3.4 Further Mechanics: Elastic Strings, Springs and Elastic Energy: 1-hour Pack

AQA 7367 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Formulae and modelling assumptions used in this worksheet

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Unless a question states otherwise, take g = 9.8 m s^-2 and model each object as a particle, and each string or spring as light. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when it is extended (or compressed) by x, has magnitude T = lambda*x/l; an elastic string can only pull (it exerts no force, and is described as slack, whenever its length is less than l), while a spring can both pull and push. The elastic potential energy (EPE) stored in a stretched or compressed string or spring is EPE = lambda*x^2/(2*l). The principle of conservation of mechanical energy states that, provided no energy is lost to resistances such as friction, the sum of the kinetic energy, gravitational potential energy and elastic potential energy of a system remains constant; more generally, the work-energy principle states that the total work done by all the forces acting on a particle (including any work done against resistances) equals its change in kinetic energy.

1 A particle of mass 2 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N, the other end of the string being attached to a fixed point O. Take g = 9.8 m/s^2.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.2 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 1 is 7 marks)

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2 A particle of mass 3 kg hangs in equilibrium at the end of a light elastic string of natural length 0.8 m and modulus of elasticity 60 N, the other end of the string being attached to a fixed point O. Take $g = 9.8\ m/s^2$.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

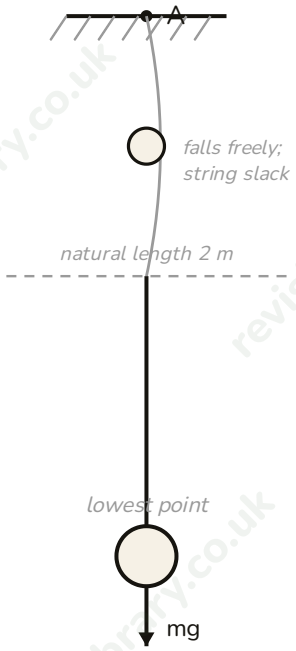
(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.05 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 2 is 7 marks)

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3 A particle P of mass 5 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 588 N. The other end of the string is attached to a fixed point A. Initially P is held at the same level as A and is released from rest, falling vertically downwards in a straight line.



(a) Explain why the string does not begin to exert any force on P until P has fallen a distance of 2 m. **(1)**

(b) Find the speed of P at the instant the string becomes taut. **(3)**

(c) Show that the maximum extension of the string is 1 m. **(6)**

(d) Find the maximum tension in the string during the motion. **(2)**

(Total for Question 3 is 12 marks)

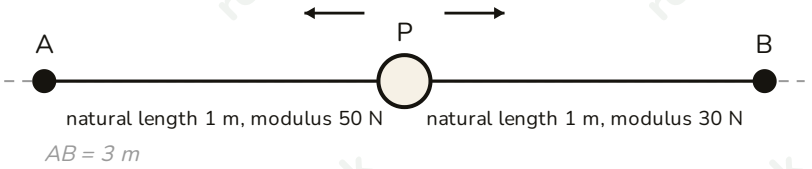
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4 The tension in a light elastic string or spring of natural length l and modulus of elasticity λ, when extended (or compressed) by an amount s, is $T = \lambda * s/l$. Starting from the fact that the work done in stretching the string by a further small amount ds against this tension is T ds, use integration to show that the elastic potential energy stored when the total extension is x is $EPE = \lambda * x^2/(2*l)$.

(Total for Question 4 is 4 marks)

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5 A particle P of mass 1.2 kg rests in equilibrium on a smooth horizontal table. P is connected to two fixed points, A and B, on the table, with A, P and B lying in a straight line and AB = 3 m. P is attached to A by a light elastic string of natural length 1 m and modulus of elasticity 50 N, and to B by a second light elastic string of natural length 1 m and modulus of elasticity 30 N. Both strings are taut.



(a) Using the fact that AB = 3 m, write down an equation connecting the extension x_A of string AP and the extension x_B of string PB. **(2)**

(b) By equating the tensions in the two strings, find the values of x_A and x_B. **(5)**

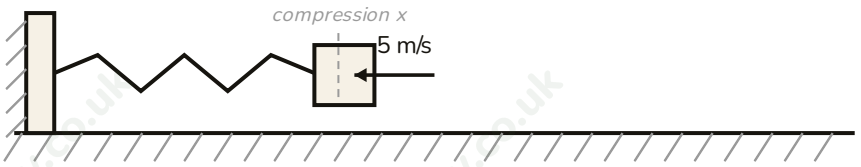
(c) Find the tension in each string. **(1)**

(d) Explain why the mass of P does not appear anywhere in the working used to find x_A and x_B. **(2)**

(Total for Question 5 is 10 marks)

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6 A block of mass 0.8 kg is moving at 5 m/s on a rough horizontal floor when it strikes one end of a light spring of natural length 1 m and modulus of elasticity 200 N, whose other end is fixed to a wall. The coefficient of friction between the block and the floor is 0.2, and this friction acts throughout the block's contact with the spring.



(a) Show that the maximum compression x of the spring satisfies the equation $100*x^2 + 1.568*x - 10 = 0$. **(4)**

(b) Solve this equation to find the maximum compression of the spring, giving your answer to 3 significant figures. **(2)**

(c) Given that the same frictional force acts over the same distance as the block moves back from the point of maximum compression to the point where the spring returns to its natural length, find the speed of the block at that point. **(4)**

(d) State one modelling assumption, other than the coefficient of friction being constant, that has been made in this problem, and briefly explain how it might affect the accuracy of the calculated speeds. **(2)**

(Total for Question 6 is 12 marks)

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Mark scheme · 3.4 Further Mechanics: Elastic Strings, Springs and Elastic Energy: 1-hour Pack

Question 1

(a) Hooke's law: $T = \lambda x/l = mg$ **M1**

(a) $x = 2(9.8)(0.4)/20 = 0.392$ m **A1**

(a) Answer: 0.392 m

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $20(0.3920)^2/(2 \times 0.4) = 3.84$ J **A1**

(b) Answer: 3.84 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.5920)^2 - (0.3920)^2]/(2l) - mg(0.2)$ **M1**

(c) $v = 1.00$ m/s **A1**

(c) Answer: 1.00 m/s

Question 2

(a) Hooke's law: $T = \lambda x/l = mg$ **M1**

(a) $x = 3(9.8)(0.8)/60 = 0.392$ m **A1**

(a) Answer: 0.392 m

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $60(0.3920)^2/(2 \times 0.8) = 5.76$ J **A1**

(b) Answer: 5.76 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.4420)^2 - (0.3920)^2]/(2l) - mg(0.05)$ **M1**

(c) $v = 0.250$ m/s **A1**

(c) Answer: 0.250 m/s

Question 3

(a) an elastic string can only pull (exert tension) and cannot push, so it is slack and exerts no force while its length is less than its natural length of 2 m, oe **B1**

(a) Answer: The string is slack (and so exerts no tension) until P has fallen its natural length of 2 m, since a string cannot push.

(b) uses $v^2 = u^2 + 2*a*s$ with $u = 0$, $a = g = 9.8$, $s = 2$ **M1**

(b) $v^2 = 39.2$ **A1**

(b) $v = 6.26$ m/s, awrt (or exact $\sqrt{39.2}$) **A1**

(b) Answer: v = awrt 6.26 m/s

(c) uses conservation of energy from release to the lowest point (speed = 0 at both ends): loss in GPE = EPE gained **M1**

(c) correct equation: $mg*(2+x) = 588*x^2/(2*2)$, i.e. $49*(2+x) = 147*x^2$ **A1**

(c) rearranges to a 3-term quadratic: $147*x^2 - 49*x - 98 = 0$ (or a simplified equivalent, e.g. $3*x^2 - x - 2 = 0$) **M1**

(c) solves the quadratic (by factorising or the quadratic formula) **M1**

(c) rejects the negative root $x = -2/3$ as not physically valid **A1**

(c) $x = 1$ m, cso **A1**

(c) Answer: Maximum extension = 1 m (shown)

(d) uses $T = \lambda*x/l$ at maximum extension, ft (c) **M1**

(d) $T = 294$ N, cao **A1**

(d) Answer: T = 294 N

Question 4

sets up EPE = integral from 0 to x of T ds = integral from 0 to x of $(\lambda*s/l)$ ds **M1**

integrates correctly: $(\lambda/l)*[s^2/2]$ (+ c) **M1**

substitutes limits: $(\lambda/(2*l))*x^2 - (\lambda/(2*l))*0^2$ **A1**

EPE = $\lambda*x^2/(2*l)$, cso **A1**

Answer: EPE = $\lambda*x^2/(2*l)$ (shown)

Question 5

(a) recognises that the sum of the natural lengths is 1 + 1 = 2 m **B1**

(a) $x_A + x_B = 3 - 2 = 1$ (m), oe **B1**

(a) Answer: $x_A + x_B = 1$

(b) writes tension in each string using Hooke's law: $T_A = 50*x_A/1$, $T_B = 30*x_B/1$ **M1**

(b) equates the tensions (P is in equilibrium with no other horizontal forces): $50*x_A = 30*x_B$ **M1**

(b) solves simultaneously with $x_A + x_B = 1$ from (a), ft **M1**

(b) $x_A = 0.375$ m, cao **A1**

(b) $x_B = 0.625$ m, cao **A1**

(b) Answer: $x_A = 0.375$ m, $x_B = 0.625$ m

(c) T = 18.75 N (both strings), ft (b) **B1**

(c) Answer: T = 18.75 N

(d) the table is smooth and horizontal, so P's weight is balanced vertically by the normal reaction from the table, and does not act along the line AB **B1**

(d) the only horizontal forces on P are the two tensions, so the equilibrium equation $50*x_A = 30*x_B$ involves only the string properties, not the mass, oe **B1**

(d) Answer: The weight of P acts vertically and is balanced by the normal reaction from the table; the only forces acting along AB are the two tensions, so the horizontal equilibrium equation does not involve P's mass.

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Question 6

(a) initial KE = $(1/2)*0.8*5^2$ = 10 (J) **M1**

(a) EPE stored at compression x = $200*x^2/(2*1) = 100*x^2$ **M1**

(a) work done against friction over compression x = $\mu*m*g*x = 0.2*0.8*9.8*x = 1.568*x$ **M1**

(a) applies the work-energy principle (KE = EPE + friction work) and rearranges to the given equation, cso **A1**

(a) Answer: $100*x^2 + 1.568*x - 10 = 0$ (shown)

(b) uses the quadratic formula (or equivalent method) to solve, ft (a) **M1**

(b) x = 0.308 m, awrt, cao **A1**

(b) Answer: x = awrt 0.308 m

(c) EPE released = $100*x^2$, ft (b) **M1**

(c) KE on leaving spring = (their EPE) - (friction work over x = 1.568*x), ft **M1**

(c) rearranges $(1/2)*0.8*v^2$ = (their KE) for v **M1**

(c) v = 4.75 m/s, awrt, cao **A1**

(c) Answer: v = awrt 4.75 m/s

(d) valid assumption, e.g. the spring is modelled as light (massless), so its own kinetic energy during compression/extension is ignored **B1**

(d) explains the likely effect, e.g. if the spring has mass, some of the initial kinetic energy would be transferred to the spring itself, so the calculated speeds would be slightly too high, oe **B1**

(d) Answer: The spring is modelled as light; if it actually has mass, some energy would go into moving the spring itself, so the true final speed would be slightly lower than calculated.

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