3.4 Further Mechanics: Elastic Strings, Springs and Elastic Energy: 30-minute Pack

AQA 7367 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Formulae and modelling assumptions used in this worksheet

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Unless a question states otherwise, take g = 9.8 m s^-2 and model each object as a particle, and each string or spring as light. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when it is extended (or compressed) by x, has magnitude T = lambda*x/l; an elastic string can only pull (it exerts no force, and is described as slack, whenever its length is less than l), while a spring can both pull and push. The elastic potential energy (EPE) stored in a stretched or compressed string or spring is EPE = lambda*x^2/(2*l). The principle of conservation of mechanical energy states that, provided no energy is lost to resistances such as friction, the sum of the kinetic energy, gravitational potential energy and elastic potential energy of a system remains constant; more generally, the work-energy principle states that the total work done by all the forces acting on a particle (including any work done against resistances) equals its change in kinetic energy.

1 A particle P of mass 0.5 kg lies on a smooth horizontal table. It is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 20 N. The other end of the string is fixed to a point O on the table. P is held at a distance 1.2 m from O and released from rest.

(a) Find the tension in the string, and the initial acceleration of P. **(3)**

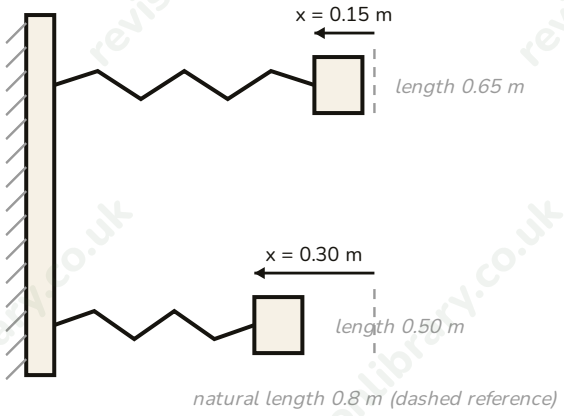
(b) Find the elastic potential energy stored in the string initially. **(2)**

(c) Find the speed of P at the instant when the string becomes slack. **(3)**

(Total for Question 1 is 8 marks)

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2 A light spring has natural length 0.8 m and modulus of elasticity 60 N. It is compressed, first to a length of 0.65 m, and then further to a length of 0.50 m.



(a) Find the elastic potential energy (EPE) stored in the spring when it is compressed to a length of 0.65 m. **(3)**

(b) Find the EPE stored in the spring when it is compressed further, to a length of 0.50 m. **(2)**

(c) Hence find the additional work done in compressing the spring from a length of 0.65 m to a length of 0.50 m. **(3)**

(Total for Question 2 is 8 marks)

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3 A fitness website advertises a resistance band, modelled as a light elastic string, with natural length 0.5 m and modulus of elasticity 80 N. The advert claims that when the band is stretched to a length of 0.75 m, the elastic potential energy stored in it is 5 J. Verify, showing your working clearly, whether this claim is correct.

(Total for Question 3 is 4 marks)

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4 Aisha claims that, for a given light elastic string, doubling the extension (while keeping the modulus of elasticity and natural length unchanged) will double the elastic potential energy stored in the string.

(a) By considering the formula for elastic potential energy, explain algebraically whether Aisha's claim is correct. **(3)**

(b) Illustrate this result using a string of natural length 1.2 m and modulus of elasticity 60 N, by calculating the EPE stored for extensions of 0.2 m and 0.4 m. **(3)**

(Total for Question 4 is 6 marks)

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Mark scheme · 3.4 Further Mechanics: Elastic Strings, Springs and Elastic Energy: 30-minute Pack

Question 1

(a) $T = \lambda x/l$ **M1**

(a) $T = 20 \times 0.4/0.8 = 10.0$ N **A1**

(a) $a = T/m = 20.0$ m/s^2 **A1**

(a) Answer: T = 10.0 N, a = 20.0 m/s^2

(b) $EPE = \lambda x^2/(2l)$ **M1**

(b) $20(0.4)^2/(2 \times 0.8) = 2.00$ J **A1**

(b) Answer: 2.00 J

(c) conservation of energy: EPE = KE **M1**

(c) $(1/2)(0.5)v^2 = 2.0000$ **A1**

(c) $v = 2.83$ m/s **A1**

(c) Answer: 2.83 m/s

Question 2

(a) uses $EPE = \lambda * x^2/(2*l)$ with $x = 0.8 - 0.65 = 0.15$ (m) **B1**

(a) substitutes correctly: $60*0.15^2/(2*0.8)$ **M1**

(a) EPE = awrt 0.844 J (exact value 0.84375 J), cao **A1**

(a) Answer: EPE = awrt 0.844 J (exact 0.84375 J)

(b) substitutes $x = 0.8 - 0.5 = 0.3$ into $EPE = \lambda * x^2/(2*l)$ **M1**

(b) EPE = 3.375 J, cao **A1**

(b) Answer: EPE = 3.375 J

(c) recognises the additional work done equals the difference in EPE values, ft (a) and (b) **M1**

(c) 3.375 - 0.84375, oe, ft **A1**

(c) awrt 2.53 J (exact 2.53125 J), cao **A1**

(c) Answer: Additional work done = awrt 2.53 J (exact 2.53125 J)

Question 3

finds the extension $x = 0.75 - 0.5 = 0.25$ (m) **M1**

substitutes into $EPE = \lambda * x^2/(2*l)$: $80*0.25^2/(2*0.5)$ **M1**

EPE = 5 J, cao **A1**

concludes the claim is correct, since the calculated value equals the advertised 5 J, cso **A1**

Answer: EPE = 5 J; the claim is correct.

Question 4

(a) states $EPE = \lambda * x^2/(2*l)$, so EPE is proportional to x^2 for fixed λ and l **M1**

(a) if x is doubled, EPE increases by a factor of $2^2 = 4$, not 2 **A1**

(a) concludes Aisha's claim is incorrect, oe **A1**

(a) Answer: Aisha's claim is incorrect: since EPE is proportional to x^2, doubling x quadruples the EPE, rather than doubling it.

(b) substitutes $x = 0.2$: $EPE = 60*0.2^2/(2*1.2)$ **M1**

(b) EPE = 1 J, cao **A1**

(b) substitutes $x = 0.4$ to find EPE = 4 J, confirming the factor of 4 (not 2), cao **A1**

(b) Answer: EPE(0.2 m) = 1 J; EPE(0.4 m) = 4 J (a factor of 4, confirming part (a))

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