3.3 Further Mechanics: Circular Motion: Test Pack

AQA 7367 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

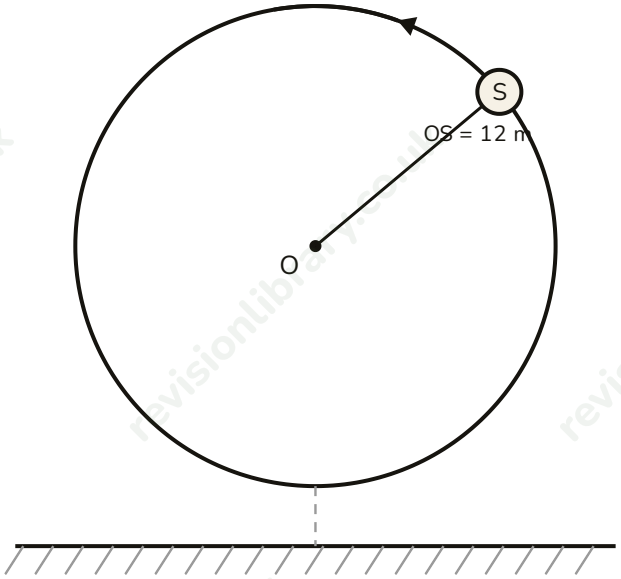
Formulae and modelling assumptions used in this worksheet

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Unless a question states otherwise, take g = 9.8 m/s^2 and model each object as a particle. Strings are light and inextensible; rods are light and rigid; all surfaces, tracks, hemispheres and hinges are smooth (frictionless) unless friction is explicitly stated. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega, and the magnitude of the centripetal (radial) acceleration, directed towards the centre of the circle, is a = v^2/r = r*omega^2. For a conical pendulum and for a vehicle on a banked track, resolve the forces vertically and horizontally (or perpendicular and parallel to the surface) to form simultaneous equations linking the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with conservation of energy; the critical condition for a particle on a string, or moving on the inside of a smooth circular track or the outside of a smooth sphere, occurs where the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when its speed is instantaneously zero.

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1 At a funfair in Brighton, a big wheel of diameter 24 m rotates at a constant angular speed, completing one full revolution every 30 seconds. Sam sits in a pod fixed to the rim of the wheel.



(a) Show that the angular speed of the wheel is $\pi/15$ rad/s. **(2)**

(b) Find the speed of Sam's pod, giving your answer in m/s to 3 significant figures. **(2)**

(c) Calculate the magnitude of Sam's centripetal acceleration using the formula $a = v^2/r$, and also using the formula $a = r*\omega^2$, showing that the two methods give the same value to 3 significant figures. **(4)**

(Total for Question 1 is 8 marks)

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2 A particle of mass 1 kg is attached by a light inextensible string of length 2 m to a fixed point O. The particle moves in a horizontal circle with the string taut, making a constant angle θ with the downward vertical, where $\cos\theta = 4/5$. Take $g = 9.8$ m/s^2.

(a) Find the tension in the string, to 3 significant figures. **(2)**

(b) Find the radius of the circle and the speed of the particle, to 3 significant figures. **(4)**

(c) Find the angular speed of the particle and the time taken for one complete revolution, to 3 significant figures. **(3)**

(Total for Question 2 is 9 marks)

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3 A car of mass 800 kg moves at constant speed on a horizontal circular bend of radius 30 m. The coefficient of friction between the tyres and the road is 0.3, and the friction is at its maximum value when the car is at the greatest speed for which it can take the bend without slipping. Model the car as a particle. Take $g = 9.8$ m/s^2.

(a) Write down the greatest value of the sideways friction force that the road can provide. **(2)**

(b) Find the greatest speed at which the car can take the bend without slipping, in m/s and in km/h, to 3 significant figures. **(4)**

(Total for Question 3 is 6 marks)

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4 A particle of mass 1 kg is attached to one end of a light inextensible string of length 0.4 m. The other end is fixed to a point O on a smooth horizontal table. The particle moves on the table in a circle with centre O, with constant angular speed 5 rad/s.

(a) Find the speed of the particle. **(2)**

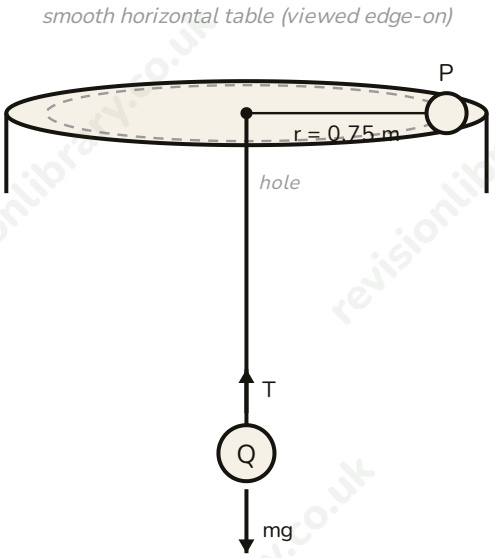
(b) Find the tension in the string. **(2)**

(c) The string will break if the tension exceeds 20 N. Find the greatest speed at which the particle can move in the circle without the string breaking, to 3 significant figures. **(3)**

(Total for Question 4 is 7 marks)

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5 In a school physics lab, Aisha sets up an experiment in which a particle P of mass 0.4 kg lies on a smooth horizontal table and is attached to a light inextensible string. The string passes through a smooth hole at the centre of the table and hangs vertically below the table, where it is attached to a particle Q of mass 1.2 kg. Q hangs in equilibrium, and P moves in a horizontal circle of radius 0.75 m at constant angular speed.



(a) Find the tension in the string. **(2)**

(b) Find the angular speed of P. **(3)**

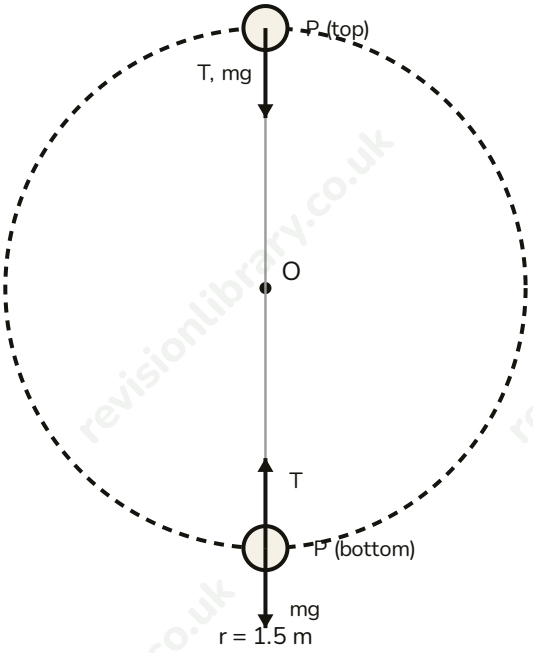
(c) Find the speed of P. **(2)**

(d) Find the time taken for P to complete one full revolution. **(2)**

(Total for Question 5 is 9 marks)

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6 A particle P of mass 0.6 kg is attached to one end of a light inextensible string of length 1.5 m. The other end of the string is fixed at a point O, and P moves in a vertical circle of radius 1.5 m with centre O.



(a) Given that P is on the point of the string becoming slack when P is at the top of the circle, show that the speed of P at the top of the circle, v_{top}, satisfies $v_{top}^2 = g*r$. **(3)**

(b) Find the speed of P at the top of the circle in this case. **(2)**

(c) Using conservation of energy, find the speed of P at the bottom of the circle in this case. **(4)**

(d) Find the tension in the string when P is at the bottom of the circle. **(3)**

(Total for Question 6 is 12 marks)

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Mark scheme · 3.3 Further Mechanics: Circular Motion: Test Pack

Question 1

(a) uses $\omega = 2*\pi/T$ with T = 30 (s) **M1**

(a) $\omega = 2*\pi/30 = \pi/15$ rad/s, cso **A1**

(a) Answer: $\omega = \pi/15$ rad/s (exact), = awrt 0.209 rad/s

(b) uses $v = r*\omega$ with r = 12 (m), ft (a) **M1**

(b) v = 2.51 m/s, awrt, cao **A1**

(b) Answer: v = awrt 2.51 m/s

(c) substitutes into $a = v^2/r$ using their v from (b) and r = 12, ft **M1**

(c) a = 0.526 m/s^2, awrt **A1**

(c) substitutes into $a = r*\omega^2$ using $\omega = \pi/15$ and r = 12 **M1**

(c) a = 0.526 m/s^2 (awrt), matching the value found by the first method, cso **A1**

(c) Answer: a = awrt 0.526 m/s^2 by both methods

Question 2

(a) resolves vertically: T cos θ = mg **M1**

(a) T = 1(9.8)/(4/5) = 12.25 = 12.3 N **A1**

(a) Answer: 12.3 N

(b) radius r = 2 sin θ = 1.20 m **B1**

(b) resolves horizontally: T sin θ = m v^2/r **M1**

(b) v^2 = r g tan θ = 8.8200 **A1**

(b) v = 2.97 m/s **A1**

(b) Answer: r = 1.20 m, v = 2.97 m/s

(c) ω = v/r **M1**

(c) ω = 2.47 rad/s **A1**

(c) period = 2 π/ω = 2.54 s **A1**

(c) Answer: ω = 2.47 rad/s, period 2.54 s

Question 3

(a) $F_{max} = \mu R$ with R = mg **M1**

(a) 0.3 x 800 x 9.8 = 2350 N **A1**

(a) Answer: 2350 N

(b) F = m v^2/r with F = μ m g **M1**

(b) v^2 = μ g r = 88.20 **A1**

(b) v = 9.39 m/s **A1**

(b) 33.8 km/h **A1**

(b) Answer: 9.39 m/s = 33.8 km/h

Question 4

(a) v = r ω **M1**

(a) 0.4 x 5 = 2.00 m/s **A1**

(a) Answer: 2.00 m/s

(b) T = m r ω^2 **M1**

(b) 1 x 0.4 x 5^2 = 10.0 N **A1**

(b) Answer: 10.0 N

(c) 20 = m v^2/r **M1**

(c) v^2 = 20 x 0.4 **A1**

(c) v = 2.83 m/s **A1**

(c) Answer: 2.83 m/s

Question 5

(a) resolves vertically for Q in equilibrium: $T = m_Q * g$ **M1**

(a) T = 11.76 N, cao **A1**

(a) Answer: T = 11.76 N

(b) uses the centripetal force equation for P: $T = m_P * r * \omega^2$, ft (a) **M1**

(b) ω^2 = 11.76/(0.4*0.75) = 39.2, ft **A1**

(b) ω = 6.26 rad/s, awrt, cao **A1**

(b) Answer: ω = awrt 6.26 rad/s

(c) uses v = r*ω, ft (b) **M1**

(c) v = 4.70 m/s, awrt, cao **A1**

(c) Answer: v = awrt 4.70 m/s

(d) uses period = 2*π/ω, ft (b) **M1**

(d) period = 1.00 s, awrt, cao **A1**

(d) Answer: period = awrt 1.00 s

Question 6

(a) applies Newton's second law in the radial direction at the top of the circle: $T + m*g = m*v_{top}^2/r$ **M1**

(a) uses the critical condition that the string is on the point of becoming slack, i.e. T = 0 **M1**

(a) $v_{top}^2 = g*r$, cso **A1**

(a) Answer: $v_{top}^2 = g*r$ (shown)

(b) substitutes r = 1.5 and g = 9.8 into $v_{top}^2 = g*r$, ft (a) **M1**

(b) v_{top} = 3.83 m/s, awrt, cao **A1**

(b) Answer: v_{top} = awrt 3.83 m/s

(c) uses conservation of energy between the bottom and top of the circle, with height gained 2*r **M1**

(c) $(1/2)*m*v_bottom^2 = (1/2)*m*v_{top}^2 + m*g*(2*r)$, ft (a) **M1**

(c) $v_bottom^2 = v_{top}^2 + 4*g*r = 5*g*r$ = 73.5, ft **A1**

(c) v_bottom = 8.57 m/s, awrt, cao **A1**

(c) Answer: v_bottom = awrt 8.57 m/s

(d) applies Newton's second law in the radial direction at the bottom of the circle: $T - m*g = m*v_bottom^2/r$, ft **M1**

(d) substitutes $v_bottom^2/r = 73.5/1.5 = 49$ (= 5*g) **M1**

(d) T = 35.28 N, cao **A1**

(d) Answer: T = 35.28 N

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