3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: 1-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: work-energy principle, elastic strings/springs and circular motion

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and treat every string or spring as light; strings are inextensible while rods are light and rigid. This worksheet is a depth paper combining three areas of Further Mechanics, momentum, impulse and collisions are assumed known from elsewhere and are not the focus here: the work-energy principle, elastic strings and springs, and circular motion, and several questions require you to combine two of these ideas within a single question. Kinetic energy is KE = (1/2)*m*v^2 and gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle (including work done against resistances such as friction) equals its change in kinetic energy. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when extended (or compressed) by x, has magnitude T = lambda*x/l, and the elastic potential energy stored is EPE = lambda*x^2/(2*l); an elastic string can only pull, and goes slack once its length is less than l, whereas a spring can both pull and push. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega and the centripetal acceleration, directed towards the centre, has magnitude a = v^2/r = r*omega^2; for a conical pendulum, or a vehicle on a banked track, resolve forces vertically and horizontally to link the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with energy conservation; for a particle on a string, or on the inside of a smooth track, the critical condition is that the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when this condition would otherwise be violated, in which case the rod is in thrust (compression) rather than tension.

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1 A particle P of mass 0.6 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 36 N. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves on the table in a horizontal circle of radius 0.65 m, with O at the centre of the circle, and with constant angular speed ω.

(a) Find the extension of the string. **(1)**

(b) Use Hooke's law to find the tension in the string. **(3)**

(c) By considering the equation of motion of P towards the centre of the circle, find the value of ω, giving your answer to 3 significant figures. **(4)**

(d) Find the time taken for P to complete one full revolution, giving your answer to 3 significant figures, ft your answer to (c). **(2)**

(e) Find the speed of P, giving your answer to 3 significant figures, ft your answer to (c). **(2)**

(Total for Question 1 is 12 marks)

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2 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 5/13$. The length of the plane is 12 m and the coefficient of friction between the particle and the plane is 0.2. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8 \text{ m/s}^2$.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 2 is 9 marks)

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3 A metal ball of mass 4 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 3 is 5 marks)

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4 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 1.0 m. The other end of the string is attached to a fixed point O. P is held at O and released from rest, falling vertically under gravity. The modulus of elasticity of the string is 20 N. Take $g = 9.8\ m/s^2$.

(a) Find the speed of P at the instant the string first becomes taut. **(3)**

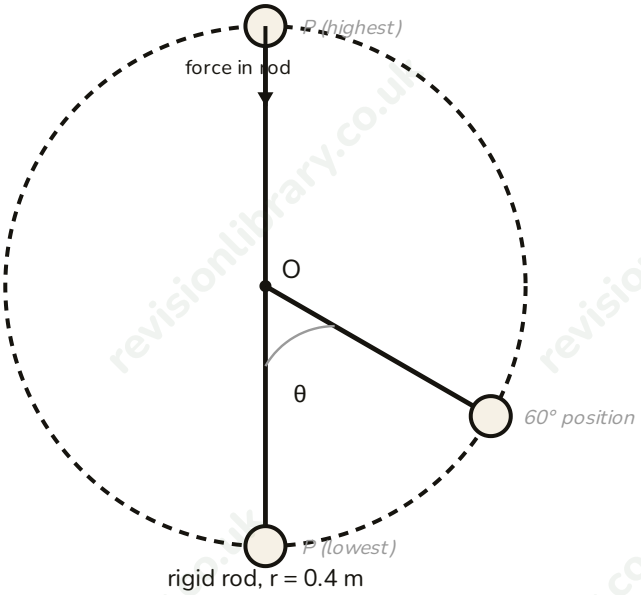
(b) Given that P first comes to instantaneous rest when the string is stretched by a further extension of e metres beyond its natural length, use energy conservation to show that e satisfies $10*e^2 - 4.9*e - 4.9 = 0$, and hence find e, giving your answer to 3 significant figures. **(6)**

(c) Hence find the total distance fallen by P when it is first at instantaneous rest. **(2)**

(Total for Question 4 is 11 marks)

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5 At an engineering workshop in Coventry, a technician called Dele is testing a model rotating-arm ride. A particle P of mass 0.25 kg is attached to one end of a light rigid rod OP of length 0.4 m; the other end of the rod is freely hinged at a fixed point O, and P moves in a complete vertical circle of radius 0.4 m about O. At the lowest point of the circle, P has speed 4 m/s. Take $g = 9.8$ m/s^2.



(a) Using the work-energy principle, find the speed of P at the point where OP makes an angle of 60 degrees with the downward vertical through O, giving your answer to 3 significant figures. **(4)**

(b) Find the speed of P at the highest point of the circle, giving your answer to 3 significant figures. **(3)**

(c) Find the magnitude of the force in the rod when P is at the highest point of the circle, and state whether the rod is in tension or in thrust (compression), ft your answer to (b). **(4)**

(Total for Question 5 is 11 marks)

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Mark scheme · 3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: 1-hour Pack

Question 1

(a) extension = 0.65 - 0.5 = 0.15 m **B1**

(a) Answer: 0.15 m

(b) correct formula T = λ*x/l **M1**

(b) correct substitution: 36*0.15/0.5, ft (a) **M1**

(b) T = 10.8 N, cao **A1**

(b) Answer: T = 10.8 N

(c) writes the equation of motion towards the centre: T = $m*\omega^2*r$, ft (b) **M1**

(c) rearranges to $\omega^2 = T/(m*r)$ **M1**

(c) substitutes: $\omega^2 = 10.8/(0.6*0.65) = 27.7$ (awrt) **M1**

(c) ω = awrt 5.26 rad/s, cao **A1**

(c) Answer: ω = awrt 5.26 rad/s

(d) uses time period = 2*π/ω, ft (c) **M1**

(d) awrt 1.19 s **A1**

(d) Answer: awrt 1.19 s

(e) uses v = ω*r, ft (c) **M1**

(e) awrt 3.42 m/s **A1**

(e) Answer: awrt 3.42 m/s

Question 2

(a) friction = μ R = μ mg cos α **M1**

(a) work = friction x distance **M1**

(a) 43.4 J **A1**

(a) Answer: 43.4 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)2v^2 = 2(9.8)(12)(5/13) - 43.422$ **A1**

(b) v = 6.86 m/s **A1**

(b) Answer: 6.86 m/s

(c) work done against friction = initial KE **M1**

(c) 0.3 x 2 x 9.8 x s = $(1/2)2(6.859)^2$ **M1**

(c) s = 8.00 m **A1**

(c) Answer: 8.00 m

Question 3

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s **A1**

(a) Answer: 5.42 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) 4(9.8)(1.5 + 0.1) = R x 0.1 **A1**

(b) R = 627 N **A1**

(b) Answer: 627 N

Question 4

(a) recognises P falls freely (the string is slack) for the first 1.0 m, and uses $v^2 = u^2 + 2*g*s$ (or an energy method) with u = 0, s = 1.0 **M1**

(a) substitutes: $v^2 = 2*9.8*1.0 = 19.6$ **M1**

(a) v = awrt 4.43 m/s, cao **A1**

(a) Answer: v = awrt 4.43 m/s

(b) uses energy conservation between the release point and the lowest point, where both the initial and final kinetic energies are zero: loss in GPE = gain in EPE **M1**

(b) writes the total distance fallen as (1.0 + e), giving loss in GPE = mg*(1+e) = 4.9*(1+e) **M1**

(b) writes the EPE gained as $\lambda*e^2/(2*l) = 20*e^2/2 = 10*e^2$ **M1**

(b) forms the equation $4.9*(1+e) = 10*e^2$, i.e. $10*e^2 - 4.9*e - 4.9 = 0$, cso **A1**

(b) solves the quadratic using the quadratic formula, $e = (4.9 \pm \sqrt{4.9^2 + 4*10*4.9})/20$, and rejects the negative root **M1**

(b) e = awrt 0.987 m **A1**

(b) Answer: $10*e^2 - 4.9*e - 4.9 = 0$; e = awrt 0.987 m

(c) total distance = 1.0 + e, ft (b) **M1**

(c) awrt 1.99 m **A1**

(c) Answer: awrt 1.99 m

Question 5

(a) finds the height risen above the lowest point: h = r*(1 - cos(60)) = 0.2 m **M1**

(a) uses energy conservation: $(1/2)*v^2 = (1/2)*v0^2 - g*h$, i.e. $v^2 = v0^2 - 2*g*h$ **M1**

(a) substitutes: $v^2 = 16 - 2*9.8*0.2 = 12.08$ **M1**

(a) v = awrt 3.48 m/s, cao **A1**

(a) Answer: v = awrt 3.48 m/s

(b) uses the height risen to the top, h = 2r = 0.8 m **M1**

(b) substitutes: $v_{top}^2 = 16 - 2*9.8*0.8 = 0.32$ **M1**

(b) v_{top} = awrt 0.566 m/s, cao **A1**

(b) Answer: v_{top} = awrt 0.566 m/s

(c) Newton's second law towards the centre (downward) at the top: $mg + F = m*v_{top}^2/r$, where F is the force in the rod taken as positive towards the centre (tension) **M1**

(c) substitutes: $F = m*v_{top}^2/r - mg = 0.25*0.32/0.4 - 0.25*9.8$, ft (b) **M1**

(c) F = -2.25 N, so the magnitude of the force in the rod is 2.25 N **A1**

(c) since F is negative (the weight alone exceeds the force needed for circular motion at this speed), the rod pushes P outward, so the rod is in thrust (compression), not tension **B1**

(c) Answer: 2.25 N of thrust (the rod is in compression)

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