3.4 Further Mechanics: Elastic Strings, Springs and Elastic Energy: Test Pack

AQA 7367 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

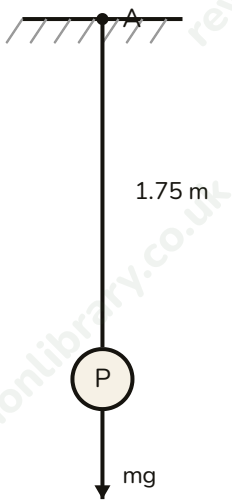
Formulae and modelling assumptions used in this worksheet

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Unless a question states otherwise, take g = 9.8 m s^-2 and model each object as a particle, and each string or spring as light. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when it is extended (or compressed) by x, has magnitude T = lambda*x/l; an elastic string can only pull (it exerts no force, and is described as slack, whenever its length is less than l), while a spring can both pull and push. The elastic potential energy (EPE) stored in a stretched or compressed string or spring is EPE = lambda*x^2/(2*l). The principle of conservation of mechanical energy states that, provided no energy is lost to resistances such as friction, the sum of the kinetic energy, gravitational potential energy and elastic potential energy of a system remains constant; more generally, the work-energy principle states that the total work done by all the forces acting on a particle (including any work done against resistances) equals its change in kinetic energy.

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1 One end of a light elastic string of natural length 1.4 m is attached to a fixed point A. A particle P is attached to the other end of the string and hangs in equilibrium vertically below A, with the string stretched to a length of 1.75 m. The modulus of elasticity of the string is 98 N.



(a) Calculate the tension in the string. **(2)**

(b) Given that P is in equilibrium, find the mass of P. **(2)**

(Total for Question 1 is 4 marks)

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2 A particle of mass 2 kg hangs in equilibrium at the end of a light elastic string of natural length 0.5 m and modulus of elasticity 30 N, the other end of the string being attached to a fixed point O. Take $g = 9.8\ m/s^2$.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

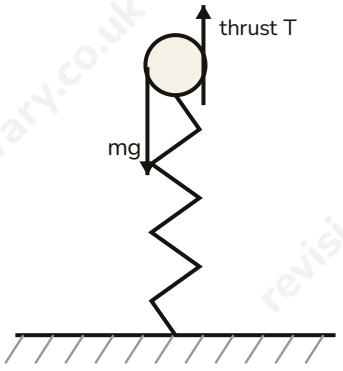
(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.2 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 2 is 7 marks)

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3 A particle of mass 2 kg rests in equilibrium on top of a light vertical spring of natural length 0.6 m and modulus of elasticity 147 N, which stands on horizontal ground.



natural length 0.6 m

(a) Find the compression of the spring. **(3)**

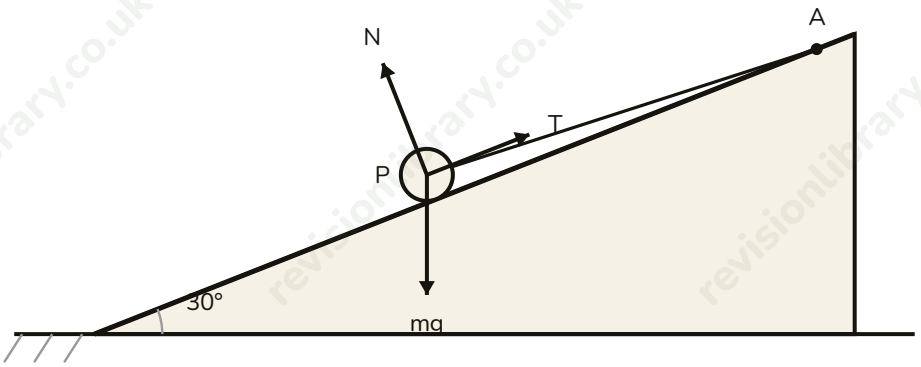
(b) Find the length of the spring in this position. **(1)**

(c) State one assumption made about the spring in this model, and briefly explain why it is needed. **(1)**

(Total for Question 3 is 5 marks)

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4 A particle P of mass 4 kg rests in equilibrium on a smooth plane inclined at 30 degrees to the horizontal. P is attached to one end of a light elastic string of natural length 1.0 m, lying along a line of greatest slope, whose other end is fixed to a point A at the top of the plane. The string is stretched to a length of 1.2 m.



string natural length 1.0 m, stretched to 1.2 m

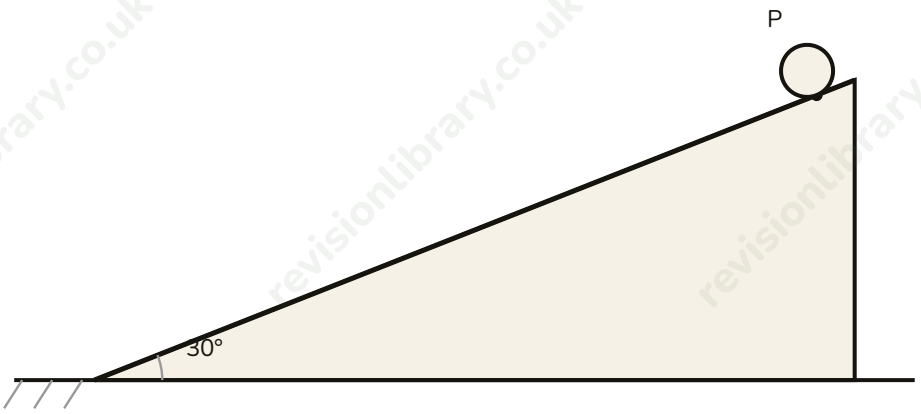
(a) Show that the modulus of elasticity of the string is 98 N. **(4)**

(b) The particle P is now replaced by a different particle, of mass 6 kg, attached to the same string. Find the new extension of the string when the system is again in equilibrium. **(3)**

(Total for Question 4 is 7 marks)

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5 A particle P of mass 2 kg lies on a smooth plane inclined at 30 degrees to the horizontal. P is attached to one end of a light elastic string of natural length 1.6 m and modulus of elasticity 120 N, lying along a line of greatest slope; the other end of the string is fixed to a point A at the top of the plane. Initially P is held at A (so the string is not stretched) and is released from rest, sliding down the line of greatest slope.



P released from rest at A; string is slack until P has slid 1.6 m

(a) Find the speed of P at the instant the string becomes taut. **(3)**

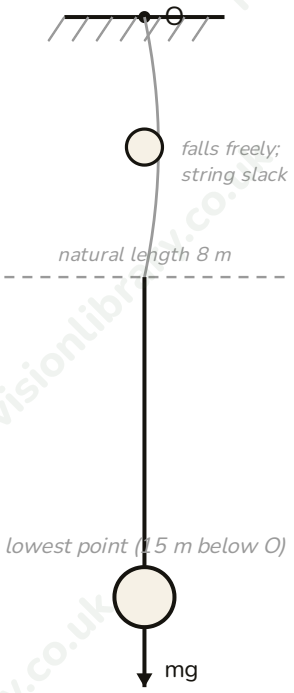
(b) Show that the maximum extension x of the string satisfies the equation $37.5*x^2 - 9.8*x - 15.68 = 0$, and hence find x, giving your answer to 3 significant figures. **(6)**

(c) Find the maximum tension in the string during the motion. **(3)**

(Total for Question 5 is 12 marks)

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6 A bungee-style fairground ride uses a ball of mass 4 kg attached to one end of a long light elastic string of natural length 8 m. The other end of the string is fixed to a point O at the top of a tower. The ball is released from rest at O and falls vertically. The ball comes to instantaneous rest at its lowest point, having fallen a total distance of 15 m below O.



(a) Find the speed of the ball at the instant the string becomes taut. **(3)**

(b) Given that the ball falls a total distance of 15 m before coming to instantaneous rest, find the modulus of elasticity of the string. **(5)**

(c) Find the maximum tension in the string during the motion, and state at what point of the motion this maximum occurs. **(3)**

(d) The ball subsequently moves back upwards. State, with a reason, the condition needed for this subsequent motion to be modelled using simple harmonic motion, and explain why this model would not apply for the whole of the return journey. **(2)**

(Total for Question 6 is 13 marks)

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Mark scheme · 3.4 Further Mechanics: Elastic Strings, Springs and Elastic Energy: Test Pack

Question 1

(a) uses Hooke's law T = λ*x/l with x = 1.75 - 1.4 = 0.35 (m) **M1**

(a) T = 24.5 N, cao **A1**

(a) Answer: T = 24.5 N

(b) equates the tension to the weight: mg = T, ft (a) **M1**

(b) m = 2.5 kg, cao **A1**

(b) Answer: m = 2.5 kg

Question 2

(a) Hooke's law: T = λ x/l = mg **M1**

(a) x = 2(9.8)(0.5)/30 = 0.327 m **A1**

(a) Answer: 0.327 m

(b) EPE = λ x^2/(2l) **M1**

(b) $30(0.3267)^2$/(2 x 0.5) = 3.20 J **A1**

(b) Answer: 3.20 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.5267)^2 - (0.3267)^2]/(2l) - mg(0.2)$ **M1**

(c) v = 1.10 m/s **A1**

(c) Answer: 1.10 m/s

Question 3

(a) at equilibrium, thrust in spring equals weight of particle: T = mg = 2*9.8 **M1**

(a) uses Hooke's law T = λ*x/l, rearranged for x **M1**

(a) x = 0.08 m, cao **A1**

(a) Answer: x = 0.08 m

(b) length = 0.6 - 0.08 = 0.52 m, ft (a) **B1**

(b) Answer: 0.52 m

(c) valid assumption with reason, e.g. the spring is modelled as light (of negligible mass), so its own weight does not need to be included in the equilibrium equation; or the spring has not exceeded its elastic limit, so T = λ*x/l remains valid, oe **B1**

(c) Answer: The spring is modelled as light, so its own weight does not contribute to the equilibrium equation.

Question 4

(a) resolves forces along the plane at equilibrium: T = mg*sin(30 degrees) **M1**

(a) T = 19.6 N, cao **A1**

(a) uses Hooke's law rearranged for λ: λ = T*l/x, with x = 1.2 - 1.0 = 0.2 **M1**

(a) λ = 98 N, cso **A1**

(a) Answer: λ = 98 N (shown)

(b) new tension T = 6*9.8*sin(30 degrees) **M1**

(b) uses Hooke's law rearranged for x: x = T*l/λ, ft (a) **M1**

(b) x = 0.3 m, cao **A1**

(b) Answer: x = 0.3 m

Question 5

(a) uses $v^2 = 2*(g*\sin(30 \text{ degrees}))*1.6$, or an equivalent energy method **M1**

(a) $v^2 = 15.68$ **A1**

(a) v = 3.96 m/s, awrt **A1**

(a) Answer: v = awrt 3.96 m/s

(b) uses conservation of energy from release to the point of maximum extension (speed = 0 at both ends): loss in GPE = EPE gained **M1**

(b) correct equation: $2*9.8*\sin(30 \text{ degrees})*(1.6+x) = 120*x^2/(2*1.6)$, i.e. $9.8*(1.6+x) = 37.5*x^2$ **A1**

(b) rearranges to the given quadratic form **M1**

(b) solves using the quadratic formula **M1**

(b) rejects the negative root as not physically valid **B1**

(b) x = 0.790 m, awrt, cao **A1**

(b) Answer: x = awrt 0.790 m

(c) uses T = λ*x/l, ft (b) **M1**

(c) substitutes λ = 120, l = 1.6, x from (b) **M1**

(c) T = 59.3 N, awrt, cao **A1**

(c) Answer: T = awrt 59.3 N

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Question 6

(a) uses $v^2 = 2*g*8$ (free fall over the natural length before the string becomes taut) **M1**

(a) $v^2 = 156.8$ **A1**

(a) v = 12.5 m/s, awrt **A1**

(a) Answer: v = awrt 12.5 m/s

(b) identifies the extension of the string at the lowest point: x = 15 - 8 = 7 (m) **M1**

(b) uses conservation of energy over the whole fall (speed = 0 at O and at the lowest point): loss in GPE = EPE gained **M1**

(b) correct equation: $4*9.8*15 = \lambda*7^2/(2*8)$ **A1**

(b) rearranges for λ **M1**

(b) λ = 192 N, cao **A1**

(b) Answer: λ = 192 N

(c) uses $T = \lambda*x/l$ with x = 7, ft (b) **M1**

(c) T = 168 N, cao **A1**

(c) occurs at the lowest point of the motion (i.e. at the point of maximum extension), oe **B1**

(c) Answer: T_{max} = 168 N, occurring at the lowest point (maximum extension).

(d) SHM requires the string to remain taut throughout, i.e. its extension $x \geq 0$, so that the restoring force is proportional to the displacement from the equilibrium position at all times considered **B1**

(d) once the ball rises above the point where the string is at its natural length, the string would need to push the ball to continue the SHM model, but a string cannot exert thrust, so it goes slack and the SHM model breaks down beyond this point, oe **B1**

(d) Answer: SHM requires the string to stay taut (extension $x \geq 0$); once the ball rises past the natural-length point the string would need to push, which it cannot do, so it goes slack and the SHM model no longer applies.

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