3.5 Further Mechanics: Momentum, Impulse and Collisions Depth: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 70 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: momentum, impulse and Newton's law of restitution

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and take all strings as light and inextensible, all pulleys and surfaces as smooth, and all spheres as smooth (so there is no friction, and no tangential impulse, at any point of contact). The impulse-momentum principle states that the impulse J exerted on a particle of mass m equals its change in momentum, J = m*(v - u); when a force F varies with time t, the impulse delivered over an interval is the integral of F with respect to t over that interval. Provided no external horizontal force acts on a system of colliding or jerked particles, total momentum is conserved: m1*u1 + m2*u2 = m1*v1 + m2*v2, with velocities signed along the line of centres (or along the string). Newton's experimental law of restitution states that the speed of separation is e times the speed of approach, v2 - v1 = -e*(u2 - u1), where 0 <= e <= 1 is the coefficient of restitution between the two surfaces; e = 1 gives a perfectly elastic collision (no kinetic energy lost) and e = 0 gives a perfectly inelastic collision (the particles coalesce, or move off together immediately after a string jerks taut). The loss in kinetic energy due to a direct collision is m1*m2*(1-e^2)*(u1-u2)^2 / (2*(m1+m2)). When a sphere strikes a fixed smooth surface obliquely, the component of its velocity parallel to the surface is unchanged by the impact, while the component perpendicular to the surface obeys Newton's law of restitution.

1 A particle P of mass 0.4 kg is moving with velocity (3i - 2j) m/s when it receives an impulse of (2i + 5j) Ns.

(a) Find the velocity of P immediately after the impulse, giving your answer in the form (ai + bj) m/s. **(3)**

(b) Find the speed of P immediately after the impulse, giving your answer to 3 significant figures. **(2)**

(Total for Question 1 is 5 marks)

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2 Three particles, of masses 2 kg, 3 kg and 5 kg, have velocities (4i - j) m/s, (-2i + 3j) m/s and (i + 2j) m/s respectively.

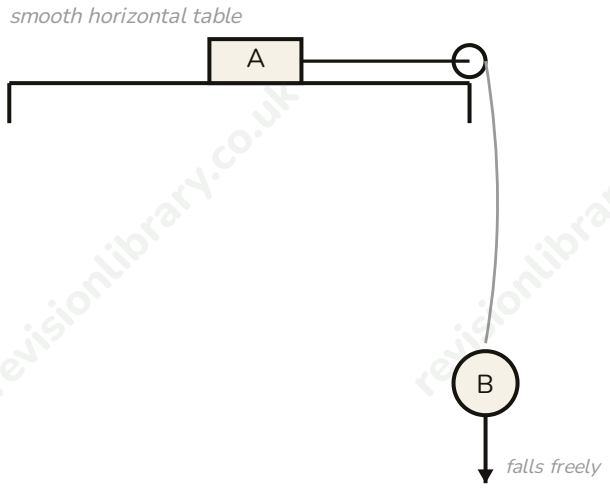
(a) Find the total momentum of the system, giving your answer in the form (ai + bj) Ns. **(3)**

(b) Hence find the magnitude of the total momentum, giving your answer to 3 significant figures. **(2)**

(Total for Question 2 is 5 marks)

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3 Two particles, A of mass 4 kg and B of mass 6 kg, are connected by a light inextensible string which passes over a small smooth pulley fixed at the edge of a smooth horizontal table. Particle A rests on the table, and particle B hangs freely, with the string initially slack. Particle B is released from rest and falls freely (A remaining at rest, since the slack string exerts no force on it) through a distance of 5 m, at which point the string becomes taut.



(a) Find, as an exact surd, the speed of B immediately before the string becomes taut. **(2)**

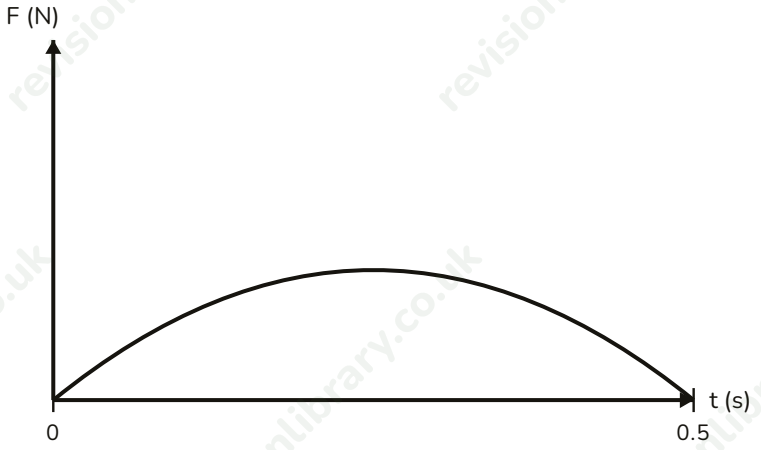
(b) Using the impulse-momentum principle, find the common speed of A and B immediately after the string jerks taut, and the magnitude of the impulsive tension in the string. **(4)**

(c) Find the loss in kinetic energy due to the string jerking taut, giving your answer to 3 significant figures. **(2)**

(Total for Question 3 is 8 marks)

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4 During an impact lasting 0.5 seconds, the force F newtons exerted on a particle of mass 2 kg varies with time t seconds ($0 \leq t \leq 0.5$) according to $F = 480*t - 960*t^2$.



$F = 480t - 960t^2$, for $0 \leq t \leq 0.5$

(a) Show that the impulse exerted on the particle during the whole impact is 20 Ns. **(3)**

(b) Given that the particle is initially at rest, find its speed immediately after the impact. **(2)**

(c) Find the time at which the force is greatest during the impact, and this greatest force, justifying that it is a maximum. **(3)**

(Total for Question 4 is 8 marks)

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5 A small ball of mass 0.5 kg hits a smooth fixed vertical wall at right angles with speed 11 m/s. The coefficient of restitution between the ball and the wall is 2/3.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 1 m/s. **(4)**

(Total for Question 5 is 10 marks)

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6 Two smooth spheres, P and Q, of masses m and km respectively (where $k > 0$ is a constant), move towards each other in the same straight line with speeds u and v respectively ($u > 0$, $v > 0$), and collide directly. The coefficient of restitution between P and Q is e, where $0 \leq e \leq 1$.

(a) Taking the direction of P's initial motion as positive, show that the velocities of P and Q immediately after the collision are $v_P = [(1-ek)u - k(1+e)v]/(1+k)$ and $v_Q = [(1+e)u + (e-k)v]/(1+k)$. **(6)**

(b) Hence show that, in the special case $k = 1$ and $e = 1$ (equal masses, a perfectly elastic collision), the two spheres exactly exchange velocities. **(3)**

(c) Hence show that, in the special case $e = 0$, the two spheres move on with a common velocity equal to $(u-kv)/(1+k)$, and verify that this agrees with the value obtained directly from conservation of momentum alone. **(2)**

(Total for Question 6 is 11 marks)

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7 Two smooth spheres, of masses m1 and m2, collide directly with signed velocities u1 and u2 respectively, separating with velocities v1 and v2 and coefficient of restitution e. Prove that the loss in kinetic energy due to the collision is $m1*m2*(1-e^2)*(u1-u2)^2 / (2*(m1+m2))$.

(Total for Question 7 is 7 marks)

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Mark scheme · 3.5 Further Mechanics: Momentum, Impulse and Collisions Depth: Test Pack

Question 1

(a) rearranges the impulse-momentum principle to v = u + I/m **M1**

(a) correct unsimplified substitution, e.g. (3i-2j) + (2i+5j)/0.4 **A1**

(a) v = (8i + 10.5j) m/s, cao **A1**

(a) Answer: v = (8i + 10.5j) m/s

(b) speed = $\sqrt{8^2 + 10.5^2}$, using their velocity from part (a) (ft) **M1**

(b) 13.2 m/s (awrt 3 s.f.), cao **A1**

(b) Answer: 13.2 m/s (3 s.f.)

Question 2

(a) attempts the component-wise sum of m*v for all three particles **M1**

(a) i-component: 2(4)+3(-2)+5(1) = 7 **A1**

(a) j-component: 2(-1)+3(3)+5(2) = 17, giving total momentum (7i+17j) Ns **A1**

(a) Answer: (7i + 17j) Ns

(b) magnitude = $\sqrt{7^2 + 17^2}$, using their values from part (a) (ft) **M1**

(b) 18.4 Ns (awrt 3 s.f.), cao **A1**

(b) Answer: 18.4 Ns (3 s.f.)

Question 3

(a) uses $v^2 = 2gh$ with g=9.8, h=5, i.e. $v^2 = 98$ **M1**

(a) $v = \sqrt{98} = 7*\sqrt{2}$ m/s (awrt 9.90 m/s), cao **A1**

(a) Answer: $7*\sqrt{2}$ m/s (awrt 9.90 m/s)

(b) momentum conserved along the string during the jerk (only B has momentum beforehand): $6*(7*\sqrt{2}) = (4+6)*V$, using their speed from part (a) (ft) **M1**

(b) $V = 4.2*\sqrt{2}$ m/s (awrt 5.94 m/s), cao **A1**

(b) impulsive tension found by applying the impulse-momentum principle to A alone: impulse = $4*(4.2*\sqrt{2} - 0)$, using their V (ft) **M1**

(b) $16.8*\sqrt{2}$ Ns (awrt 23.8 Ns), cao **A1**

(b) Answer: V = $4.2*\sqrt{2}$ m/s (awrt 5.94 m/s); impulsive tension = $16.8*\sqrt{2}$ Ns (awrt 23.8 Ns)

(c) KE before = $0.5(6)(7*\sqrt{2})^2$ [A contributes 0]; KE after = $0.5(10)(4.2*\sqrt{2})^2$, using their V from part (b) (ft) **M1**

(c) loss = 294 - 176.4 = 118 J (awrt 3 s.f.; exact value 117.6 J), cao **A1**

(c) Answer: 118 J (3 s.f.); exact value 117.6 J

Question 4

(a) sets up the impulse as the definite integral of F with respect to t from 0 to 0.5 **M1**

(a) correct antiderivative: $240*t^2 - 320*t^3$ **A1**

(a) evaluates to 240(0.25) - 320(0.125) = 60 - 40 = 20 Ns, cso **A1**

(a) Answer: Impulse = 20 Ns

(b) impulse = m*v, i.e. 2v = 20, using their impulse from part (a) (ft) **M1**

(b) v = 10 m/s, cao **A1**

(b) Answer: 10 m/s

(c) differentiates and sets dF/dt=0: 480 - 1920*t = 0 **M1**

(c) t = 0.25 s (within the interval $0 \le t \le 0.5$), and justifies it is a maximum since F is a downward-opening quadratic in t (the coefficient of t^2 is negative) **A1**

(c) $F_{max} = 480(0.25) - 960(0.25)^2 = 120 - 60 = 60$ N, cao **A1**

(c) Answer: t = 0.25 s; greatest force = 60 N

Question 5

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 2/3 x 11 = 7.333 m/s **A1**

(a) Answer: 7.333 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.5(11 + 7.333) = 9.17 N s **A1**

(b) Answer: 9.17 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.5)(11^2)(1 - 4/9) = 16.8$ J **A1**

(c) Answer: 16.8 J

(d) forms 11 x $(2/3)^n < 1$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) n > 5.914 **A1**

(d) n = 6 **A1**

(d) Answer: 6

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Question 6

(a) momentum equation (dividing throughout by m): $u - kv = v_P + k*v_Q$, since Q's initial velocity is -v — **M1**

(a) restitution equation: $v_Q - v_P = -e(-v - u)$, simplifying to $v_Q - v_P = e(u+v)$ — **M1**

(a) eliminates v_P (or v_Q) by substitution, e.g. $v_P = v_Q - e(u+v)$, into the momentum equation — **dM1**

(a) correctly derives $v_Q(1+k) = (1+e)u + (e-k)v$, i.e. $v_Q = [(1+e)u+(e-k)v]/(1+k)$ — **A1**

(a) back-substitutes to derive $v_P = [(1-ek)u - k(1+e)v]/(1+k)$ — **A1**

(a) presents both final results together, fully simplified, cso — **A1**

(a) Answer: $v_P = [(1-ek)u - k(1+e)v]/(1+k)$; $v_Q = [(1+e)u + (e-k)v]/(1+k)$

(b) substitutes k=1, e=1 into the formula for v_P: $v_P = [(1-1)u - 1(2)v]/2 = -v$ — **B1**

(b) substitutes k=1, e=1 into the formula for v_Q: $v_Q = [(2)u + (0)v]/2 = u$ — **B1**

(b) concludes that P now moves with the velocity Q originally had (-v), and Q now moves with the velocity P originally had (u), i.e. the spheres exactly exchange velocities, cso — **B1**

(b) Answer: $v_P = -v$ and $v_Q = u$, so P and Q exchange velocities exactly

(c) substitutes e=0 into both formulae to obtain $v_P = v_Q = (u-kv)/(1+k)$, cso — **B1**

(c) verifies against momentum conservation directly: $m*u + km*(-v) = (m+km)*V$ gives $V = (u-kv)/(1+k)$, identical to the value above — **B1**

(c) Answer: Common velocity = $(u-kv)/(1+k)$, matching the momentum-conservation result

Question 7

writes the loss as $0.5*m1(u1-v1)(u1+v1) + 0.5*m2(u2-v2)(u2+v2)$, using the difference-of-two-squares factorisation of $u1^2-v1^2$ and $u2^2-v2^2$ — **M1**

uses momentum conservation $m1*u1+m2*u2=m1*v1+m2*v2$ to show $m1(u1-v1) = m2(v2-u2)$ — **M1**

substitutes to reduce the loss to $0.5*m1(u1-v1)*[(u1+v1)-(u2+v2)]$ — **A1**

uses the restitution equation $v1-v2=-e(u1-u2)$ to simplify $(u1+v1)-(u2+v2) = (u1-u2)+(v1-v2) = (1-e)(u1-u2)$ — **M1**

solves the momentum and restitution equations simultaneously for v1 to show $m1(u1-v1) = m1*m2*(1+e)*(u1-u2)/(m1+m2)$ — **dM1**

combines both results to obtain loss $= m1*m2*(1-e^2)*(u1-u2)^2 / (2*(m1+m2))$, cso — **A1**

notes that since $0 \le e \le 1$, $(1-e^2) \ge 0$, so the loss is always non-negative, confirming kinetic energy cannot be created in a collision — **B1**

Answer: Loss $= m1*m2*(1-e^2)*(u1-u2)^2 / (2*(m1+m2))$, which is always greater than or equal to 0 since $0 \le e \le 1$

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