3.5 Further Mechanics: Momentum, Impulse and Collisions Depth: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

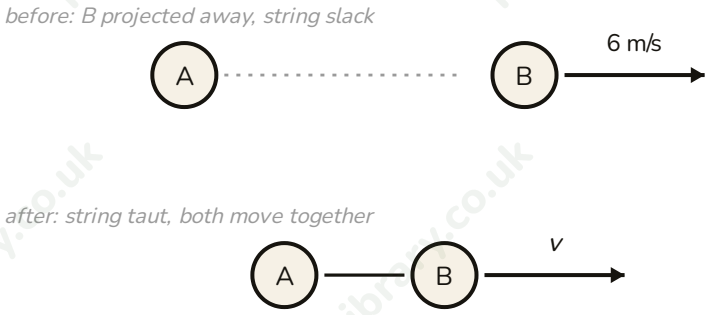
Key results: momentum, impulse and Newton's law of restitution

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and take all strings as light and inextensible, all pulleys and surfaces as smooth, and all spheres as smooth (so there is no friction, and no tangential impulse, at any point of contact). The impulse-momentum principle states that the impulse J exerted on a particle of mass m equals its change in momentum, J = m*(v - u); when a force F varies with time t, the impulse delivered over an interval is the integral of F with respect to t over that interval. Provided no external horizontal force acts on a system of colliding or jerked particles, total momentum is conserved: m1*u1 + m2*u2 = m1*v1 + m2*v2, with velocities signed along the line of centres (or along the string). Newton's experimental law of restitution states that the speed of separation is e times the speed of approach, v2 - v1 = -e*(u2 - u1), where 0 <= e <= 1 is the coefficient of restitution between the two surfaces; e = 1 gives a perfectly elastic collision (no kinetic energy lost) and e = 0 gives a perfectly inelastic collision (the particles coalesce, or move off together immediately after a string jerks taut). The loss in kinetic energy due to a direct collision is m1*m2*(1-e^2)*(u1-u2)^2 / (2*(m1+m2)). When a sphere strikes a fixed smooth surface obliquely, the component of its velocity parallel to the surface is unchanged by the impact, while the component perpendicular to the surface obeys Newton's law of restitution.

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1 Two particles, A of mass 3 kg and B of mass 2 kg, lie at rest on a smooth horizontal plane, connected by a light inextensible string which is initially slack. Particle B is then projected directly away from A with speed 6 m/s, and travels in a straight line until the string becomes taut.



(a) Find the common speed of A and B immediately after the string becomes taut. **(3)**

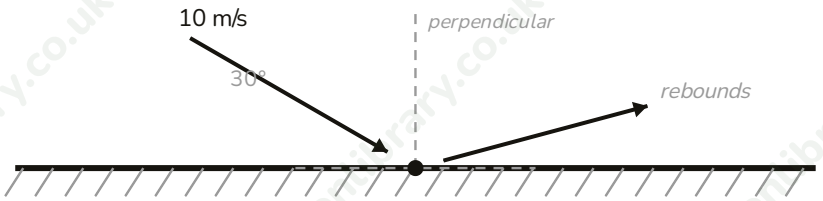
(b) Find the magnitude of the impulsive tension in the string as it jerks taut. **(3)**

(c) Show that kinetic energy is lost when the string jerks taut, and find this loss. **(3)**

(Total for Question 1 is 9 marks)

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2 A smooth sphere strikes a fixed smooth plane with speed 10 m/s, travelling at an angle of 30 degrees to the plane, and rebounds. The coefficient of restitution between the sphere and the plane is e = 0.6.



(a) Find the components of the sphere's velocity, resolved perpendicular and parallel to the plane, immediately before impact. **(3)**

(b) State the parallel component of velocity immediately after impact, giving a reason, and find the perpendicular component of velocity immediately after impact. **(3)**

(c) Find the speed of the sphere, and the angle its path makes with the plane, immediately after impact, giving each answer to 3 significant figures. **(4)**

(d) Given that the mass of the sphere is 0.3 kg, find the magnitude of the impulse exerted on the sphere by the plane during the impact. **(3)**

(Total for Question 2 is 13 marks)

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3 Three smooth spheres A, B and C, of masses 2 kg, 3 kg and m kg respectively, lie at rest in a straight line on a smooth horizontal table, with B between A and C. Sphere A is projected towards B with speed 8 m/s. The coefficient of restitution between A and B is 0.5, and the coefficient of restitution between B and C is 0.75.



(a) Find the velocities of A and B immediately after their collision. **(5)**

(b) Sphere B then collides directly with sphere C, which is also initially at rest. Given that B's velocity immediately after this second collision is 1.44 m/s (still in A's original direction of motion), find the value of m. **(5)**

(c) Determine, with justification, whether any further collisions occur between the three spheres. **(3)**

(Total for Question 3 is 13 marks)

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4 A small ball of mass 0.2 kg hits a smooth fixed vertical wall at right angles with speed 12 m/s. The coefficient of restitution between the ball and the wall is 3/4.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 2 m/s. **(4)**

(Total for Question 4 is 10 marks)

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5 A particle P of mass 5 kg is moving with speed 5 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 8 kg, which is moving along the same line with velocity -7 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.1 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 5 is 10 marks)

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Mark scheme · 3.5 Further Mechanics: Momentum, Impulse and Collisions Depth: 1-hour Pack

Question 1

(a) uses conservation of momentum for the system: 3(0) + 2(6) = (3+2)v **M1**

(a) 12 = 5v (oe) **A1**

(a) v = 2.4 m/s, cao **A1**

(a) Answer: 2.4 m/s

(b) applies the impulse-momentum principle to A alone: impulsive tension = 3(2.4 - 0), using their v from part (a) (ft) **M1**

(b) correct unsimplified expression, e.g. 3 x 2.4 **A1**

(b) 7.2 Ns, cao **A1**

(b) Answer: 7.2 Ns

(c) KE before the jerk = $0.5(2)(6^2)$, since A is at rest and contributes no kinetic energy **M1**

(c) KE after the jerk = $0.5(5)(2.4^2)$, using their v from part (a) (ft) **A1**

(c) loss = 36 - 14.4 = 21.6 J, cao (a positive loss confirms energy is lost) **A1**

(c) Answer: 21.6 J lost

Question 2

(a) resolves the velocity using the 30 degree angle to the plane: perpendicular component = 10 sin(30), parallel component = 10 cos(30) **M1**

(a) perpendicular component = 5 m/s (towards the plane) **A1**

(a) parallel component = $5*\sqrt{3}$ m/s (awrt 8.66 m/s), cao **A1**

(a) Answer: Perpendicular: 5 m/s (towards the plane); parallel: $5*\sqrt{3}$ m/s (awrt 8.66 m/s)

(b) states the parallel component is unchanged, since the plane is smooth so no impulse acts parallel to it: $5*\sqrt{3}$ m/s (ft) **B1**

(b) perpendicular component after impact = e x (perpendicular component before) = 0.6 x 5, using their value from part (a) (ft) **M1**

(b) 3 m/s (away from the plane), cao **A1**

(b) Answer: Parallel: $5*\sqrt{3}$ m/s (unchanged); perpendicular: 3 m/s (away from the plane)

(c) speed = $\sqrt{3^2 + (5*\sqrt{3})^2}$, using their components from part (b) (ft) **M1**

(c) 9.17 m/s (awrt 3 s.f.), cao **A1**

(c) angle to the plane: $\tan(\alpha) = 3/(5*\sqrt{3})$ **M1**

(c) 19.1 degrees (awrt 3 s.f.), cao **A1**

(c) Answer: Speed = 9.17 m/s (3 s.f.); angle to the plane = 19.1 degrees (3 s.f.)

(d) recognises the impulse acts only perpendicular to the plane (the parallel component of velocity is unchanged, so no impulse acts parallel to the plane) **M1**

(d) impulse = 0.3 x (3 - (-5)), using their perpendicular components from parts (a) and (b) (ft) **M1**

(d) 2.4 Ns, cao **A1**

(d) Answer: 2.4 Ns

Question 3

(a) momentum equation: $2(8) + 3(0) = 2*v_A + 3*v_B$ **M1**

(a) simplified to $2*v_A + 3*v_B = 16$ **A1**

(a) restitution equation: $v_B - v_A = -0.5(0-8)$ **M1**

(a) simplified to $v_B - v_A = 4$ **A1**

(a) solves to $v_A = 0.8$ m/s and $v_B = 4.8$ m/s, both correct, cao **A1**

(a) Answer: $v_A = 0.8$ m/s; $v_B = 4.8$ m/s

(b) momentum equation for the B-C collision, using B's speed from part (a) (ft): $3(4.8) + m(0) = 3(1.44) + m*v_C$ **M1**

(b) restitution equation: $v_C - 1.44 = 0.75(4.8 - 0)$ **M1**

(b) $v_C = 5.04$ m/s **A1**

(b) substitutes v_C into the momentum equation to solve for m: $14.4 = 4.32 + m(5.04)$ **dM1**

(b) m = 2 kg, cao **A1**

(b) Answer: m = 2 kg

(c) states A's velocity remains 0.8 m/s (unaffected by later events) while B moves at 1.44 m/s; since 0.8 < 1.44, A cannot catch up with B, so the gap between A and B does not close **B1**

(c) states B's velocity after its collision with C is 1.44 m/s while C moves off at 5.04 m/s; since 1.44 < 5.04, B cannot catch up with C, so the gap between B and C does not close **B1**

(c) concludes that, since each sphere in the line A, B, C moves more slowly than the one ahead of it, no further collisions occur between any pair **B1**

(c) Answer: No further collisions occur, since 0.8 < 1.44 < 5.04 with the spheres in the order A, B, C

Question 4

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 3/4 x 12 = 9.000 m/s **A1**

(a) Answer: 9.000 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.2(12 + 9.000) = 4.20 N s **A1**

(b) Answer: 4.20 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.2)(12^2)(1 - 9/16) = 6.30$ J **A1**

(c) Answer: 6.30 J

(d) forms $12 \times (3/4)^n < 2$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) n > 6.228 **A1**

(d) n = 7 **A1**

(d) Answer: 7

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Question 5

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 25 + (-56) = 13v **A1**

(a) v = -31/13 m/s **A1**

(a) Answer: -31/13 m/s

(b) change of momentum of P **M1**

(b) 5(-31/13 - 5) = -480/13 N s **A1**

(b) Answer: 480/13 N s

(c) KE before - KE after **M1**

(c) 517/2 - 961/26 **A1**

(c) 2880/13 J **A1**

(c) Answer: 2880/13 J

(d) force = impulse/time **M1**

(d) 369 N **A1**

(d) Answer: 369 N

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