3.3 Further Mechanics: Circular Motion: 1-hour Pack

AQA 7367 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Formulae and modelling assumptions used in this worksheet

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Unless a question states otherwise, take g = 9.8 m/s^2 and model each object as a particle. Strings are light and inextensible; rods are light and rigid; all surfaces, tracks, hemispheres and hinges are smooth (frictionless) unless friction is explicitly stated. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega, and the magnitude of the centripetal (radial) acceleration, directed towards the centre of the circle, is a = v^2/r = r*omega^2. For a conical pendulum and for a vehicle on a banked track, resolve the forces vertically and horizontally (or perpendicular and parallel to the surface) to form simultaneous equations linking the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with conservation of energy; the critical condition for a particle on a string, or moving on the inside of a smooth circular track or the outside of a smooth sphere, occurs where the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when its speed is instantaneously zero.

1 A car of mass 800 kg moves at constant speed on a horizontal circular bend of radius 20 m. The coefficient of friction between the tyres and the road is 0.3, and the friction is at its maximum value when the car is at the greatest speed for which it can take the bend without slipping. Model the car as a particle. Take $g = 9.8\ m/s^2$.

(a) Write down the greatest value of the sideways friction force that the road can provide. **(2)**

(b) Find the greatest speed at which the car can take the bend without slipping, in m/s and in km/h, to 3 significant figures. **(4)**

(Total for Question 1 is 6 marks)

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2 A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is fixed at a point O. The particle is held at rest vertically below O and then projected horizontally with speed u m/s, where $u^2 = 9gr$, so that it moves in a vertical circle. Take $g = 9.8$ m/s^2.

(a) Find the value of u, to 3 significant figures. **(2)**

(b) Find the tension in the string when the particle is at its lowest point, immediately after projection. **(2)**

(c) Find the speed of the particle when it is at the highest point of the circle, and the tension in the string at that point. **(4)**

(Total for Question 2 is 8 marks)

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3 A particle of mass 1 kg is attached by a light inextensible string of length 0.5 m to a fixed point O. The particle moves in a horizontal circle with the string taut, making a constant angle θ with the downward vertical, where $\cos\theta = 5/13$. Take $g = 9.8$ m/s^2.

(a) Find the tension in the string, to 3 significant figures. **(2)**

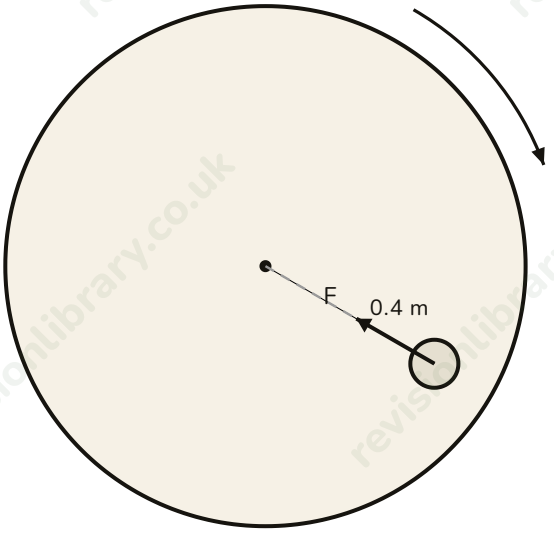
(b) Find the radius of the circle and the speed of the particle, to 3 significant figures. **(4)**

(c) Find the angular speed of the particle and the time taken for one complete revolution, to 3 significant figures. **(3)**

(Total for Question 3 is 9 marks)

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4 At a funfair in Blackpool, a small 50p coin of mass 0.05 kg rests on a rotating horizontal disc at a distance of 0.4 m from the centre. The coefficient of friction between the coin and the disc is 0.25. The disc's angular speed is slowly increased from rest.



disc viewed from above

(a) Find the maximum angular speed of the disc for which the coin does not slide. **(4)**

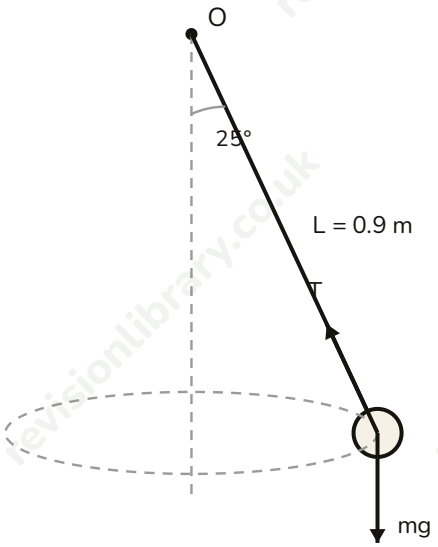
(b) Find the corresponding linear speed of the coin at this angular speed. **(2)**

(c) The coin is now moved to a distance of 0.6 m from the centre of the disc, which continues to rotate at the angular speed found in part (a). Determine, with justification, whether the coin remains in place at this new radius. **(4)**

(Total for Question 4 is 10 marks)

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5 An engineering student, Kwame, models a fairground ride using a conical pendulum: a light inextensible string of length 0.9 m has one end fixed at a point O, and a small ball of mass 0.25 kg is attached to the other end. The ball moves in a horizontal circle at constant speed, with the string making a constant angle of 25 degrees with the vertical.



(a) Show that the tension in the string is awrt 2.70 N. **(3)**

(b) Find the radius of the circle in which the ball moves. **(2)**

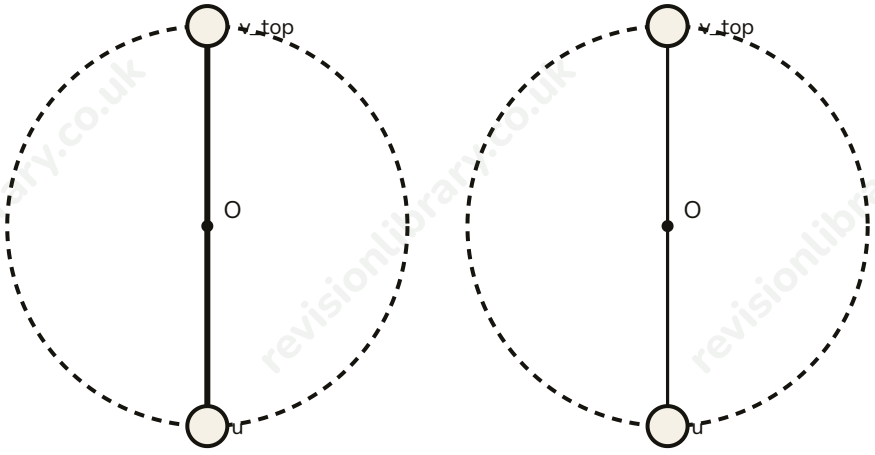
(c) Find the speed of the ball. **(4)**

(d) Find the time taken for the ball to complete one full revolution. **(3)**

(Total for Question 5 is 12 marks)

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6 In a mechanics practical, Ravi compares two versions of the same apparatus. In each version, a small ball of mass 0.3 kg is attached to the end of a light connecting piece of length 0.8 m, the other end of which is fixed at a point O, and the ball is projected from the lowest point of a vertical circle of radius 0.8 m with speed u. In version 1, the connecting piece is a light rigid rod; in version 2, it is a light inextensible string.



version 1: rigid rod *version 2: string*

(a) Explain why the minimum value of u needed for the ball to complete full circles is different for the rod (version 1) and the string (version 2). **(2)**

(b) For version 1 (the rigid rod), find the minimum speed of projection, u, at the lowest point for the ball to complete full circles. **(4)**

(c) For version 2 (the string), find the minimum speed of projection, u, at the lowest point for the ball to complete full circles. **(4)**

(d) State the ratio of the minimum value of u^2 for the rod to the minimum value of u^2 for the string, giving your answer in its simplest form. **(1)**

(Total for Question 6 is 11 marks)

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Mark scheme · 3.3 Further Mechanics: Circular Motion: 1-hour Pack

Question 1

(a) $F_{max} = \mu R$ with $R = mg$ **M1**

(a) 0.3 x 800 x 9.8 = 2350 N **A1**

(a) Answer: 2350 N

(b) $F = m v^2/r$ with $F = \mu m g$ **M1**

(b) $v^2 = \mu g r = 58.80$ **A1**

(b) v = 7.67 m/s **A1**

(b) 27.6 km/h **A1**

(b) Answer: 7.67 m/s = 27.6 km/h

Question 2

(a) $u^2 = 9 \times 9.8 \times 0.5$ **M1**

(a) u = 6.64 m/s **A1**

(a) Answer: 6.64 m/s

(b) resolves towards the centre: $T - mg = m u^2/r$ **M1**

(b) T = 2(9.8) + 2(9)(9.8) = 196 N **A1**

(b) Answer: 196 N

(c) energy: $(1/2)mv^2 = (1/2)\mu^2 - mg(2r)$ **M1**

(c) $v^2 = (9 - 4)gr$, v = 4.95 m/s **A1**

(c) resolves towards the centre: $T + mg = m v^2/r$ **M1**

(c) T = 78.4 N **A1**

(c) Answer: v = 4.95 m/s, T = 78.4 N

Question 3

(a) resolves vertically: $T \cos \theta = mg$ **M1**

(a) T = 1(9.8)/(5/13) = 25.5 N **A1**

(a) Answer: 25.5 N

(b) radius $r = 0.5 \sin \theta = 0.462$ m **B1**

(b) resolves horizontally: $T \sin \theta = m v^2/r$ **M1**

(b) $v^2 = r g \tan \theta = 10.8554$ **A1**

(b) v = 3.29 m/s **A1**

(b) Answer: r = 0.462 m, v = 3.29 m/s

(c) $\omega = v/r$ **M1**

(c) $\omega = 7.14$ rad/s **A1**

(c) period $= 2\pi/\omega = 0.880$ s **A1**

(c) Answer: ω = 7.14 rad/s, period 0.880 s

Question 4

(a) identifies that at maximum angular speed, friction is at its limiting value: $F = \mu * m * g$ **M1**

(a) sets the limiting friction equal to the required centripetal force: $\mu * m * g = m * r * \omega^2$ **M1**

(a) $\omega^2 = \mu * g / r = 6.125$, cao **A1**

(a) $\omega = 2.47$ rad/s, awrt, cao **A1**

(a) Answer: ω = awrt 2.47 rad/s

(b) uses $v = r * \omega$, ft (a) **M1**

(b) v = 0.990 m/s, awrt, cao **A1**

(b) Answer: v = awrt 0.990 m/s

(c) finds the friction force required to keep the coin moving in a circle at radius 0.6 m at this angular speed: F_required = $m * r * \omega^2$, ft **M1**

(c) F_required = 0.184 N, awrt, ft **A1**

(c) finds the maximum friction force available: $F_{max} = \mu * m * g$ **M1**

(c) $F_{max} = 0.1225$ N, and since F_required > F_{max}, the disc cannot supply enough friction, so the coin slides (does not remain in place), cao with correct conclusion **A1**

(c) Answer: The coin slides, since the required friction (awrt 0.184 N) exceeds the maximum available friction (0.1225 N).

Question 5

(a) resolves vertically for the ball: $T * \cos(25 \text{ degrees}) = m * g$ **M1**

(a) correct unsimplified expression, $T = (0.25 * 9.8)/\cos(25 \text{ degrees})$ **A1**

(a) T = 2.70 N, cso **A1**

(a) Answer: T = awrt 2.70 N (shown)

(b) uses $r = L * \sin(25 \text{ degrees})$ with L = 0.9 **M1**

(b) r = 0.380 m, awrt, cao **A1**

(b) Answer: r = awrt 0.380 m

(c) resolves horizontally for the ball: $T * \sin(25 \text{ degrees}) = m * v^2/r$, ft (a) and (b) **M1**

(c) rearranges for v^2, ft **M1**

(c) $v^2 = 1.74$, awrt, ft **A1**

(c) v = 1.32 m/s, awrt, cao **A1**

(c) Answer: v = awrt 1.32 m/s

(d) uses period $= 2 * \pi * r/v$, ft (b) and (c) **M1**

(d) substitutes values correctly **M1**

(d) period = 1.81 s, awrt, cao **A1**

(d) Answer: period = awrt 1.81 s

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Question 6

(a) a rigid rod can push as well as pull (it can act in compression as well as tension), so it can supply the centripetal force required at the top of the circle even when the ball's speed there is very small (or zero), oe **B1**

(a) a string can only pull (it can only be in tension), so if the string is to remain taut the weight of the ball alone must not exceed the centripetal force required at the top; this requires a strictly positive minimum speed at the top, oe **B1**

(a) Answer: A rod can push or pull, so it can maintain the circular path even when the ball's speed at the top is zero; a string can only pull, so it would go slack unless the ball has enough speed at the top for its weight alone to provide the required centripetal force.

(b) uses conservation of energy between the bottom and top of the circle: $(1/2)*u^2 = (1/2)*v_{top}^2 + g*(2*r)$ **M1**

(b) uses the critical condition for a rod, $v_{top} = 0$, since the rod can maintain the circular path down to zero speed at the top **M1**

(b) $u^2 = 4*g*r = 31.36$, ft **A1**

(b) u = 5.6 m/s, cao **A1**

(b) Answer: u = 5.6 m/s

(c) uses the critical condition for a string, $v_{top}^2 = g*r$ (tension zero at the top) **M1**

(c) uses conservation of energy: $u^2 = v_{top}^2 + 4*g*r$ **M1**

(c) $u^2 = 5*g*r = 39.2$, ft **A1**

(c) u = 6.26 m/s, awrt, cao **A1**

(c) Answer: u = awrt 6.26 m/s

(d) ratio = 4:5, cao (independent of the values of g and r, since the ratio is $4*g*r : 5*g*r$) **B1**

(d) Answer: 4:5

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