3.3 Further Mechanics: Circular Motion

AQA 7367 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Formulae and modelling assumptions used in this worksheet

Revision Library

Unless a question states otherwise, take g = 9.8 m/s^2 and model each object as a particle. Strings are light and inextensible; rods are light and rigid; all surfaces, tracks, hemispheres and hinges are smooth (frictionless) unless friction is explicitly stated. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega, and the magnitude of the centripetal (radial) acceleration, directed towards the centre of the circle, is a = v^2/r = r*omega^2. For a conical pendulum and for a vehicle on a banked track, resolve the forces vertically and horizontally (or perpendicular and parallel to the surface) to form simultaneous equations linking the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with conservation of energy; the critical condition for a particle on a string, or moving on the inside of a smooth circular track or the outside of a smooth sphere, occurs where the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when its speed is instantaneously zero.

1 A small ball of mass 0.2 kg lies on a smooth horizontal table and is attached to a fixed point on the table by a light inextensible string of length 0.5 m. The ball moves in a horizontal circle of radius 0.5 m at a constant speed of 3 m/s.

(a) Find the angular speed of the ball. **(2)**

(b) Find the tension in the string. **(2)**

(Total for Question 1 is 4 marks)

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2 A particle of mass 1 kg is attached to one end of a light inextensible string of length 1.5 m. The other end is fixed to a point O on a smooth horizontal table. The particle moves on the table in a circle with centre O, with constant angular speed 6 rad/s.

(a) Find the speed of the particle. **(2)**

(b) Find the tension in the string. **(2)**

(c) The string will break if the tension exceeds 30 N. Find the greatest speed at which the particle can move in the circle without the string breaking, to 3 significant figures. **(3)**

(Total for Question 2 is 7 marks)

3 A car of mass 1000 kg moves at constant speed on a horizontal circular bend of radius 30 m. The coefficient of friction between the tyres and the road is 0.4, and the friction is at its maximum value when the car is at the greatest speed for which it can take the bend without slipping. Model the car as a particle. Take $g = 9.8$ m/s^2.

(a) Write down the greatest value of the sideways friction force that the road can provide. **(2)**

(b) Find the greatest speed at which the car can take the bend without slipping, in m/s and in km/h, to 3 significant figures. **(4)**

(Total for Question 3 is 6 marks)

4 A particle of mass 0.5 kg is attached by a light inextensible string of length 1.5 m to a fixed point O. The particle moves in a horizontal circle with the string taut, making a constant angle θ with the downward vertical, where $\cos\theta = 3/5$. Take $g = 9.8$ m/s^2.

(a) Find the tension in the string, to 3 significant figures. **(2)**

(b) Find the radius of the circle and the speed of the particle, to 3 significant figures. **(4)**

(c) Find the angular speed of the particle and the time taken for one complete revolution, to 3 significant figures. **(3)**

(Total for Question 4 is 9 marks)

5 A particle of mass 0.2 kg is attached to one end of a light inextensible string of length 0.4 m. The other end is fixed to a point O on a smooth horizontal table. The particle moves on the table in a circle with centre O, with constant angular speed 8 rad/s.

(a) Find the speed of the particle. **(2)**

(b) Find the tension in the string. **(2)**

(c) The string will break if the tension exceeds 30 N. Find the greatest speed at which the particle can move in the circle without the string breaking, to 3 significant figures. **(3)**

(Total for Question 5 is 7 marks)

6 A car of mass 800 kg moves at constant speed on a horizontal circular bend of radius 20 m. The coefficient of friction between the tyres and the road is 0.3, and the friction is at its maximum value when the car is at the greatest speed for which it can take the bend without slipping. Model the car as a particle. Take $g = 9.8$ m/s^2.

(a) Write down the greatest value of the sideways friction force that the road can provide. **(2)**

(b) Find the greatest speed at which the car can take the bend without slipping, in m/s and in km/h, to 3 significant figures. **(4)**

(Total for Question 6 is 6 marks)

7 A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is fixed at a point O. The particle is held at rest vertically below O and then projected horizontally with speed u m/s, where $u^2 = 9gr$, so that it moves in a vertical circle. Take $g = 9.8$ m/s^2.

(a) Find the value of u, to 3 significant figures. **(2)**

(b) Find the tension in the string when the particle is at its lowest point, immediately after projection. **(2)**

(c) Find the speed of the particle when it is at the highest point of the circle, and the tension in the string at that point. **(4)**

(Total for Question 7 is 8 marks)

8 A particle of mass 0.2 kg is attached to one end of a light inextensible string of length 0.4 m. The other end is fixed to a point O on a smooth horizontal table. The particle moves on the table in a circle with centre O, with constant angular speed 6 rad/s.

(a) Find the speed of the particle. **(2)**

(b) Find the tension in the string. **(2)**

(c) The string will break if the tension exceeds 20 N. Find the greatest speed at which the particle can move in the circle without the string breaking, to 3 significant figures. **(3)**

(Total for Question 8 is 7 marks)

9 A particle of mass 1 kg is attached by a light inextensible string of length 0.5 m to a fixed point O. The particle moves in a horizontal circle with the string taut, making a constant angle θ with the downward vertical, where $\cos\theta = 5/13$. Take $g = 9.8\ m/s^2$.

(a) Find the tension in the string, to 3 significant figures. **(2)**

(b) Find the radius of the circle and the speed of the particle, to 3 significant figures. **(4)**

(c) Find the angular speed of the particle and the time taken for one complete revolution, to 3 significant figures. **(3)**

(Total for Question 9 is 9 marks)

10 A particle of mass 1 kg is attached to one end of a light inextensible string of length 1 m. The other end of the string is fixed at a point O. The particle is held at rest vertically below O and then projected horizontally with speed u m/s, where $u^2 = 6gr$, so that it moves in a vertical circle. Take $g = 9.8$ m/s^2.

(a) Find the value of u, to 3 significant figures. **(2)**

(b) Find the tension in the string when the particle is at its lowest point, immediately after projection. **(2)**

(c) Find the speed of the particle when it is at the highest point of the circle, and the tension in the string at that point. **(4)**

(Total for Question 10 is 8 marks)

11 A car of mass 800 kg moves at constant speed on a horizontal circular bend of radius 80 m. The coefficient of friction between the tyres and the road is 0.6, and the friction is at its maximum value when the car is at the greatest speed for which it can take the bend without slipping. Model the car as a particle. Take $g = 9.8$ m/s^2.

(a) Write down the greatest value of the sideways friction force that the road can provide. **(2)**

(b) Find the greatest speed at which the car can take the bend without slipping, in m/s and in km/h, to 3 significant figures. **(4)**

(Total for Question 11 is 6 marks)

12 A particle of mass 1 kg is attached to one end of a light inextensible string of length 0.4 m. The other end is fixed to a point O on a smooth horizontal table. The particle moves on the table in a circle with centre O, with constant angular speed 9 rad/s.

(a) Find the speed of the particle. **(2)**

(b) Find the tension in the string. **(2)**

(c) The string will break if the tension exceeds 30 N. Find the greatest speed at which the particle can move in the circle without the string breaking, to 3 significant figures. **(3)**

(Total for Question 12 is 7 marks)

13 A particle of mass 2 kg is attached by a light inextensible string of length 1 m to a fixed point O. The particle moves in a horizontal circle with the string taut, making a constant angle θ with the downward vertical, where $\cos\theta = 3/5$. Take $g = 9.8\ m/s^2$.

(a) Find the tension in the string, to 3 significant figures. **(2)**

(b) Find the radius of the circle and the speed of the particle, to 3 significant figures. **(4)**

(c) Find the angular speed of the particle and the time taken for one complete revolution, to 3 significant figures. **(3)**

(Total for Question 13 is 9 marks)

14 A particle of mass 1 kg is attached to one end of a light inextensible string of length 0.8 m. The other end of the string is fixed at a point O. The particle is held at rest vertically below O and then projected horizontally with speed u m/s, where $u^2 = 10gr$, so that it moves in a vertical circle. Take $g = 9.8$ m/s^2.

(a) Find the value of u, to 3 significant figures. **(2)**

(b) Find the tension in the string when the particle is at its lowest point, immediately after projection. **(2)**

(c) Find the speed of the particle when it is at the highest point of the circle, and the tension in the string at that point. **(4)**

(Total for Question 14 is 8 marks)

15 A particle of mass 1 kg is attached by a light inextensible string of length 1 m to a fixed point O. The particle moves in a horizontal circle with the string taut, making a constant angle θ with the downward vertical, where $\cos\theta = 5/13$. Take $g = 9.8\ m/s^2$.

(a) Find the tension in the string, to 3 significant figures. **(2)**

(b) Find the radius of the circle and the speed of the particle, to 3 significant figures. **(4)**

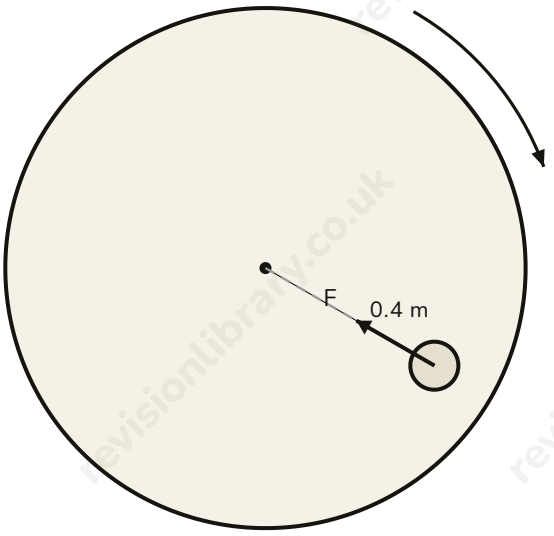
(c) Find the angular speed of the particle and the time taken for one complete revolution, to 3 significant figures. **(3)**

(Total for Question 15 is 9 marks)

16 A wheel rotates at a constant angular speed of 8 rad/s. Find the angular speed of the wheel in revolutions per minute, to the nearest whole number.

(Total for Question 16 is 3 marks)

17 At a funfair in Blackpool, a small 50p coin of mass 0.05 kg rests on a rotating horizontal disc at a distance of 0.4 m from the centre. The coefficient of friction between the coin and the disc is 0.25. The disc's angular speed is slowly increased from rest.



disc viewed from above

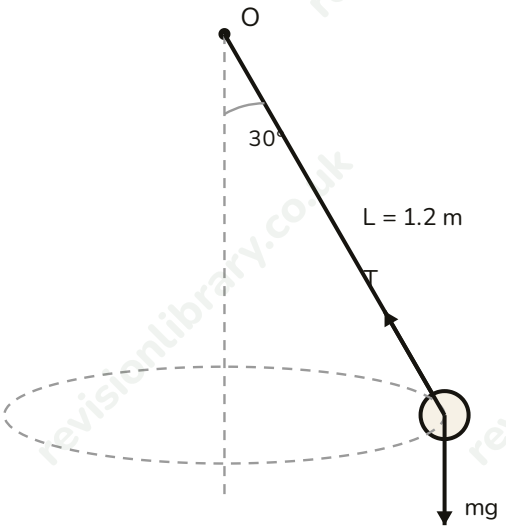
(a) Find the maximum angular speed of the disc for which the coin does not slide. **(4)**

(b) Find the corresponding linear speed of the coin at this angular speed. **(2)**

(c) The coin is now moved to a distance of 0.6 m from the centre of the disc, which continues to rotate at the angular speed found in part (a). Determine, with justification, whether the coin remains in place at this new radius. **(4)**

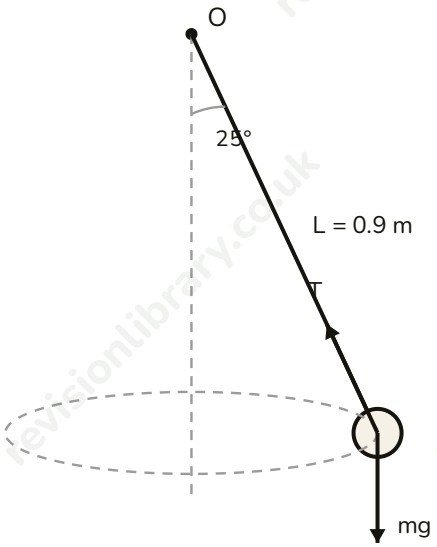
(Total for Question 17 is 10 marks)

18 During a practical, Sofia sets up a conical pendulum using a string of length 1.2 m, with the string making a constant angle of 30 degrees with the vertical. She measures the time for one complete revolution and records a period of 1.6 seconds. Verify whether Sofia's recorded period is consistent with the given length and angle.



(Total for Question 18 is 4 marks)

19 An engineering student, Kwame, models a fairground ride using a conical pendulum: a light inextensible string of length 0.9 m has one end fixed at a point O, and a small ball of mass 0.25 kg is attached to the other end. The ball moves in a horizontal circle at constant speed, with the string making a constant angle of 25 degrees with the vertical.



(a) Show that the tension in the string is awrt 2.70 N. **(3)**

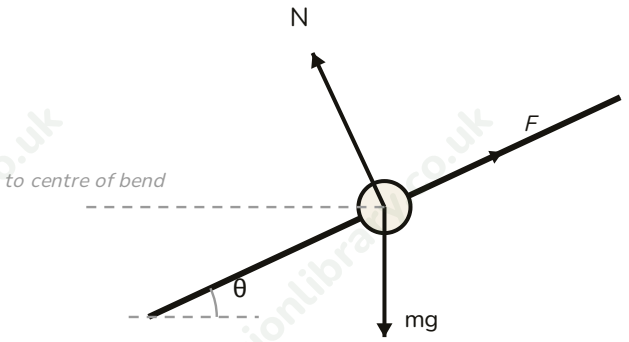
(b) Find the radius of the circle in which the ball moves. **(2)**

(c) Find the speed of the ball. **(4)**

(d) Find the time taken for the ball to complete one full revolution. **(3)**

(Total for Question 19 is 12 marks)

20 A go-kart track designer, Olivia, is designing a bend of radius of curvature 120 m, banked at an angle θ to the horizontal. The bend is designed so that a car travelling at a constant 108 km/h experiences no sideways friction force from the track surface.



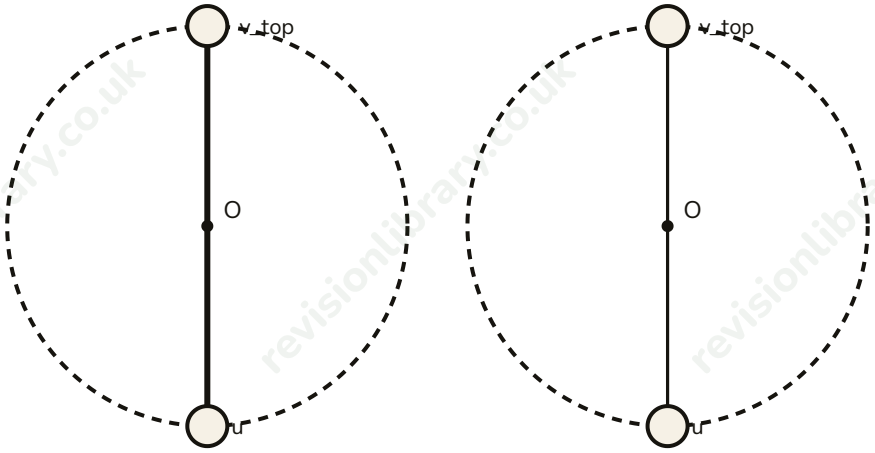
cross-section of the banked bend, radius 120 m

(a) Show that $\tan(\theta) = v^2/(r*g)$, where v is the car's speed and r is the radius of curvature, and hence find θ to 1 decimal place. (Use v = 108 km/h = 30 m/s.) **(4)**

(b) The bend is banked at the angle θ found in part (a). Given that the coefficient of friction between a car's tyres and the track surface is 0.3, find the range of speeds at which a car can travel around the bend without sliding up or down the slope. **(8)**

(Total for Question 20 is 12 marks)

21 In a mechanics practical, Ravi compares two versions of the same apparatus. In each version, a small ball of mass 0.3 kg is attached to the end of a light connecting piece of length 0.8 m, the other end of which is fixed at a point O, and the ball is projected from the lowest point of a vertical circle of radius 0.8 m with speed u. In version 1, the connecting piece is a light rigid rod; in version 2, it is a light inextensible string.



version 1: rigid rod *version 2: string*

(a) Explain why the minimum value of u needed for the ball to complete full circles is different for the rod (version 1) and the string (version 2). **(2)**

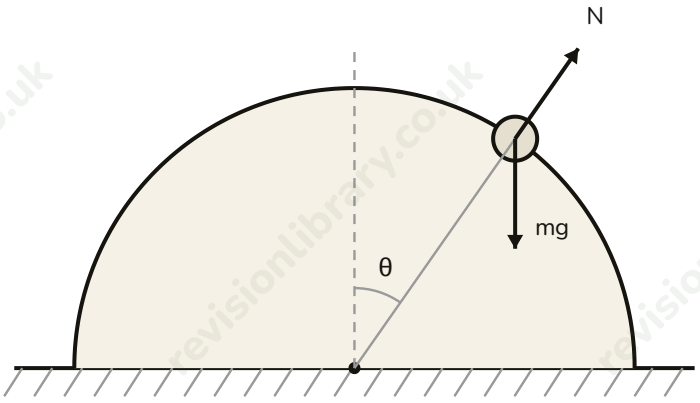
(b) For version 1 (the rigid rod), find the minimum speed of projection, u, at the lowest point for the ball to complete full circles. **(4)**

(c) For version 2 (the string), find the minimum speed of projection, u, at the lowest point for the ball to complete full circles. **(4)**

(d) State the ratio of the minimum value of u^2 for the rod to the minimum value of u^2 for the string, giving your answer in its simplest form. **(1)**

(Total for Question 21 is 11 marks)

22 A smooth solid hemisphere of radius 0.6 m rests with its flat face on horizontal ground. A small particle is placed at the highest point of the hemisphere and given a negligible speed to disturb it from rest, after which it slides on the smooth outer curved surface of the hemisphere. Let θ be the angle between the radius from the centre of the hemisphere to the particle and the upward vertical, and let v be the particle's speed when it has turned through angle θ.



smooth hemisphere, radius 0.6 m

(a) Using conservation of energy, show that $v^2 = 2*g*r*(1-\cos(\theta))$, where r is the radius of the hemisphere. **(3)**

(b) By applying Newton's second law in the radial direction, show that the particle leaves the surface of the hemisphere when $\cos(\theta) = v^2/(g*r)$. **(3)**

(c) Hence show that, at the point where the particle leaves the surface, $\cos(\theta) = 2/3$. **(2)**

(d) Find the value of θ, to 1 decimal place, at which the particle leaves the surface. **(2)**

(e) Find the speed of the particle at the point where it leaves the surface. **(3)**

(Total for Question 22 is 13 marks)

Mark scheme · 3.3 Further Mechanics: Circular Motion

Question 1

(a) uses $\omega = v/r$ **M1**

(a) $\omega = 6$ rad/s, cao **A1**

(a) Answer: $\omega = 6$ rad/s

(b) uses $T = m*v^2/r$ **M1**

(b) T = 3.6 N, cao **A1**

(b) Answer: T = 3.6 N

Question 2

(a) $v = r\omega$ **M1**

(a) 1.5 x 6 = 9.00 m/s **A1**

(a) Answer: 9.00 m/s

(b) $T = m r \omega^2$ **M1**

(b) $1 \times 1.5 \times 6^2 = 54.0$ N **A1**

(b) Answer: 54.0 N

(c) $30 = m v^2/r$ **M1**

(c) $v^2 = 30 \times 1.5$ **A1**

(c) v = 6.71 m/s **A1**

(c) Answer: 6.71 m/s

Question 3

(a) $F_{max} = \mu R$ with $R = mg$ **M1**

(a) 0.4 x 1000 x 9.8 = 3920 N **A1**

(a) Answer: 3920 N

(b) $F = m v^2/r$ with $F = \mu m g$ **M1**

(b) $v^2 = \mu g r = 117.60$ **A1**

(b) v = 10.8 m/s **A1**

(b) 39.0 km/h **A1**

(b) Answer: 10.8 m/s = 39.0 km/h

Question 4

(a) resolves vertically: $T \cos\theta = mg$ **M1**

(a) T = 0.5(9.8)/(3/5) = 8.17 N **A1**

(a) Answer: 8.17 N

(b) radius $r = 1.5 \sin\theta = 1.20$ m **B1**

(b) resolves horizontally: $T \sin\theta = m v^2/r$ **M1**

(b) $v^2 = r g \tan\theta = 15.6800$ **A1**

(b) v = 3.96 m/s **A1**

(b) Answer: r = 1.20 m, v = 3.96 m/s

(c) $\omega = v/r$ **M1**

(c) $\omega = 3.30$ rad/s **A1**

(c) period $= 2\pi/\omega = 1.90$ s **A1**

(c) Answer: $\omega = 3.30$ rad/s, period 1.90 s

Question 5

(a) $v = r\omega$ **M1**

(a) 0.4 x 8 = 3.20 m/s **A1**

(a) Answer: 3.20 m/s

(b) $T = m r \omega^2$ **M1**

(b) $0.2 \times 0.4 \times 8^2 = 5.12$ N **A1**

(b) Answer: 5.12 N

(c) $30 = m v^2/r$ **M1**

(c) $v^2 = 30 \times 0.4/0.2$ **A1**

(c) v = 7.75 m/s **A1**

(c) Answer: 7.75 m/s

Question 6

(a) $F_{max} = \mu R$ with $R = mg$ **M1**

(a) 0.3 x 800 x 9.8 = 2350 N **A1**

(a) Answer: 2350 N

(b) $F = m v^2/r$ with $F = \mu m g$ **M1**

(b) $v^2 = \mu g r = 58.80$ **A1**

(b) v = 7.67 m/s **A1**

(b) 27.6 km/h **A1**

(b) Answer: 7.67 m/s = 27.6 km/h

Question 7

(a) $u^2 = 9 \times 9.8 \times 0.5$ **M1**

(a) u = 6.64 m/s **A1**

(a) Answer: 6.64 m/s

(b) resolves towards the centre: $T - mg = m u^2/r$ **M1**

(b) T = 2(9.8) + 2(9)(9.8) = 196 N **A1**

(b) Answer: 196 N

(c) energy: $(1/2)mv^2 = (1/2)\mu^2 - mg(2r)$ **M1**

(c) $v^2 = (9 - 4)gr$, v = 4.95 m/s **A1**

(c) resolves towards the centre: $T + mg = m v^2/r$ **M1**

(c) T = 78.4 N **A1**

(c) Answer: v = 4.95 m/s, T = 78.4 N

Question 8

(a) $v = r\omega$ **M1**

(a) 0.4 x 6 = 2.40 m/s **A1**

(a) Answer: 2.40 m/s

(b) $T = m r \omega^2$ **M1**

(b) $0.2 \times 0.4 \times 6^2 = 2.88$ N **A1**

(b) Answer: 2.88 N

(c) $20 = m v^2/r$ **M1**

(c) $v^2 = 20 \times 0.4/0.2$ **A1**

(c) v = 6.32 m/s **A1**

(c) Answer: 6.32 m/s

Question 9

(a) resolves vertically: $T \cos \theta = mg$ **M1**

(a) $T = 1(9.8)/(5/13) = 25.5$ N **A1**

(a) Answer: 25.5 N

(b) radius $r = 0.5 \sin \theta = 0.462$ m **B1**

(b) resolves horizontally: $T \sin \theta = m v^2/r$ **M1**

(b) $v^2 = r g \tan \theta = 10.8554$ **A1**

(b) $v = 3.29$ m/s **A1**

(b) Answer: r = 0.462 m, v = 3.29 m/s

(c) $\omega = v/r$ **M1**

(c) $\omega = 7.14$ rad/s **A1**

(c) period $= 2\pi/\omega = 0.880$ s **A1**

(c) Answer: ω = 7.14 rad/s, period 0.880 s

Question 10

(a) $u^2 = 6 \times 9.8 \times 1$ **M1**

(a) $u = 7.67$ m/s **A1**

(a) Answer: 7.67 m/s

(b) resolves towards the centre: $T - mg = m u^2/r$ **M1**

(b) $T = 1(9.8) + 1(6)(9.8) = 68.6$ N **A1**

(b) Answer: 68.6 N

(c) energy: $(1/2)mv^2 = (1/2)\mu^2 - mg(2r)$ **M1**

(c) $v^2 = (6 - 4)gr$, $v = 4.43$ m/s **A1**

(c) resolves towards the centre: $T + mg = m v^2/r$ **M1**

(c) $T = 9.80$ N **A1**

(c) Answer: v = 4.43 m/s, T = 9.80 N

Question 11

(a) $F_{max} = \mu R$ with $R = mg$ **M1**

(a) $0.6 \times 800 \times 9.8 = 4700$ N **A1**

(a) Answer: 4700 N

(b) $F = m v^2/r$ with $F = \mu m g$ **M1**

(b) $v^2 = \mu g r = 470.40$ **A1**

(b) $v = 21.7$ m/s **A1**

(b) 78.1 km/h **A1**

(b) Answer: 21.7 m/s = 78.1 km/h

Question 12

(a) $v = r \omega$ **M1**

(a) $0.4 \times 9 = 3.60$ m/s **A1**

(a) Answer: 3.60 m/s

(b) $T = m r \omega^2$ **M1**

(b) $1 \times 0.4 \times 9^2 = 32.4$ N **A1**

(b) Answer: 32.4 N

(c) $30 = m v^2/r$ **M1**

(c) $v^2 = 30 \times 0.4$ **A1**

(c) $v = 3.46$ m/s **A1**

(c) Answer: 3.46 m/s

Question 13

(a) resolves vertically: $T \cos \theta = mg$ **M1**

(a) $T = 2(9.8)/(3/5) = 32.7$ N **A1**

(a) Answer: 32.7 N

(b) radius $r = 1 \sin \theta = 0.800$ m **B1**

(b) resolves horizontally: $T \sin \theta = m v^2/r$ **M1**

(b) $v^2 = r g \tan \theta = 10.4533$ **A1**

(b) $v = 3.23$ m/s **A1**

(b) Answer: r = 0.800 m, v = 3.23 m/s

(c) $\omega = v/r$ **M1**

(c) $\omega = 4.04$ rad/s **A1**

(c) period $= 2\pi/\omega = 1.55$ s **A1**

(c) Answer: ω = 4.04 rad/s, period 1.55 s

Question 14

(a) $u^2 = 10 \times 9.8 \times 0.8$ **M1**

(a) $u = 8.85$ m/s **A1**

(a) Answer: 8.85 m/s

(b) resolves towards the centre: $T - mg = m u^2/r$ **M1**

(b) $T = 1(9.8) + 1(10)(9.8) = 108$ N **A1**

(b) Answer: 108 N

(c) energy: $(1/2)mv^2 = (1/2)\mu^2 - mg(2r)$ **M1**

(c) $v^2 = (10 - 4)gr$, $v = 6.86$ m/s **A1**

(c) resolves towards the centre: $T + mg = m v^2/r$ **M1**

(c) $T = 49.0$ N **A1**

(c) Answer: v = 6.86 m/s, T = 49.0 N

Question 15

(a) resolves vertically: $T \cos \theta = mg$ **M1**

(a) $T = 1(9.8)/(5/13) = 25.5$ N **A1**

(a) Answer: 25.5 N

(b) radius $r = 1 \sin \theta = 0.923$ m **B1**

(b) resolves horizontally: $T \sin \theta = m v^2/r$ **M1**

(b) $v^2 = r g \tan \theta = 21.7108$ **A1**

(b) $v = 4.66$ m/s **A1**

(b) Answer: r = 0.923 m, v = 4.66 m/s

(c) $\omega = v/r$ **M1**

(c) $\omega = 5.05$ rad/s **A1**

(c) period $= 2\pi/\omega = 1.24$ s **A1**

(c) Answer: ω = 5.05 rad/s, period 1.24 s

Question 16

converts angular speed to revolutions per second: revs per second $= \omega/(2*\pi)$ **M1**

multiplies by 60 to convert to revolutions per minute **M1**

76 revolutions per minute (to the nearest whole number), awrt **A1**

Answer: 76 revolutions per minute (to the nearest whole number)

Question 17

(a) identifies that at maximum angular speed, friction is at its limiting value: $F = \mu*m*g$ **M1**

(a) sets the limiting friction equal to the required centripetal force: $\mu*m*g = m*r*\omega^2$ **M1**

(a) $\omega^2 = \mu*g/r = 6.125$, cao **A1**

(a) $\omega = 2.47$ rad/s, awrt, cao **A1**

(a) Answer: ω = awrt 2.47 rad/s

(b) uses $v = r*\omega$, ft (a) **M1**

(b) v = 0.990 m/s, awrt, cao **A1**

(b) Answer: v = awrt 0.990 m/s

(c) finds the friction force required to keep the coin moving in a circle at radius 0.6 m at this angular speed: F_required = $m*r*\omega^2$, ft **M1**

(c) F_required = 0.184 N, awrt, ft **A1**

(c) finds the maximum friction force available: $F_{max} = \mu*m*g$ **M1**

(c) $F_{max} = 0.1225$ N, and since F_required > F_{max}, the disc cannot supply enough friction, so the coin slides (does not remain in place), cao with correct conclusion **A1**

(c) Answer: The coin slides, since the required friction (awrt 0.184 N) exceeds the maximum available friction (0.1225 N).

Question 18

uses the conical pendulum result $\omega^2 = g/(L*\cos(\theta))$ with L = 1.2 and θ = 30 degrees **M1**

substitutes values correctly and finds the corresponding period, $period = 2*\pi/\omega$ **M1**

period = 2.05 s, awrt **A1**

compares this calculated period with the recorded value of 1.6 s and correctly concludes that Sofia's measurements are not consistent with a half-angle of 30 degrees, since 2.05 s does not equal 1.6 s, oe **A1**

Answer: The theoretical period for a half-angle of 30 degrees is awrt 2.05 s, which does not match Sofia's recorded period of 1.6 s, so the measurements are not consistent.

Question 19

(a) resolves vertically for the ball: $T*\cos(25 \text{ degrees}) = m*g$ **M1**

(a) correct unsimplified expression, $T = (0.25*9.8)/\cos(25 \text{ degrees})$ **A1**

(a) T = 2.70 N, cso **A1**

(a) Answer: T = awrt 2.70 N (shown)

(b) uses $r = L*\sin(25 \text{ degrees})$ with L = 0.9 **M1**

(b) r = 0.380 m, awrt, cao **A1**

(b) Answer: r = awrt 0.380 m

(c) resolves horizontally for the ball: $T*\sin(25 \text{ degrees}) = m*v^2/r$, ft (a) and (b) **M1**

(c) rearranges for v^2, ft **M1**

(c) $v^2 = 1.74$, awrt, ft **A1**

(c) v = 1.32 m/s, awrt, cao **A1**

(c) Answer: v = awrt 1.32 m/s

(d) uses $period = 2*\pi*r/v$, ft (b) and (c) **M1**

(d) substitutes values correctly **M1**

(d) period = 1.81 s, awrt, cao **A1**

(d) Answer: period = awrt 1.81 s

Question 20

(a) resolves vertically: $N*\cos(\theta) = m*g$ **M1**

(a) resolves horizontally (centripetal direction): $N*\sin(\theta) = m*v^2/r$ **M1**

(a) divides the two equations to eliminate N and m, giving $\tan(\theta) = v^2/(r*g)$, cso **A1**

(a) θ = 37.4 degrees, awrt, cao **A1**

(a) Answer: $\tan(\theta) = v^2/(r*g)$ (shown); θ = awrt 37.4 degrees

(b) for the maximum speed (car about to slide up), resolves vertically with friction acting down the slope: $N*\cos(\theta) - \mu*N*\sin(\theta) = m*g$ **M1**

(b) resolves horizontally for the maximum speed: $N*\sin(\theta) + \mu*N*\cos(\theta) = m*v_{max}^2/r$ **M1**

(b) combines to give $v_{max}^2 = r*g*(\sin(\theta)+\mu*\cos(\theta))/(\cos(\theta)-\mu*\sin(\theta))$, oe (e.g. the equivalent tan(θ) form) **A1**

(b) $v_{max} = 40.3$ m/s, awrt, cao **A1**

(b) for the minimum speed (car about to slide down), resolves vertically with friction acting up the slope: $N*\cos(\theta) + \mu*N*\sin(\theta) = m*g$ **M1**

(b) resolves horizontally for the minimum speed: $N*\sin(\theta) - \mu*N*\cos(\theta) = m*v_{min}^2/r$ **M1**

(b) combines to give $v_{min}^2 = r*g*(\sin(\theta)-\mu*\cos(\theta))/(\cos(\theta)+\mu*\sin(\theta))$, oe **A1**

(b) $v_{min} = 21.1$ m/s, awrt, cao **A1**

(b) Answer: 21.1 m/s ≤ v ≤ 40.3 m/s (awrt)

Question 21

(a) a rigid rod can push as well as pull (it can act in compression as well as tension), so it can supply the centripetal force required at the top of the circle even when the ball's speed there is very small (or zero), oe **B1**

(a) a string can only pull (it can only be in tension), so if the string is to remain taut the weight of the ball alone must not exceed the centripetal force required at the top; this requires a strictly positive minimum speed at the top, oe **B1**

(a) Answer: A rod can push or pull, so it can maintain the circular path even when the ball's speed at the top is zero; a string can only pull, so it would go slack unless the ball has enough speed at the top for its weight alone to provide the required centripetal force.

(b) uses conservation of energy between the bottom and top of the circle: $(1/2)*u^2 = (1/2)*v_{top}^2 + g*(2*r)$ **M1**

(b) uses the critical condition for a rod, $v_{top} = 0$, since the rod can maintain the circular path down to zero speed at the top **M1**

(b) $u^2 = 4*g*r = 31.36$, ft **A1**

(b) u = 5.6 m/s, cao **A1**

(b) Answer: u = 5.6 m/s

(c) uses the critical condition for a string, $v_{top}^2 = g*r$ (tension zero at the top) **M1**

(c) uses conservation of energy: $u^2 = v_{top}^2 + 4*g*r$ **M1**

(c) $u^2 = 5*g*r = 39.2$, ft **A1**

(c) u = 6.26 m/s, awrt, cao **A1**

(c) Answer: u = awrt 6.26 m/s

(d) ratio = 4:5, cao (independent of the values of g and r, since the ratio is $4*g*r : 5*g*r$) **B1**

(d) Answer: 4:5

Question 22

(a) finds the vertical height fallen when the particle has turned through angle θ: h = r*(1-cos(θ)) **M1**

(a) uses conservation of energy: $(1/2)*m*v^2 - 0 = m*g*r*(1-\cos(\theta))$ **M1**

(a) $v^2 = 2*g*r*(1-\cos(\theta))$, cso **A1**

(a) Answer: $v^2 = 2*g*r*(1-\cos(\theta))$ (shown)

(b) applies Newton's second law towards the centre: $m*g*\cos(\theta) - N = m*v^2/r$ **M1**

(b) uses the condition that the particle leaves the surface when the normal reaction N = 0 **M1**

(b) $\cos(\theta) = v^2/(g*r)$, cso **A1**

(b) Answer: $\cos(\theta) = v^2/(g*r)$ (shown)

(c) substitutes $v^2 = 2*g*r*(1-\cos(\theta))$ from part (a) into $\cos(\theta) = v^2/(g*r)$ from part (b) **M1**

(c) rearranges $3*\cos(\theta) = 2$ to give $\cos(\theta) = 2/3$, cso **A1**

(c) Answer: $\cos(\theta) = 2/3$ (shown)

(d) uses θ = arccos(2/3), ft (c) **M1**

(d) θ = 48.2 degrees, awrt, cao **A1**

(d) Answer: θ = awrt 48.2 degrees

(e) substitutes $\cos(\theta) = 2/3$ into $v^2 = 2*g*r*(1-\cos(\theta))$, ft (a) and (c) **M1**

(e) $v^2 = (2/3)*g*r = 3.92$, cao **A1**

(e) v = 1.98 m/s, awrt, cao **A1**

(e) Answer: v = awrt 1.98 m/s