3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: 30-minute Pack

EDEXCEL 9FM0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: work-energy principle, elastic strings/springs and circular motion

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and treat every string or spring as light; strings are inextensible while rods are light and rigid. This worksheet is a depth paper combining three areas of Further Mechanics, momentum, impulse and collisions are assumed known from elsewhere and are not the focus here: the work-energy principle, elastic strings and springs, and circular motion, and several questions require you to combine two of these ideas within a single question. Kinetic energy is KE = (1/2)*m*v^2 and gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle (including work done against resistances such as friction) equals its change in kinetic energy. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when extended (or compressed) by x, has magnitude T = lambda*x/l, and the elastic potential energy stored is EPE = lambda*x^2/(2*l); an elastic string can only pull, and goes slack once its length is less than l, whereas a spring can both pull and push. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega and the centripetal acceleration, directed towards the centre, has magnitude a = v^2/r = r*omega^2; for a conical pendulum, or a vehicle on a banked track, resolve forces vertically and horizontally to link the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with energy conservation; for a particle on a string, or on the inside of a smooth track, the critical condition is that the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when this condition would otherwise be violated, in which case the rod is in thrust (compression) rather than tension.

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1 A particle P of mass 0.4 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point O on a smooth horizontal table. P is held on the table at the point where OP is a straight line and the string is stretched to a length of 0.9 m, and is then released from rest.

(a) Find the extension of the string in P's initial position. **(1)**

(b) Find the elastic potential energy (EPE) stored in the string initially. **(3)**

(c) Given that the table is smooth, find the speed of P at the instant the string first becomes slack, ft their answer to (b). **(3)**

(Total for Question 1 is 7 marks)

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2 A particle of mass 0.5 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 40 N, the other end of the string being attached to a fixed point O. Take $g = 9.8\ m/s^2$.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.15 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 2 is 7 marks)

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3 A metal ball of mass 2 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

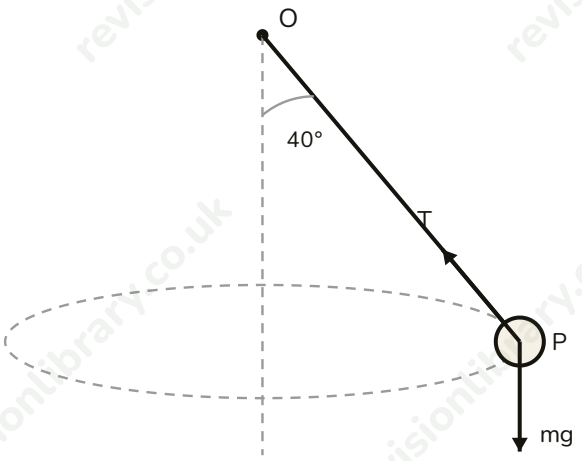
(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 3 is 5 marks)

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4 A particle P of mass 0.3 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point O. P moves with constant angular speed ω in a horizontal circle, with the string at a constant angle of 40 degrees to the downward vertical through O (a conical pendulum). Take $g = 9.8\ m/s^2$.



L = 0.5 m

(a) By resolving vertically, show that the tension in the string is given by T = mg/cos(40 degrees), and find the value of T, giving your answer to 3 significant figures. **(3)**

(b) Find the radius r of the circle in which P moves, giving your answer to 3 significant figures. **(2)**

(c) By considering the horizontal equation of motion of P, find the value of ω, and hence find the speed of P, giving each answer to 3 significant figures. **(4)**

(Total for Question 4 is 9 marks)

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Mark scheme · 3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: 30-minute Pack

Question 1

(a) extension = 0.9 - 0.6 = 0.3 m **B1**

(a) Answer: 0.3 m

(b) correct formula EPE = $\lambda * x^2/(2*l)$ **M1**

(b) correct substitution: $24*0.3^2/(2*0.6)$ **M1**

(b) EPE = 1.8 J, cao **A1**

(b) Answer: EPE = 1.8 J

(c) uses energy conservation: EPE = KE gained, since the table is smooth and no other horizontal force does work, ft (b) **M1**

(c) substitutes: $(1/2)(0.4)v^2 = 1.8$ **M1**

(c) v = 3 m/s, cao **A1**

(c) Answer: v = 3 m/s

Question 2

(a) Hooke's law: $T = \lambda x/l = mg$ **M1**

(a) $x = 0.5(9.8)(0.4)/40 = 0.0490$ m **A1**

(a) Answer: 0.0490 m

(b) EPE = $\lambda x^2/(2l)$ **M1**

(b) $40(0.0490)^2/(2 \times 0.4) = 0.120$ J **A1**

(b) Answer: 0.120 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.1990)^2 - (0.0490)^2]/(2l) - mg(0.15)$ **M1**

(c) v = 2.12 m/s **A1**

(c) Answer: 2.12 m/s

Question 3

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s **A1**

(a) Answer: 5.42 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $2(9.8)(1.5 + 0.1) = R \times 0.1$ **A1**

(b) R = 314 N **A1**

(b) Answer: 314 N

Question 4

(a) resolves vertically: $T*\cos(40) = mg$ **M1**

(a) rearranges to $T = mg/\cos(40)$, cso **A1**

(a) T = awrt 3.84 N **A1**

(a) Answer: T = mg/cos(40); T = awrt 3.84 N

(b) uses $r = L*\sin(40)$ with L = 0.5 m **M1**

(b) r = awrt 0.321 m **A1**

(b) Answer: r = awrt 0.321 m

(c) writes the horizontal equation of motion: $T*\sin(40) = m*\omega^2*r$, ft (a) and (b) **M1**

(c) rearranges and substitutes to find $\omega^2 = T*\sin(40)/(m*r)$ **M1**

(c) ω = awrt 5.06 rad/s **A1**

(c) speed $v = \omega*r$ = awrt 1.63 m/s, ft their ω and r **A1**

(c) Answer: ω = awrt 5.06 rad/s; v = awrt 1.63 m/s

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