3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: 2-hour Pack

EDEXCEL 9FM0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: work-energy principle, elastic strings/springs and circular motion

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and treat every string or spring as light; strings are inextensible while rods are light and rigid. This worksheet is a depth paper combining three areas of Further Mechanics, momentum, impulse and collisions are assumed known from elsewhere and are not the focus here: the work-energy principle, elastic strings and springs, and circular motion, and several questions require you to combine two of these ideas within a single question. Kinetic energy is KE = (1/2)*m*v^2 and gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle (including work done against resistances such as friction) equals its change in kinetic energy. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when extended (or compressed) by x, has magnitude T = lambda*x/l, and the elastic potential energy stored is EPE = lambda*x^2/(2*l); an elastic string can only pull, and goes slack once its length is less than l, whereas a spring can both pull and push. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega and the centripetal acceleration, directed towards the centre, has magnitude a = v^2/r = r*omega^2; for a conical pendulum, or a vehicle on a banked track, resolve forces vertically and horizontally to link the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with energy conservation; for a particle on a string, or on the inside of a smooth track, the critical condition is that the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when this condition would otherwise be violated, in which case the rod is in thrust (compression) rather than tension.

1 A particle of mass 0.5 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N, the other end of the string being attached to a fixed point O. Take g = 9.8 m/s^2.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.05 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 1 is 7 marks)

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2 A particle P of mass 0.5 kg lies on a smooth horizontal table. It is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 10 N. The other end of the string is fixed to a point O on the table. P is held at a distance 1.3 m from O and released from rest.

(a) Find the tension in the string, and the initial acceleration of P. **(3)**

(b) Find the elastic potential energy stored in the string initially. **(2)**

(c) Find the speed of P at the instant when the string becomes slack. **(3)**

(Total for Question 2 is 8 marks)

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3 A particle of mass 4 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 6 m and the coefficient of friction between the particle and the plane is 0.2. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 3 is 9 marks)

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4 A particle P of mass 1 kg lies on a smooth horizontal table. It is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 30 N. The other end of the string is fixed to a point O on the table. P is held at a distance 0.7 m from O and released from rest.

(a) Find the tension in the string, and the initial acceleration of P. **(3)**

(b) Find the elastic potential energy stored in the string initially. **(2)**

(c) Find the speed of P at the instant when the string becomes slack. **(3)**

(Total for Question 4 is 8 marks)

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5 A metal ball of mass 1 kg is released from rest at a height of 2.5 m above the surface of soft sand. It penetrates a distance of 0.05 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8$ m/s^2.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 5 is 5 marks)

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6 A car of mass 1500 kg has an engine working at a constant rate of 40 kW. The resistance to motion is a constant 600 N whatever the speed. Take $g = 9.8\ m/s^2$.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 14 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/40$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 6 is 9 marks)

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7 A metal ball of mass 1 kg is released from rest at a height of 3 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 7 is 5 marks)

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8 A particle of mass 0.5 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 50 N, the other end of the string being attached to a fixed point O. Take $g = 9.8\ m/s^2$.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.1 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 8 is 7 marks)

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9 A car of mass 1500 kg has an engine working at a constant rate of 30 kW. The resistance to motion is a constant 600 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 10 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/40$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 9 is 9 marks)

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10 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 5/13$. The length of the plane is 8 m and the coefficient of friction between the particle and the plane is 0.1. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.4. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 10 is 9 marks)

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11 A car of mass 800 kg has an engine working at a constant rate of 30 kW. The resistance to motion is a constant 500 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

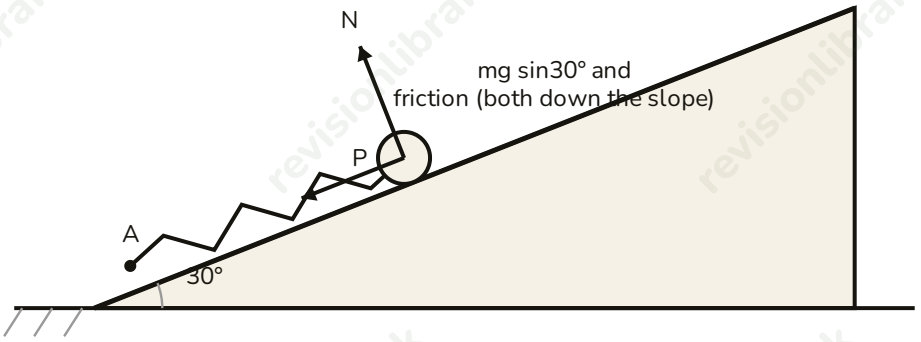
(b) Find the acceleration of the car on the horizontal road when its speed is 17 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/25$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 11 is 9 marks)

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12 A particle P of mass 1.5 kg lies on a rough plane inclined at 30 degrees to the horizontal. The coefficient of friction between P and the plane is 0.2. A light spring of natural length 0.9 m and modulus of elasticity 90 N lies along a line of greatest slope of the plane, with its lower end fixed to a point A at the bottom of the plane. P is attached to the upper end of the spring and is held at rest at the point where the spring is compressed to a length of 0.6 m. P is then released from rest and moves up the plane. Take $g = 9.8$ m/s^2.



spring natural length 0.9 m; P held at compressed length 0.6 m

(a) Find the elastic potential energy (EPE) stored in the spring in P's initial position. **(3)**

(b) Find the work done against friction, and the work done against gravity, as P moves from its initial position to the point where the spring first reaches its natural length (a distance of 0.3 m up the slope). **(4)**

(c) Use the work-energy principle to find the speed of P at the instant the spring reaches its natural length, giving your answer to 3 significant figures, ft your answers to (a) and (b). **(3)**

(Total for Question 12 is 10 marks)

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Mark scheme · 3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: 2-hour Pack

Question 1

(a) Hooke's law: $T = \lambda x/l = mg$ **M1**

(a) $x = 0.5(9.8)(0.4)/20 = 0.0980$ m **A1**

(a) Answer: 0.0980 m

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $20(0.0980)^2/(2 \times 0.4) = 0.240$ J **A1**

(b) Answer: 0.240 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.1480)^2 - (0.0980)^2]/(2l) - mg(0.05)$ **M1**

(c) $v = 0.500$ m/s **A1**

(c) Answer: 0.500 m/s

Question 2

(a) $T = \lambda x/l$ **M1**

(a) $T = 10 \times 0.3 = 3.00$ N **A1**

(a) $a = T/m = 6.00$ m/s^2 **A1**

(a) Answer: T = 3.00 N, a = 6.00 m/s^2

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $10(0.3)^2/(2 \times 1) = 0.450$ J **A1**

(b) Answer: 0.450 J

(c) conservation of energy: EPE = KE **M1**

(c) $(1/2)(0.5)v^2 = 0.4500$ **A1**

(c) $v = 1.34$ m/s **A1**

(c) Answer: 1.34 m/s

Question 3

(a) friction = $\mu R = \mu mg \cos \alpha$ **M1**

(a) work = friction x distance **M1**

(a) 18.1 J **A1**

(a) Answer: 18.1 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)4v^2 = 4(9.8)(6)(12/13) - 18.092$ **A1**

(b) $v = 9.98$ m/s **A1**

(b) Answer: 9.98 m/s

(c) work done against friction = initial KE **M1**

(c) $0.3 \times 4 \times 9.8 \times s = (1/2)4(9.975)^2$ **M1**

(c) $s = 16.9$ m **A1**

(c) Answer: 16.9 m

Question 4

(a) $T = \lambda x/l$ **M1**

(a) $T = 30 \times 0.2/0.5 = 12.0$ N **A1**

(a) $a = T/m = 12.0$ m/s^2 **A1**

(a) Answer: T = 12.0 N, a = 12.0 m/s^2

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $30(0.2)^2/(2 \times 0.5) = 1.20$ J **A1**

(b) Answer: 1.20 J

(c) conservation of energy: EPE = KE **M1**

(c) $(1/2)(1)v^2 = 1.2000$ **A1**

(c) $v = 1.55$ m/s **A1**

(c) Answer: 1.55 m/s

Question 5

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 2.5} = 7.00$ m/s **A1**

(a) Answer: 7.00 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $1(9.8)(2.5 + 0.05) = R \times 0.05$ **A1**

(b) $R = 500$ N **A1**

(b) Answer: 500 N

Question 6

(a) at maximum speed the driving force equals the resistance: $F = R$ **M1**

(a) uses $P = Fv$ **M1**

(a) $v = 40000/600 = 66.7$ m/s **A1**

(a) Answer: 66.7 m/s

(b) driving force = 40000/14 **M1**

(b) applies $F = ma$: driving force - resistance = ma **M1**

(b) $a = 1.5$ m/s^2 **A1**

(b) Answer: 1.5 m/s^2

(c) resolves along the slope: $F = R + mg \sin \alpha$ **M1**

(c) uses $P = Fv$ **M1**

(c) $v = 40000/(600 + 1500(9.8)/40) = 41.3$ m/s **A1**

(c) Answer: 41.3 m/s

Question 7

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 3} = 7.67$ m/s **A1**

(a) Answer: 7.67 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $1(9.8)(3 + 0.1) = R \times 0.1$ **A1**

(b) $R = 304$ N **A1**

(b) Answer: 304 N

Question 8

(a) Hooke's law: $T = \lambda x/l = mg$ **M1**

(a) $x = 0.5(9.8)(0.4)/50 = 0.0392$ m **A1**

(a) Answer: 0.0392 m

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $50(0.0392)^2/(2 \times 0.4) = 0.0960$ J **A1**

(b) Answer: 0.0960 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.1392)^2 - (0.0392)^2]/(2l) - mg(0.1)$ **M1**

(c) $v = 1.58$ m/s **A1**

(c) Answer: 1.58 m/s

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Question 9

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 30000/600 = 50.0 m/s **A1**

(a) Answer: 50.0 m/s

(b) driving force = 30000/10 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) a = 1.6 m/s^2 **A1**

(b) Answer: 1.6 m/s^2

(c) resolves along the slope: F = R + mg sin α **M1**

(c) uses P = Fv **M1**

(c) v = 30000/(600 + 1500(9.8)/40) = 31.0 m/s **A1**

(c) Answer: 31.0 m/s

Question 10

(a) friction = μ R = μ mg cos α **M1**

(a) work = friction x distance **M1**

(a) 14.5 J **A1**

(a) Answer: 14.5 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) (1/2)2v^2 = 2(9.8)(8)(5/13) - 14.474 **A1**

(b) v = 6.77 m/s **A1**

(b) Answer: 6.77 m/s

(c) work done against friction = initial KE **M1**

(c) 0.4 x 2 x 9.8 x s = (1/2)2(6.770)^2 **M1**

(c) s = 5.85 m **A1**

(c) Answer: 5.85 m

Question 11

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 30000/500 = 60.0 m/s **A1**

(a) Answer: 60.0 m/s

(b) driving force = 30000/17 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) a = 1.6 m/s^2 **A1**

(b) Answer: 1.6 m/s^2

(c) resolves along the slope: F = R + mg sin α **M1**

(c) uses P = Fv **M1**

(c) v = 30000/(500 + 800(9.8)/25) = 36.9 m/s **A1**

(c) Answer: 36.9 m/s

Question 12

(a) correct formula EPE = λ*x^2/(2*l) with x = 0.9 - 0.6 = 0.3 m **M1**

(a) correct substitution: 90*0.3^2/(2*0.9) **M1**

(a) EPE = 4.5 J, cao **A1**

(a) Answer: EPE = 4.5 J

(b) finds the normal reaction R = mg*cos(30) = awrt 12.7 N **M1**

(b) finds the friction force F = μ*R, then work done against friction = F*0.3 **M1**

(b) work done against friction = awrt 0.764 J **A1**

(b) work done against gravity = mg*0.3*sin(30) = 2.205 J **B1**

(b) Answer: Work against friction = awrt 0.764 J; work against gravity = 2.205 J

(c) sets up the energy equation: EPE = KE gained + work done against gravity + work done against friction, ft (a) and (b) **M1**

(c) substitutes: (1/2)(1.5)v^2 = 4.5 - 2.205 - 0.764 **M1**

(c) v = awrt 1.43 m/s, cao **A1**

(c) Answer: v = awrt 1.43 m/s

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