \times 2 \times 9.8 \times s = (1/2)2(11.649)^2$ **M1**

(c) s = 34.6 m **A1**

(c) Answer: 34.6 m

Question 5

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 2} = 6.26$ m/s **A1**

(a) Answer: 6.26 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) 0.5(9.8)(2 + 0.08) = R x 0.08 **A1**

(b) R = 127 N **A1**

(b) Answer: 127 N

Question 6

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 20000/400 = 50.0 m/s **A1**

(a) Answer: 50.0 m/s

(b) driving force = 20000/15 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) $a = 1.2$ m/s^2 **A1**

(b) Answer: 1.2 m/s^2

(c) resolves along the slope: F = R + mg sin α **M1**

(c) uses P = Fv **M1**

(c) v = 20000/(400 + 800(9.8)/25) = 28.0 m/s **A1**

(c) Answer: 28.0 m/s

Question 7

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s **A1**

(a) Answer: 5.42 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) 1(9.8)(1.5 + 0.2) = R x 0.2 **A1**

(b) R = 83.3 N **A1**

(b) Answer: 83.3 N

Question 8

(a) friction = μ R = μ mg cos α **M1**

(a) work = friction x distance **M1**

(a) 39.2 J **A1**

(a) Answer: 39.2 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)2v^2 = 2(9.8)(10)(3/5) - 39.200$ **A1**

(b) v = 8.85 m/s **A1**

(b) Answer: 8.85 m/s

(c) work done against friction = initial KE **M1**

(c) $0.3 \times 2 \times 9.8 \times s = (1/2)2(8.854)^2$ **M1**

(c) s = 13.3 m **A1**

(c) Answer: 13.3 m

Question 9

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s **A1**

(a) Answer: 5.42 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) 4(9.8)(1.5 + 0.05) = R x 0.05 **A1**

(b) R = 1220 N **A1**

(b) Answer: 1220 N

Question 10

(a) friction = $\mu R = \mu mg \cos \alpha$ **M1**

(a) work = friction x distance **M1**

(a) 4.52 J **A1**

(a) Answer: 4.52 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)2v^2 = 2(9.8)(6)(12/13) - 4.523$ **A1**

(b) v = 10.2 m/s **A1**

(b) Answer: 10.2 m/s

(c) work done against friction = initial KE **M1**

(c) $0.3 \times 2 \times 9.8 \times s = (1/2)2(10.200)^2$ **M1**

(c) s = 17.7 m **A1**

(c) Answer: 17.7 m

Question 11

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 2.5} = 7.00$ m/s **A1**

(a) Answer: 7.00 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $4(9.8)(2.5 + 0.2) = R \times 0.2$ **A1**

(b) R = 529 N **A1**

(b) Answer: 529 N

Question 12

(a) friction = $\mu R = \mu mg \cos \alpha$ **M1**

(a) work = friction x distance **M1**

(a) 22.6 J **A1**

(a) Answer: 22.6 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)2v^2 = 2(9.8)(12)(12/13) - 22.615$ **A1**

(b) v = 13.9 m/s **A1**

(b) Answer: 13.9 m/s

(c) work done against friction = initial KE **M1**

(c) $0.3 \times 2 \times 9.8 \times s = (1/2)2(13.946)^2$ **M1**

(c) s = 33.1 m **A1**

(c) Answer: 33.1 m

Question 13

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 3} = 7.67$ m/s **A1**

(a) Answer: 7.67 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $1(9.8)(3 + 0.2) = R \times 0.2$ **A1**

(b) R = 157 N **A1**

(b) Answer: 157 N

Question 14

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 3} = 7.67$ m/s **A1**

(a) Answer: 7.67 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $2(9.8)(3 + 0.08) = R \times 0.08$ **A1**

(b) R = 755 N **A1**

(b) Answer: 755 N

Question 15

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 20000/500 = 40.0 m/s **A1**

(a) Answer: 40.0 m/s

(b) driving force = 20000/10 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) $a = 1.0$ m/s^2 **A1**

(b) Answer: 1.0 m/s^2

(c) resolves along the slope: $F = R + mg \sin \alpha$ **M1**

(c) uses P = Fv **M1**

(c) v = 20000/(500 + 1500(9.8)/20) = 16.2 m/s **A1**

(c) Answer: 16.2 m/s

Question 16

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 60000/400 = 150 m/s **A1**

(a) Answer: 150 m/s

(b) driving force = 60000/19 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) $a = 3.4$ m/s^2 **A1**

(b) Answer: 3.4 m/s^2

(c) resolves along the slope: $F = R + mg \sin \alpha$ **M1**

(c) uses P = Fv **M1**

(c) v = 60000/(400 + 800(9.8)/25) = 84.1 m/s **A1**

(c) Answer: 84.1 m/s

Question 17

(a) friction = $\mu R = \mu mg \cos \alpha$ **M1**

(a) work = friction x distance **M1**

(a) 26.5 J **A1**

(a) Answer: 26.5 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)5v^2 = 5(9.8)(6)(4/5) - 26.460$ **A1**

(b) v = 9.14 m/s **A1**

(b) Answer: 9.14 m/s

(c) work done against friction = initial KE **M1**

(c) $0.4 \times 5 \times 9.8 \times s = (1/2)5(9.138)^2$ **M1**

(c) s = 10.65 = 10.7 m **A1**

(c) Answer: 10.7 m

Question 18

(a) vertical height dropped, h = 6*sin(20 degrees) = 2.05 (m), awrt — **M1**

(a) equates GPE lost to KE gained: $m*g*h = (1/2)*m*v^2$ — **M1**

(a) v = 6.34 m/s, awrt, cao — **A1**

(a) Answer: v = awrt 6.34 m/s

(b) resolves along the incline: m*a = m*g*sin(20 degrees), giving a = g*sin(20 degrees) — **M1**

(b) uses $v^2 = u^2 + 2*a*s$ with u = 0, a = 3.35 (m/s^2, awrt), s = 6 — **M1**

(b) v = 6.34 m/s (awrt), matching part (a), cso — **A1**

(b) Answer: v = awrt 6.34 m/s, matching part (a) (shown)

Question 19

(a) friction = $\mu R = \mu mg \cos \alpha$ — **M1**

(a) work = friction x distance — **M1**

(a) 22.6 J — **A1**

(a) Answer: 22.6 J

(b) KE gained = loss in GPE - work done against friction — **M1**

(b) $(1/2)5v^2 = 5(9.8)(12)(12/13) - 22.615$ — **A1**

(b) v = 14.4 m/s — **A1**

(b) Answer: 14.4 m/s

(c) work done against friction = initial KE — **M1**

(c) $0.2 \times 5 \times 9.8 \times s = (1/2)5(14.424)^2$ — **M1**

(c) s = 53.1 m — **A1**

(c) Answer: 53.1 m

Question 20

(a) at maximum speed the driving force equals the resistance: F = R — **M1**

(a) uses P = Fv — **M1**

(a) v = 20000/400 = 50.0 m/s — **A1**

(a) Answer: 50.0 m/s

(b) driving force = 20000/10 — **M1**

(b) applies F = ma: driving force - resistance = ma — **M1**

(b) a = 1.1 m/s^2 — **A1**

(b) Answer: 1.1 m/s^2

(c) resolves along the slope: F = R + mg sin α — **M1**

(c) uses P = Fv — **M1**

(c) v = 20000/(400 + 1500(9.8)/40) = 26.1 m/s — **A1**

(c) Answer: 26.1 m/s

Question 21

(a) friction = $\mu R = \mu mg \cos \alpha$ — **M1**

(a) work = friction x distance — **M1**

(a) 6.78 J — **A1**

(a) Answer: 6.78 J

(b) KE gained = loss in GPE - work done against friction — **M1**

(b) $(1/2)2v^2 = 2(9.8)(6)(12/13) - 6.785$ — **A1**

(b) v = 10.1 m/s — **A1**

(b) Answer: 10.1 m/s

(c) work done against friction = initial KE — **M1**

(c) $0.3 \times 2 \times 9.8 \times s = (1/2)2(10.088)^2$ — **M1**

(c) s = 17.3 m — **A1**

(c) Answer: 17.3 m

Question 22

(a) uses P = F*v = 40*6, with F = R at constant speed — **M1**

(a) P = 240 W, cao — **A1**

(a) Answer: P = 240 W

(b) resolves along the slope at constant speed: driving force = R + m*g*sin(5 degrees) — **M1**

(b) m*g*sin(5 degrees) = 68.3 N, awrt — **A1**

(b) total resisting force = 40 + 68.3 = 108 (N), ft — **M1**

(b) uses P = F*v with the constant power P = 240 W from (a), ft — **M1**

(b) v = 2.22 m/s, awrt, cao — **A1**

(b) Answer: v = awrt 2.22 m/s

Question 23

(a) sets up W = integral from 0 to 8 of (300 + 40t) dt — **M1**

(a) integrates correctly to $300t + 20t^2$, and substitutes limits — **M1**

(a) W = 3680 J, cao — **A1**

(a) Answer: W = 3680 J

(b) applies the work-energy principle: $(1/2)*120*v^2 - 0 =$ 3680, ft (a) — **M1**

(b) rearranges to $v^2 = 3680/60$ — **M1**

(b) v = 7.83 m/s, awrt, cao — **A1**

(b) Answer: v = awrt 7.83 m/s

Question 24

(a) initial KE = $(1/2)*0.5*8^2 = 16$ (J) — **M1**

(a) identifies the resisting force acting down the slope as m*g*sin(20 degrees) + μ*m*g*cos(20 degrees) — **M1**

(a) resisting force = 2.83 N, awrt — **A1**

(a) equates initial KE to (resisting force)*d and rearranges for d, ft — **M1**

(a) d = 5.66 m, awrt, cao — **A1**

(a) Answer: d = awrt 5.66 m

(b) compares tan(20 degrees) with the coefficient of friction μ = 0.25 (or equivalently compares m*g*sin(20 degrees) with μ*m*g*cos(20 degrees)) — **M1**

(b) tan(20 degrees) = 0.364, awrt — **A1**

(b) since tan(20 degrees) > μ (0.364 > 0.25), the component of gravity down the slope exceeds the maximum available friction force, so the particle does slide back down, oe — **A1**

(b) Answer: Yes, the particle slides back down, since tan(20 degrees) = awrt 0.364 > μ = 0.25.

Question 25

(a) sets up work against resistance = integral from 0 to 12 of (15 + 2x) dx — **M1**

(a) integrates correctly to $15x + x^2$, and substitutes limits — **M1**

(a) 324 J, cao — **A1**

(a) Answer: Work against resistance = 324 J

(b) work done by the tension = 350*12 = 4200 (J) — **M1**

(b) work done against gravity = m*g*sin(10 degrees)*12 — **M1**

(b) work done against gravity = 817 J, awrt — **A1**

(b) work done against resistance = 324 J, ft (a) — **B1**

(b) applies the work-energy principle: $(1/2)*40*v^2 - 0 =$ 4200 - (their gravity work) - (their resistance work), ft — **M1**

(b) v = 12.4 m/s, awrt, cao — **A1**

(b) Answer: v = awrt 12.4 m/s