3.5 Further Mechanics: Momentum, Impulse and Collisions Depth

EDEXCEL 9FM0 · Calculator allowed · about 225 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

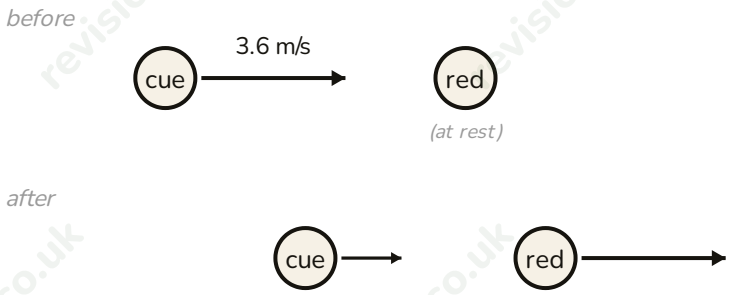
Answer ALL questions. Show all your working.

Key results: momentum, impulse and Newton's law of restitution

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and take all strings as light and inextensible, all pulleys and surfaces as smooth, and all spheres as smooth (so there is no friction, and no tangential impulse, at any point of contact). The impulse-momentum principle states that the impulse J exerted on a particle of mass m equals its change in momentum, J = m*(v - u); when a force F varies with time t, the impulse delivered over an interval is the integral of F with respect to t over that interval. Provided no external horizontal force acts on a system of colliding or jerked particles, total momentum is conserved: m1*u1 + m2*u2 = m1*v1 + m2*v2, with velocities signed along the line of centres (or along the string). Newton's experimental law of restitution states that the speed of separation is e times the speed of approach, v2 - v1 = -e*(u2 - u1), where 0 <= e <= 1 is the coefficient of restitution between the two surfaces; e = 1 gives a perfectly elastic collision (no kinetic energy lost) and e = 0 gives a perfectly inelastic collision (the particles coalesce, or move off together immediately after a string jerks taut). The loss in kinetic energy due to a direct collision is m1*m2*(1-e^2)*(u1-u2)^2 / (2*(m1+m2)). When a sphere strikes a fixed smooth surface obliquely, the component of its velocity parallel to the surface is unchanged by the impact, while the component perpendicular to the surface obeys Newton's law of restitution.

1 In snooker, a cue ball of mass 0.17 kg travelling at 3.6 m/s strikes a stationary red ball of the same mass directly. The coefficient of restitution between the balls is 0.9.

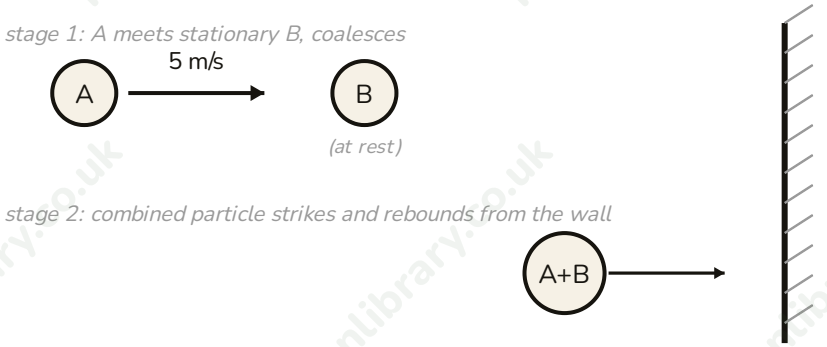


(a) Find the velocities of both balls immediately after the collision. **(4)**

(b) Calculate the percentage of the cue ball's original kinetic energy that is lost due to the collision, giving your answer to 3 significant figures. **(3)**

(Total for Question 1 is 7 marks)

2 A particle A of mass 3 kg, moving with speed 5 m/s, collides directly with a stationary particle B of mass 2 kg, and the two particles coalesce on impact to form a single combined particle. This combined particle then travels on and strikes a fixed smooth wall at right angles, rebounding with coefficient of restitution 0.4 between the combined particle and the wall.



(a) Find the common speed of the combined particle immediately after the first collision (with B). **(2)**

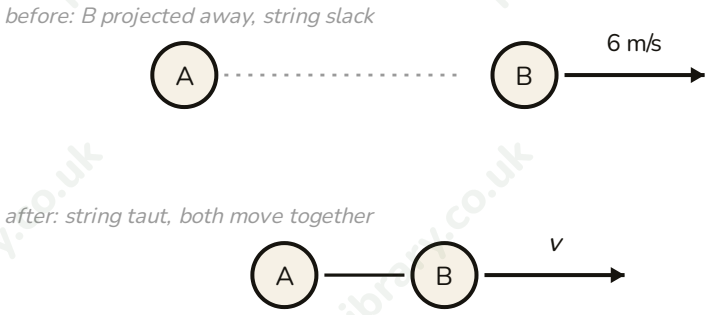
(b) Find the speed of the combined particle immediately after it rebounds from the wall. **(2)**

(c) Find the total loss in kinetic energy, from immediately before the first collision to immediately after the rebound from the wall. **(3)**

(d) Find, to 3 significant figures, the percentage of the original kinetic energy that has been lost overall. **(2)**

(Total for Question 2 is 9 marks)

3 Two particles, A of mass 3 kg and B of mass 2 kg, lie at rest on a smooth horizontal plane, connected by a light inextensible string which is initially slack. Particle B is then projected directly away from A with speed 6 m/s, and travels in a straight line until the string becomes taut.



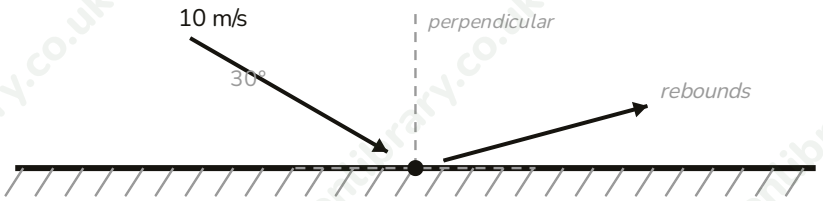
(a) Find the common speed of A and B immediately after the string becomes taut. **(3)**

(b) Find the magnitude of the impulsive tension in the string as it jerks taut. **(3)**

(c) Show that kinetic energy is lost when the string jerks taut, and find this loss. **(3)**

(Total for Question 3 is 9 marks)

4 A smooth sphere strikes a fixed smooth plane with speed 10 m/s, travelling at an angle of 30 degrees to the plane, and rebounds. The coefficient of restitution between the sphere and the plane is e = 0.6.



(a) Find the components of the sphere's velocity, resolved perpendicular and parallel to the plane, immediately before impact. **(3)**

(b) State the parallel component of velocity immediately after impact, giving a reason, and find the perpendicular component of velocity immediately after impact. **(3)**

(c) Find the speed of the sphere, and the angle its path makes with the plane, immediately after impact, giving each answer to 3 significant figures. **(4)**

(d) Given that the mass of the sphere is 0.3 kg, find the magnitude of the impulse exerted on the sphere by the plane during the impact. **(3)**

(Total for Question 4 is 13 marks)

5 Three smooth spheres A, B and C, of masses 2 kg, 3 kg and m kg respectively, lie at rest in a straight line on a smooth horizontal table, with B between A and C. Sphere A is projected towards B with speed 8 m/s. The coefficient of restitution between A and B is 0.5, and the coefficient of restitution between B and C is 0.75.



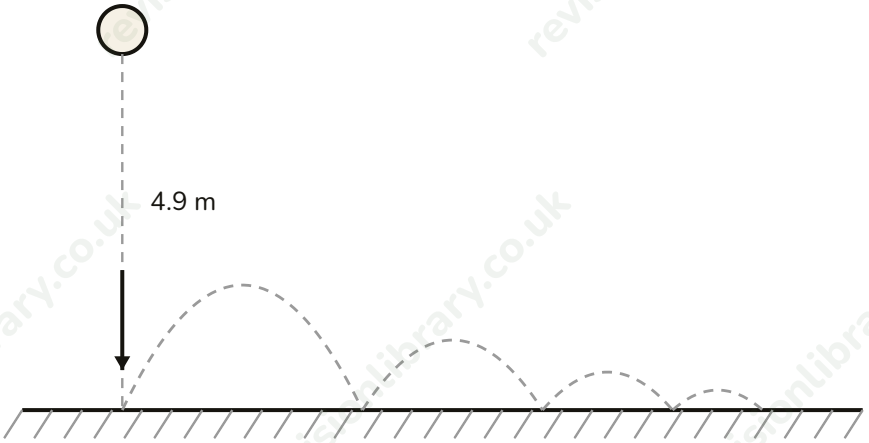
(a) Find the velocities of A and B immediately after their collision. **(5)**

(b) Sphere B then collides directly with sphere C, which is also initially at rest. Given that B's velocity immediately after this second collision is 1.44 m/s (still in A's original direction of motion), find the value of m. **(5)**

(c) Determine, with justification, whether any further collisions occur between the three spheres. **(3)**

(Total for Question 5 is 13 marks)

6 A ball is released from rest at a height of 4.9 m above smooth horizontal ground and allowed to bounce repeatedly. The coefficient of restitution between the ball and the ground is e = 0.5. Air resistance is negligible, and g = 9.8 m/s^2 throughout this question.



(a) Show that the ball takes exactly 1 second to fall to the ground for the first time, and find its speed immediately before this first impact. **(3)**

(b) Show that the time taken for the ball to rise to its highest point after the nth bounce, and fall back to the ground again, is $2*e^n$ seconds. **(3)**

(c) Hence find, as an exact number of seconds, the total time from the ball's release until it comes to rest. **(3)**

(Total for Question 6 is 9 marks)

7 A particle P of mass 7 kg is moving with speed 9 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 3 kg, which is moving along the same line with velocity -8 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.5 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 7 is 10 marks)

8 A small ball of mass 0.2 kg hits a smooth fixed vertical wall at right angles with speed 12 m/s. The coefficient of restitution between the ball and the wall is 3/4.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 2 m/s. **(4)**

(Total for Question 8 is 10 marks)

9 A particle P of mass 5 kg is moving with speed 5 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 8 kg, which is moving along the same line with velocity -7 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.1 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 9 is 10 marks)

10 A particle P of mass 7 kg is moving with speed 8 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 8 kg, which is moving along the same line with velocity -4 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.25 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 10 is 10 marks)

11 A small ball of mass 0.2 kg hits a smooth fixed vertical wall at right angles with speed 14 m/s. The coefficient of restitution between the ball and the wall is 1/2.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 2 m/s. **(4)**

(Total for Question 11 is 10 marks)

12 A particle P of mass 1 kg is moving with speed 5 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 6 kg, which is moving along the same line with velocity 2 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.1 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 12 is 10 marks)

13 A small ball of mass 0.2 kg hits a smooth fixed vertical wall at right angles with speed 16 m/s. The coefficient of restitution between the ball and the wall is 3/4.

(a) Find the speed of the ball immediately after it rebounds from the wall. **(2)**

(b) Find the magnitude of the impulse exerted by the wall on the ball. **(2)**

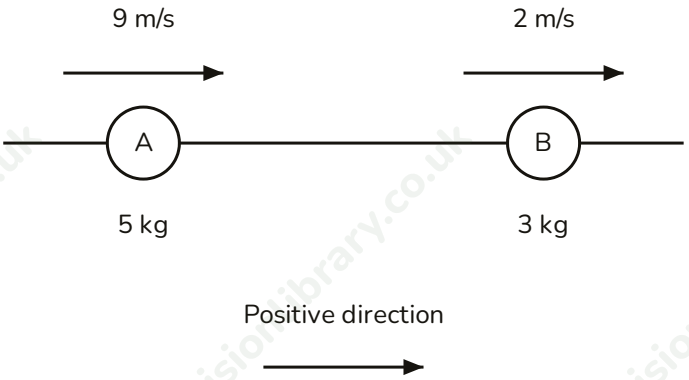
(c) Find the kinetic energy lost at the impact. **(2)**

(d) The ball bounces between the wall and a second smooth wall opposite so that each bounce reduces the speed by the same factor. Find the least number of impacts after which the speed is below 1 m/s. **(4)**

(Total for Question 13 is 10 marks)

14 Two smooth spheres A and B, of masses 5 kg and 3 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 9 m/s and B has velocity 2 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 1/4.

Diagram NOT accurately drawn



(a) Find the velocities of A and B immediately after the collision. **(4)**

(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 14 is 9 marks)

15 A particle P of mass 7 kg is moving with speed 3 m/s along a smooth horizontal straight line. It collides directly with a particle Q of mass 7 kg, which is moving along the same line with velocity -2 m/s (a negative value means moving in the opposite direction to P). After the collision P and Q coalesce.

(a) Find the common velocity of the combined particle after the collision. **(3)**

(b) Find the magnitude of the impulse exerted by Q on P. **(2)**

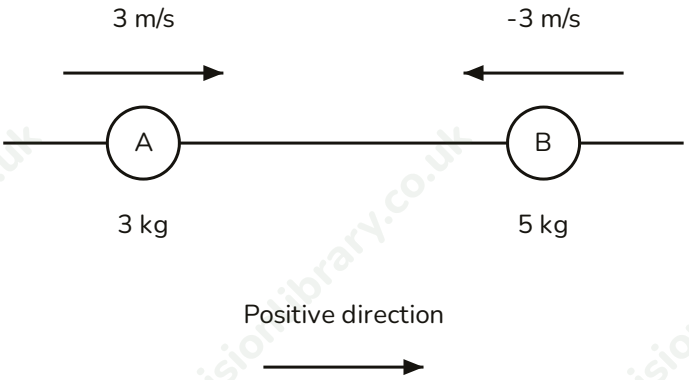
(c) Find the kinetic energy lost in the collision. **(3)**

(d) The contact between P and Q lasts 0.2 s. Find the magnitude of the average force between them, to 3 significant figures. **(2)**

(Total for Question 15 is 10 marks)

16 Two smooth spheres A and B, of masses 3 kg and 5 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 3 m/s and B has velocity -3 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 1/3.

Diagram NOT accurately drawn



(a) Find the velocities of A and B immediately after the collision. **(4)**

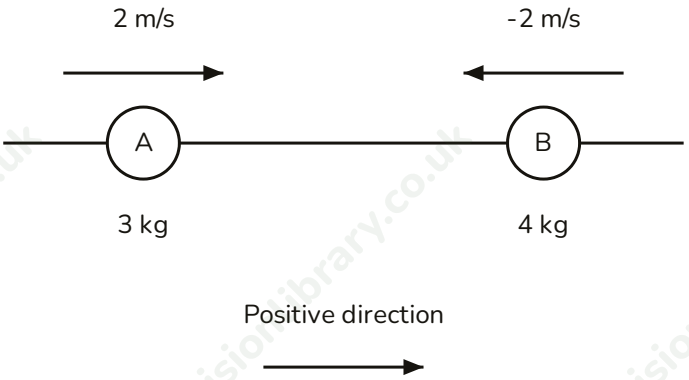
(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 16 is 9 marks)

17 Two smooth spheres A and B, of masses 3 kg and 4 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 2 m/s and B has velocity -2 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 1/4.

Diagram NOT accurately drawn



(a) Find the velocities of A and B immediately after the collision. **(4)**

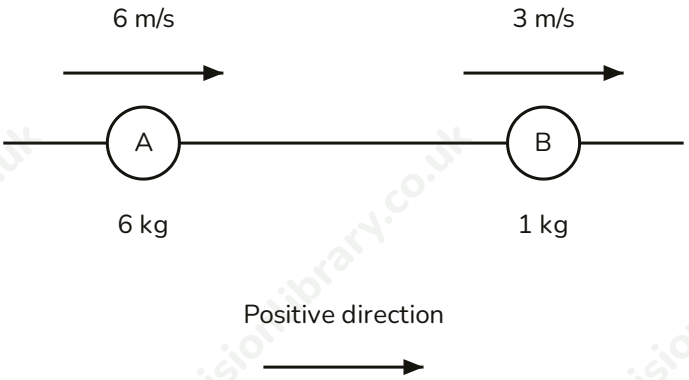
(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 17 is 9 marks)

18 Two smooth spheres A and B, of masses 6 kg and 1 kg, are moving in the same straight line on a smooth horizontal table. A has velocity 6 m/s and B has velocity 3 m/s, both measured in the same direction along the line. The spheres collide directly, and the coefficient of restitution between them is 2/3.

Diagram NOT accurately drawn



(a) Find the velocities of A and B immediately after the collision. **(4)**

(b) Find the magnitude of the impulse exerted by A on B in the collision. **(2)**

(c) Find the total kinetic energy lost in the collision. **(3)**

(Total for Question 18 is 9 marks)

Mark scheme · 3.5 Further Mechanics: Momentum, Impulse and Collisions Depth

Question 1

(a) momentum equation: 0.17(3.6) + 0.17(0) = 0.17*v1 + 0.17*v2, simplifying (dividing by 0.17) to v1+v2=3.6 **M1**

(a) restitution equation: v2-v1 = -0.9(0-3.6), simplifying to v2-v1=3.24 **M1**

(a) cue ball velocity v1 = 0.18 m/s, cao **A1**

(a) red ball velocity v2 = 3.42 m/s, cao **A1**

(a) Answer: Cue ball: 0.18 m/s; red ball: 3.42 m/s

(b) KE before = $0.5(0.17)(3.6^2)$; KE after = $0.5(0.17)(0.18^2+3.42^2)$, using their velocities from part (a) (ft) **M1**

(b) loss = 1.1016 - 0.996948 = 0.105 J (awrt 3 s.f.) **A1**

(b) percentage lost = 9.50% (awrt 3 s.f.), cao **A1**

(b) Answer: 9.50% (3 s.f.)

Question 2

(a) momentum equation: 3(5) + 2(0) = (3+2)v **M1**

(a) v = 3 m/s, cao **A1**

(a) Answer: 3 m/s

(b) speed after = e x (speed before) = 0.4 x 3, using their speed from part (a) (ft) **M1**

(b) 1.2 m/s (moving away from the wall), cao **A1**

(b) Answer: 1.2 m/s, away from the wall

(c) KE initial (before any collision) = $0.5(3)(5^2)$, since B is at rest **M1**

(c) KE final (after the wall rebound) = $0.5(5)(1.2^2)$, using their speeds from parts (a) and (b) (ft) **M1**

(c) total loss = 37.5 - 3.6 = 33.9 J, cao **A1**

(c) Answer: 33.9 J

(d) percentage = (33.9/37.5) x 100, using their total loss from part (c) (ft) **M1**

(d) 90.4% (awrt 3 s.f.), cao **A1**

(d) Answer: 90.4% (3 s.f.)

Question 3

(a) uses conservation of momentum for the system: 3(0) + 2(6) = (3+2)v **M1**

(a) 12 = 5v (oe) **A1**

(a) v = 2.4 m/s, cao **A1**

(a) Answer: 2.4 m/s

(b) applies the impulse-momentum principle to A alone: impulsive tension = 3(2.4 - 0), using their v from part (a) (ft) **M1**

(b) correct unsimplified expression, e.g. 3 x 2.4 **A1**

(b) 7.2 Ns, cao **A1**

(b) Answer: 7.2 Ns

(c) KE before the jerk = $0.5(2)(6^2)$, since A is at rest and contributes no kinetic energy **M1**

(c) KE after the jerk = $0.5(5)(2.4^2)$, using their v from part (a) (ft) **A1**

(c) loss = 36 - 14.4 = 21.6 J, cao (a positive loss confirms energy is lost) **A1**

(c) Answer: 21.6 J lost

Question 4

(a) resolves the velocity using the 30 degree angle to the plane: perpendicular component = 10 sin(30), parallel component = 10 cos(30) **M1**

(a) perpendicular component = 5 m/s (towards the plane) **A1**

(a) parallel component = $5*\sqrt{3}$ m/s (awrt 8.66 m/s), cao **A1**

(a) Answer: Perpendicular: 5 m/s (towards the plane); parallel: $5*\sqrt{3}$ m/s (awrt 8.66 m/s)

(b) states the parallel component is unchanged, since the plane is smooth so no impulse acts parallel to it: $5*\sqrt{3}$ m/s (ft) **B1**

(b) perpendicular component after impact = e x (perpendicular component before) = 0.6 x 5, using their value from part (a) (ft) **M1**

(b) 3 m/s (away from the plane), cao **A1**

(b) Answer: Parallel: $5*\sqrt{3}$ m/s (unchanged); perpendicular: 3 m/s (away from the plane)

(c) speed = $\sqrt{3^2 + (5*\sqrt{3})^2}$, using their components from part (b) (ft) **M1**

(c) 9.17 m/s (awrt 3 s.f.), cao **A1**

(c) angle to the plane: $\tan(\alpha) = 3/(5*\sqrt{3})$ **M1**

(c) 19.1 degrees (awrt 3 s.f.), cao **A1**

(c) Answer: Speed = 9.17 m/s (3 s.f.); angle to the plane = 19.1 degrees (3 s.f.)

(d) recognises the impulse acts only perpendicular to the plane (the parallel component of velocity is unchanged, so no impulse acts parallel to the plane) **M1**

(d) impulse = 0.3 x (3 - (-5)), using their perpendicular components from parts (a) and (b) (ft) **M1**

(d) 2.4 Ns, cao **A1**

(d) Answer: 2.4 Ns

Question 5

(a) momentum equation: $2(8) + 3(0) = 2*v_A + 3*v_B$ **M1**

(a) simplified to $2*v_A + 3*v_B = 16$ **A1**

(a) restitution equation: $v_B - v_A = -0.5(0-8)$ **M1**

(a) simplified to $v_B - v_A = 4$ **A1**

(a) solves to $v_A = 0.8$ m/s and $v_B = 4.8$ m/s, both correct, cao **A1**

(a) Answer: $v_A = 0.8$ m/s; $v_B = 4.8$ m/s

(b) momentum equation for the B-C collision, using B's speed from part (a) (ft): $3(4.8) + m(0) = 3(1.44) + m*v_C$ **M1**

(b) restitution equation: $v_C - 1.44 = 0.75(4.8 - 0)$ **M1**

(b) $v_C = 5.04$ m/s **A1**

(b) substitutes v_C into the momentum equation to solve for m: $14.4 = 4.32 + m(5.04)$ **dM1**

(b) m = 2 kg, cao **A1**

(b) Answer: m = 2 kg

(c) states A's velocity remains 0.8 m/s (unaffected by later events) while B moves at 1.44 m/s; since 0.8 < 1.44, A cannot catch up with B, so the gap between A and B does not close **B1**

(c) states B's velocity after its collision with C is 1.44 m/s while C moves off at 5.04 m/s; since 1.44 < 5.04, B cannot catch up with C, so the gap between B and C does not close **B1**

(c) concludes that, since each sphere in the line A, B, C moves more slowly than the one ahead of it, no further collisions occur between any pair **B1**

(c) Answer: No further collisions occur, since 0.8 < 1.44 < 5.04 with the spheres in the order A, B, C

Question 6

(a) uses $h = 0.5*g*t^2$ with h=4.9, g=9.8, i.e. $4.9 = 4.9*t^2$ **M1**

(a) t = 1 s, cso **A1**

(a) speed = g*t = 9.8*1 = 9.8 m/s, cao **A1**

(a) Answer: t = 1 s; speed = 9.8 m/s

(b) recognises the speed immediately after the nth bounce is e^n x 9.8, since each bounce multiplies the impact speed by e, starting from the first impact speed of 9.8 m/s **M1**

(b) uses the fact that, in free flight under gravity with no resistance, the time to rise to the peak equals the time to fall back, so the total flight time is 2 x (speed just after the bounce)/g **M1**

(b) = $2*(e^n*9.8)/9.8 = 2*e^n$ seconds, cso **A1**

(b) Answer: Time between the nth and (n+1)th bounce = $2*e^n$ seconds

(c) total time = (time of first fall) + sum from n=1 to infinity of $2*e^n$, using their results from parts (a) and (b) (ft) **M1**

(c) recognises this as an infinite geometric series with first term 2e and common ratio e, and uses sum to infinity = 2e/(1-e) **M1**

(c) total time = 1 + 2(0.5)/(1-0.5) = 1+2 = 3 seconds, cao **A1**

(c) Answer: Total time = 3 seconds

Question 7

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 63 + (-24) = 10v **A1**

(a) v = 39/10 m/s **A1**

(a) Answer: 39/10 m/s

(b) change of momentum of P **M1**

(b) 7(39/10 - 9) = -357/10 N s **A1**

(b) Answer: 357/10 N s

(c) KE before - KE after **M1**

(c) 759/2 - 1521/20 **A1**

(c) 6069/20 J **A1**

(c) Answer: 6069/20 J

(d) force = impulse/time **M1**

(d) 71.4 N **A1**

(d) Answer: 71.4 N

Question 8

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 3/4 x 12 = 9.000 m/s **A1**

(a) Answer: 9.000 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.2(12 + 9.000) = 4.20 N s **A1**

(b) Answer: 4.20 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.2)(12^2)(1 - 9/16) = 6.30$ J **A1**

(c) Answer: 6.30 J

(d) forms $12 \times (3/4)^n < 2$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) n > 6.228 **A1**

(d) n = 7 **A1**

(d) Answer: 7

Question 9

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 25 + (-56) = 13v **A1**

(a) v = -31/13 m/s **A1**

(a) Answer: -31/13 m/s

(b) change of momentum of P **M1**

(b) 5(-31/13 - 5) = -480/13 N s **A1**

(b) Answer: 480/13 N s

(c) KE before - KE after **M1**

(c) 517/2 - 961/26 **A1**

(c) 2880/13 J **A1**

(c) Answer: 2880/13 J

(d) force = impulse/time **M1**

(d) 369 N **A1**

(d) Answer: 369 N

Question 10

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 56 + (-32) = 15v **A1**

(a) v = 8/5 m/s **A1**

(a) Answer: 8/5 m/s

(b) change of momentum of P **M1**

(b) 7(8/5 - 8) = -224/5 N s **A1**

(b) Answer: 224/5 N s

(c) KE before - KE after **M1**

(c) 288 - 96/5 **A1**

(c) 1344/5 J **A1**

(c) Answer: 1344/5 J

(d) force = impulse/time **M1**

(d) 179 N **A1**

(d) Answer: 179 N

Question 11

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 1/2 x 14 = 7.000 m/s **A1**

(a) Answer: 7.000 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.2(14 + 7.000) = 4.20 N s **A1**

(b) Answer: 4.20 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.2)(14^2)(1 - 1/4) = 14.7$ J **A1**

(c) Answer: 14.7 J

(d) forms $14 \times (1/2)^n < 2$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) n > 2.807 **A1**

(d) n = 3 **A1**

(d) Answer: 3

Question 12

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 5 + (12) = 7v **A1**

(a) v = 17/7 m/s **A1**

(a) Answer: 17/7 m/s

(b) change of momentum of P **M1**

(b) 1(17/7 - 5) = -18/7 N s **A1**

(b) Answer: 18/7 N s

(c) KE before - KE after **M1**

(c) 49/2 - 289/14 **A1**

(c) 27/7 J **A1**

(c) Answer: 27/7 J

(d) force = impulse/time **M1**

(d) 25.7 N **A1**

(d) Answer: 25.7 N

Question 13

(a) uses the restitution law: speed after = e x speed before **M1**

(a) 3/4 x 16 = 12.00 m/s **A1**

(a) Answer: 12.00 m/s

(b) impulse = change in momentum, with the velocity reversing **M1**

(b) 0.2(16 + 12.00) = 5.60 N s **A1**

(b) Answer: 5.60 N s

(c) KE before - KE after **M1**

(c) $(1/2)(0.2)(16^2)(1 - 9/16) = 11.2$ J **A1**

(c) Answer: 11.2 J

(d) forms $16 \times (3/4)^n < 1$ **M1**

(d) takes logarithms (or tests values of n) **M1**

(d) n > 9.638 **A1**

(d) n = 10 **A1**

(d) Answer: 10

Question 14

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) **M1**

(a) v1 = 183/32 m/s **A1**

(a) v2 = 239/32 m/s **A1**

(a) Answer: $v_A = 183/32$ m/s, $v_B = 239/32$ m/s

(b) impulse = change in momentum of B **M1**

(b) 3(239/32 - (2)) = 525/32 N s **A1**

(b) Answer: 525/32 N s

(c) KE before - KE after **M1**

(c) before 417/2 J, after 42351/256 J **A1**

(c) loss = 11025/256 J **A1**

(c) Answer: 11025/256 J

Question 15

(a) conservation of momentum: m1 u1 + m2 u2 = (m1 + m2) v **M1**

(a) 21 + (-14) = 14v **A1**

(a) v = 1/2 m/s **A1**

(a) Answer: 1/2 m/s

(b) change of momentum of P **M1**

(b) 7(1/2 - 3) = -35/2 N s **A1**

(b) Answer: 35/2 N s

(c) KE before - KE after **M1**

(c) 91/2 - 7/4 **A1**

(c) 175/4 J **A1**

(c) Answer: 175/4 J

(d) force = impulse/time **M1**

(d) 87.5 N **A1**

(d) Answer: 87.5 N

Question 16

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) **M1**

(a) v1 = -2 m/s **A1**

(a) v2 = 0 m/s **A1**

(a) Answer: v_A = -2 m/s, v_B = 0 m/s

(b) impulse = change in momentum of B **M1**

(b) 5(0 - (-3)) = 15 N s **A1**

(b) Answer: 15 N s

(c) KE before - KE after **M1**

(c) before 36 J, after 6 J **A1**

(c) loss = 30 J **A1**

(c) Answer: 30 J

Question 17

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) **M1**

(a) v1 = -6/7 m/s **A1**

(a) v2 = 1/7 m/s **A1**

(a) Answer: v_A = -6/7 m/s, v_B = 1/7 m/s

(b) impulse = change in momentum of B **M1**

(b) 4(1/7 - (-2)) = 60/7 N s **A1**

(b) Answer: 60/7 N s

(c) KE before - KE after **M1**

(c) before 14 J, after 8/7 J **A1**

(c) loss = 90/7 J **A1**

(c) Answer: 90/7 J

Question 18

(a) conservation of momentum: m1 u1 + m2 u2 = m1 v1 + m2 v2 **M1**

(a) Newton's experimental law: v2 - v1 = e(u1 - u2) **M1**

(a) v1 = 37/7 m/s **A1**

(a) v2 = 51/7 m/s **A1**

(a) Answer: v_A = 37/7 m/s, v_B = 51/7 m/s

(b) impulse = change in momentum of B **M1**

(b) 1(51/7 - (3)) = 30/7 N s **A1**

(b) Answer: 30/7 N s

(c) KE before - KE after **M1**

(c) before 225/2 J, after 1545/14 J **A1**

(c) loss = 15/7 J **A1**

(c) Answer: 15/7 J