3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: Test Pack

EDEXCEL 9FM0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

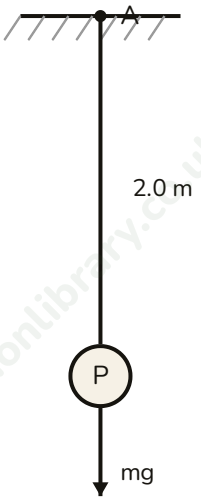
Key results: work-energy principle, elastic strings/springs and circular motion

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Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and treat every string or spring as light; strings are inextensible while rods are light and rigid. This worksheet is a depth paper combining three areas of Further Mechanics, momentum, impulse and collisions are assumed known from elsewhere and are not the focus here: the work-energy principle, elastic strings and springs, and circular motion, and several questions require you to combine two of these ideas within a single question. Kinetic energy is KE = (1/2)*m*v^2 and gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle (including work done against resistances such as friction) equals its change in kinetic energy. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when extended (or compressed) by x, has magnitude T = lambda*x/l, and the elastic potential energy stored is EPE = lambda*x^2/(2*l); an elastic string can only pull, and goes slack once its length is less than l, whereas a spring can both pull and push. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega and the centripetal acceleration, directed towards the centre, has magnitude a = v^2/r = r*omega^2; for a conical pendulum, or a vehicle on a banked track, resolve forces vertically and horizontally to link the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with energy conservation; for a particle on a string, or on the inside of a smooth track, the critical condition is that the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when this condition would otherwise be violated, in which case the rod is in thrust (compression) rather than tension.

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1 A light elastic string has natural length 1.6 m. One end of the string is attached to a fixed point A on a ceiling. A particle of mass 3 kg is attached to the other end of the string and hangs in equilibrium vertically below A, with the string stretched to a length of 2.0 m. The string obeys Hooke's law with modulus of elasticity λ newtons. Take $g = 9.8$ m/s^2 throughout.



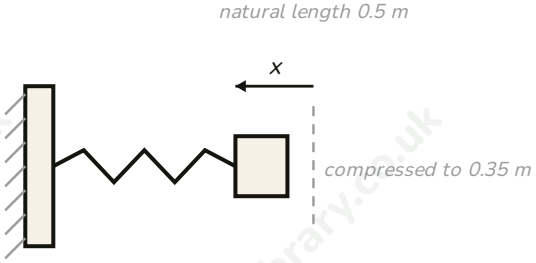
(a) Find the extension of the string in this position. **(1)**

(b) By modelling P as being in equilibrium, use Hooke's law to find the value of λ. **(3)**

(Total for Question 1 is 4 marks)

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2 A light elastic spring has natural length 0.5 m and modulus of elasticity 40 N. The spring is compressed, without buckling, to a length of 0.35 m.



(a) State the compression of the spring. **(1)**

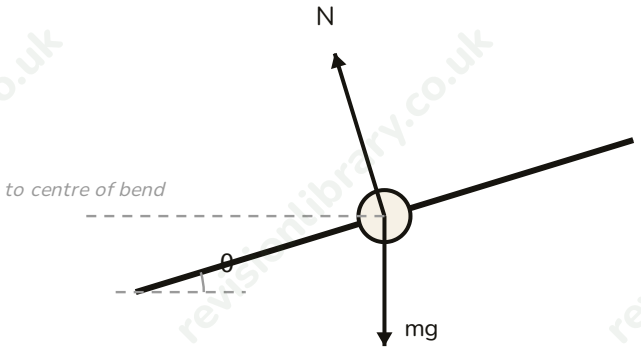
(b) Calculate the elastic potential energy (EPE) stored in the spring. **(3)**

(c) State one modelling assumption used in part (b) about the spring. **(1)**

(Total for Question 2 is 5 marks)

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3 A car of mass 900 kg, driven by Priya, travels at a constant speed round a circular bend of radius 50 m on a country road near Matlock in Derbyshire. The road is banked at an angle θ to the horizontal, where tan(θ) = 0.3. The road is modelled as smooth, and the car is modelled as a particle travelling at exactly the speed for which no sideways friction force is needed to keep it on the bend. Take g = 9.8 m/s^2.



cross-section of the banked road, radius 50 m

(a) Find the value of θ, giving your answer in degrees to 3 significant figures. **(2)**

(b) By resolving the forces on the car vertically and horizontally, show that v^2 = r*g*tan(θ), where v is the car's speed and r is the radius of the bend. **(3)**

(c) Hence find the speed, in m/s, at which Priya should drive round the bend. **(3)**

(Total for Question 3 is 8 marks)

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4 A particle of mass 1 kg hangs in equilibrium at the end of a light elastic string of natural length 0.5 m and modulus of elasticity 40 N, the other end of the string being attached to a fixed point O. Take g = 9.8 m/s^2.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

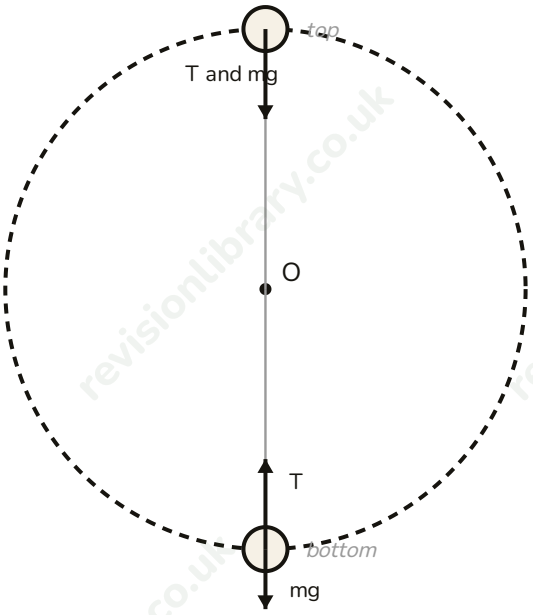
(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.2 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 4 is 7 marks)

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5 A particle of mass 0.5 kg is attached to one end of a light inextensible string of length 0.75 m. The other end of the string is attached to a fixed point O. The particle moves in a complete vertical circle of radius 0.75 m about O, with the string remaining taut throughout the motion. Take $g = 9.8\ m/s^2$.



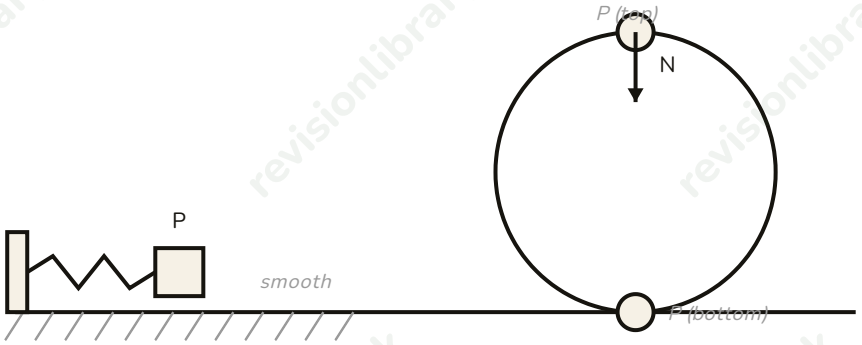
(a) By considering the particle at the top of the circle, show that the minimum possible speed there is given by $v^2 = gr$, and hence find this minimum speed, giving your answer to 3 significant figures. **(4)**

(b) Given that the particle has this minimum speed at the top of the circle, use the work-energy principle to find its speed at the bottom of the circle, giving your answer to 3 significant figures. **(4)**

(Total for Question 5 is 8 marks)

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6 In a school physics club demonstration in Leeds, run by a technician called Aisha, a particle P of mass 0.2 kg rests in contact with the free end of a light spring of natural length 0.3 m and modulus of elasticity 48 N. The spring is compressed so that its length is 0.15 m, and P is held at rest at the free end of the spring at the bottom of a smooth track. When released, the spring pushes P along a smooth horizontal section of track, after which P moves round the inside of a smooth vertical circular loop of radius 0.35 m. Take $g = 9.8\ m/s^2$.



loop radius 0.35 m

(a) Find the elastic potential energy (EPE) stored in the spring initially. **(3)**

(b) Given that all of this elastic potential energy is transferred to kinetic energy of P as it leaves the spring, find the speed of P as it leaves the spring, ft your answer to (a). **(2)**

(c) By finding the minimum speed at the bottom of the loop needed for P to maintain contact with the track throughout the loop, show that P does complete the loop without losing contact with the track. **(4)**

(d) Find the speed of P, and the normal reaction of the track on P, at the top of the loop, giving each answer to 3 significant figures. **(4)**

(Total for Question 6 is 13 marks)

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Mark scheme · 3.6 Further Mechanics: Work, Energy and Elastic Strings Depth: Test Pack

Question 1

(a) extension = 2.0 - 1.6 = 0.4 m **B1**

(a) Answer: 0.4 m

(b) uses equilibrium of P to find the tension: T = mg = 3 x 9.8 = 29.4 (N) **M1**

(b) applies Hooke's law T = λ*x/l with their T, x = 0.4 and l = 1.6 **M1**

(b) λ = 117.6 N, cao **A1**

(b) Answer: λ = 117.6 N

Question 2

(a) compression = 0.5 - 0.35 = 0.15 m **B1**

(a) Answer: 0.15 m

(b) correct formula EPE = $\lambda*x^2/(2*l)$ **M1**

(b) correct substitution: $40*0.15^2/(2*0.5)$ **M1**

(b) EPE = 0.9 J, cao **A1**

(b) Answer: EPE = 0.9 J

(c) e.g. the spring is light (has negligible mass), or the spring obeys Hooke's law up to this compression, or the spring does not buckle (oe) **B1**

(c) Answer: E.g. the spring is light (massless) and obeys Hooke's law up to this compression.

Question 3

(a) θ = arctan(0.3) **M1**

(a) θ = awrt 16.7 degrees **A1**

(a) Answer: θ = awrt 16.7 degrees

(b) resolves vertically: N*cos(θ) = mg **M1**

(b) resolves horizontally, towards the centre: $N*\sin(\theta) = m*v^2/r$ **M1**

(b) divides the two equations to eliminate N and m, giving $\tan(\theta) = v^2/(r*g)$, i.e. $v^2 = r*g*\tan(\theta)$, cso **A1**

(b) Answer: $v^2 = r*g*\tan(\theta)$

(c) substitutes into $v^2 = r*g*\tan(\theta)$: $v^2 = 50*9.8*0.3$, ft (b) **M1**

(c) $v^2 = 147$ **A1**

(c) v = awrt 12.1 m/s, cao **A1**

(c) Answer: v = awrt 12.1 m/s

Question 4

(a) Hooke's law: T = λ x/l = mg **M1**

(a) x = 1(9.8)(0.5)/40 = 0.123 m **A1**

(a) Answer: 0.123 m

(b) EPE = $\lambda\ x^2/(2l)$ **M1**

(b) $40(0.1225)^2/(2\ x\ 0.5) = 0.600$ J **A1**

(b) Answer: 0.600 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.3225)^2 - (0.1225)^2]/(2l) - mg(0.2)$ **M1**

(c) v = 1.79 m/s **A1**

(c) Answer: 1.79 m/s

Question 5

(a) Newton's second law towards the centre (downward) at the top: $T + mg = m*v^2/r$ **M1**

(a) sets T = 0 for the minimum (critical) case, the string is on the point of becoming slack **M1**

(a) $v^2 = gr$, cso **A1**

(a) $v = \sqrt{9.8*0.75}$ = awrt 2.71 m/s **A1**

(a) Answer: $v^2 = gr$; minimum speed = awrt 2.71 m/s

(b) uses energy conservation between top and bottom: $(1/2)m*v_bottom^2 = (1/2)m*v_{top}^2 + mg*h$ **M1**

(b) uses the correct height fallen, h = 2r = 1.5 m (the diameter) **M1**

(b) substitutes $v_{top}^2 = gr = 7.35$ to get $v_bottom^2 = 7.35 + 2*9.8*1.5 = 36.75$, ft (a) **M1**

(b) v_bottom = awrt 6.06 m/s, cao **A1**

(b) Answer: v_bottom = awrt 6.06 m/s

Question 6

(a) correct formula EPE = $\lambda*x^2/(2*l)$ with x = 0.3 - 0.15 = 0.15 m **M1**

(a) correct substitution: $48*0.15^2/(2*0.3)$ **M1**

(a) EPE = 1.8 J, cao **A1**

(a) Answer: EPE = 1.8 J

(b) uses $(1/2)*m*v^2$ = EPE, ft (a) **M1**

(b) $v = \sqrt{18}$ = awrt 4.24 m/s, cao **A1**

(b) Answer: v = awrt 4.24 m/s

(c) states the critical condition at the top of the loop: the normal reaction N = 0, so $v_{top}^2 = g*r$ **M1**

(c) uses energy conservation between the bottom and top of the loop (height 2r) to write the corresponding minimum speed at the bottom: $v_bottom(min)^2 = g*r + 2*g*(2r) = 5*g*r$ **M1**

(c) 5*g*r = 5*9.8*0.35 = 17.15, so the minimum speed-squared at the bottom is 17.15 $(m/s)^2$ **A1**

(c) compares with the actual value found in (b), $v^2 = 18 > 17.15$, so P has more than enough speed and maintains contact with the track throughout the loop, cso **A1**

(c) Answer: Minimum $v_bottom^2 = 17.15$; since 18 > 17.15, P maintains contact throughout the loop

(d) uses energy conservation from bottom to top: $v_{top}^2 = v_bottom^2 - 2*g*(2r)$, ft (b) **M1**

(d) v_{top} = awrt 2.07 m/s **A1**

(d) Newton's second law towards the centre (downward) at the top: $mg + N = m*v_{top}^2/r$, ft their v_{top} **M1**

(d) N = awrt 0.486 N, cao **A1**

(d) Answer: v_{top} = awrt 2.07 m/s; N = awrt 0.486 N

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