3.6 Further Mechanics: Work, Energy and Elastic Strings Depth

EDEXCEL 9FM0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: work-energy principle, elastic strings/springs and circular motion

Original content written for Revision Library.

Unless a question states otherwise, take g = 9.8 m/s^2, model each object as a particle, and treat every string or spring as light; strings are inextensible while rods are light and rigid. This worksheet is a depth paper combining three areas of Further Mechanics, momentum, impulse and collisions are assumed known from elsewhere and are not the focus here: the work-energy principle, elastic strings and springs, and circular motion, and several questions require you to combine two of these ideas within a single question. Kinetic energy is KE = (1/2)*m*v^2 and gravitational potential energy relative to a reference level is GPE = m*g*h; the work-energy principle states that the total work done by all the forces acting on a particle (including work done against resistances such as friction) equals its change in kinetic energy. Hooke's law states that the tension (or thrust) in an elastic string or spring of natural length l and modulus of elasticity lambda, when extended (or compressed) by x, has magnitude T = lambda*x/l, and the elastic potential energy stored is EPE = lambda*x^2/(2*l); an elastic string can only pull, and goes slack once its length is less than l, whereas a spring can both pull and push. For a particle moving on a circle of radius r with angular speed omega, the linear speed is v = r*omega and the centripetal acceleration, directed towards the centre, has magnitude a = v^2/r = r*omega^2; for a conical pendulum, or a vehicle on a banked track, resolve forces vertically and horizontally to link the tension or normal reaction to the speed. For vertical circular motion, combine Newton's second law in the radial direction with energy conservation; for a particle on a string, or on the inside of a smooth track, the critical condition is that the tension or normal reaction is zero, whereas a particle threaded on a rigid rod can maintain circular motion even when this condition would otherwise be violated, in which case the rod is in thrust (compression) rather than tension.

1 A block of mass 5 kg is pulled from rest along a rough horizontal floor by a constant horizontal force of 30 N. The coefficient of friction between the block and the floor is 0.25. The block travels a distance of 4 m. Take $g = 9.8\ m/s^2$.

(a) Find the work done against friction as the block travels the 4 m. **(2)**

(b) Use the work-energy principle to find the speed of the block after it has travelled the 4 m, giving your answer to 3 significant figures. **(4)**

(Total for Question 1 is 6 marks)

2 A particle P of mass 0.4 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point O on a smooth horizontal table. P is held on the table at the point where OP is a straight line and the string is stretched to a length of 0.9 m, and is then released from rest.

(a) Find the extension of the string in P's initial position. **(1)**

(b) Find the elastic potential energy (EPE) stored in the string initially. **(3)**

(c) Given that the table is smooth, find the speed of P at the instant the string first becomes slack, ft their answer to (b). **(3)**

(Total for Question 2 is 7 marks)

3 A particle P of mass 0.6 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 36 N. The other end of the string is attached to a fixed point O on a smooth horizontal table. P moves on the table in a horizontal circle of radius 0.65 m, with O at the centre of the circle, and with constant angular speed ω.

(a) Find the extension of the string. **(1)**

(b) Use Hooke's law to find the tension in the string. **(3)**

(c) By considering the equation of motion of P towards the centre of the circle, find the value of ω, giving your answer to 3 significant figures. **(4)**

(d) Find the time taken for P to complete one full revolution, giving your answer to 3 significant figures, ft your answer to (c). **(2)**

(e) Find the speed of P, giving your answer to 3 significant figures, ft your answer to (c). **(2)**

(Total for Question 3 is 12 marks)

---

4 A particle of mass 0.5 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 40 N, the other end of the string being attached to a fixed point O. Take $g = 9.8$ m/s^2.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.15 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 4 is 7 marks)

5 A metal ball of mass 2 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 5 is 5 marks)

6 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 5/13$. The length of the plane is 12 m and the coefficient of friction between the particle and the plane is 0.2. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 6 is 9 marks)

7 A particle of mass 0.5 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 20 N, the other end of the string being attached to a fixed point O. Take $g = 9.8\ m/s^2$.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.05 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 7 is 7 marks)

8 A metal ball of mass 4 kg is released from rest at a height of 1.5 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 8 is 5 marks)

9 A particle P of mass 0.5 kg lies on a smooth horizontal table. It is attached to one end of a light elastic string of natural length 1 m and modulus of elasticity 10 N. The other end of the string is fixed to a point O on the table. P is held at a distance 1.3 m from O and released from rest.

(a) Find the tension in the string, and the initial acceleration of P. **(3)**

(b) Find the elastic potential energy stored in the string initially. **(2)**

(c) Find the speed of P at the instant when the string becomes slack. **(3)**

(Total for Question 9 is 8 marks)

10 A particle of mass 4 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 12/13$. The length of the plane is 6 m and the coefficient of friction between the particle and the plane is 0.2. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.3. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 10 is 9 marks)

11 A particle P of mass 1 kg lies on a smooth horizontal table. It is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 30 N. The other end of the string is fixed to a point O on the table. P is held at a distance 0.7 m from O and released from rest.

(a) Find the tension in the string, and the initial acceleration of P. **(3)**

(b) Find the elastic potential energy stored in the string initially. **(2)**

(c) Find the speed of P at the instant when the string becomes slack. **(3)**

(Total for Question 11 is 8 marks)

12 A metal ball of mass 1 kg is released from rest at a height of 2.5 m above the surface of soft sand. It penetrates a distance of 0.05 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8\ m/s^2$.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 12 is 5 marks)

13 A car of mass 1500 kg has an engine working at a constant rate of 40 kW. The resistance to motion is a constant 600 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 14 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/40$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 13 is 9 marks)

14 A metal ball of mass 1 kg is released from rest at a height of 3 m above the surface of soft sand. It penetrates a distance of 0.1 m into the sand before coming to rest. The sand exerts a constant resisting force on the ball. Take $g = 9.8$ m/s^2.

(a) Find the speed of the ball as it hits the sand, to 3 significant figures. **(2)**

(b) Use the work-energy principle for the whole motion to find the magnitude of the resisting force, to 3 significant figures. **(3)**

(Total for Question 14 is 5 marks)

15 A particle of mass 0.5 kg hangs in equilibrium at the end of a light elastic string of natural length 0.4 m and modulus of elasticity 50 N, the other end of the string being attached to a fixed point O. Take $g = 9.8\ m/s^2$.

(a) Find the extension of the string in equilibrium, to 3 significant figures. **(2)**

(b) Find the elastic potential energy stored in the string in equilibrium, to 3 significant figures. **(2)**

(c) The particle is pulled a further 0.1 m vertically downwards and released from rest. Use conservation of energy to find the speed of the particle as it passes through the equilibrium position, to 3 significant figures. **(3)**

(Total for Question 15 is 7 marks)

16 A car of mass 1500 kg has an engine working at a constant rate of 30 kW. The resistance to motion is a constant 600 N whatever the speed. Take $g = 9.8$ m/s^2.

(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 10 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/40$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 16 is 9 marks)

17 A particle of mass 2 kg is released from rest at the top of a rough plane inclined at an angle α to the horizontal, where $\sin \alpha = 5/13$. The length of the plane is 8 m and the coefficient of friction between the particle and the plane is 0.1. At the bottom of the plane the particle moves onto a rough horizontal surface for which the coefficient of friction is 0.4. Take $g = 9.8$ m/s^2.

(a) Find the work done against friction as the particle moves down the plane, to 3 significant figures. **(3)**

(b) Use the work-energy principle to find the speed of the particle at the bottom of the plane, to 3 significant figures. **(3)**

(c) Find the distance the particle travels along the horizontal surface before coming to rest, to 3 significant figures. **(3)**

(Total for Question 17 is 9 marks)

18 A car of mass 800 kg has an engine working at a constant rate of 30 kW. The resistance to motion is a constant 500 N whatever the speed. Take $g = 9.8$ m/s^2.

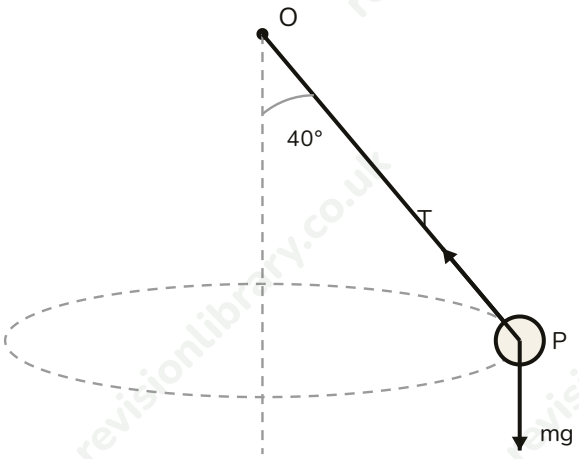
(a) Find the maximum speed of the car on a straight horizontal road, in m/s to 3 significant figures. **(3)**

(b) Find the acceleration of the car on the horizontal road when its speed is 17 m/s, to 2 significant figures. **(3)**

(c) The car now climbs a straight road inclined at an angle α to the horizontal where $\sin \alpha = 1/25$. The engine power and the resistance are unchanged. Find the maximum speed up the hill, to 3 significant figures. **(3)**

(Total for Question 18 is 9 marks)

19 A particle P of mass 0.3 kg is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point O. P moves with constant angular speed ω in a horizontal circle, with the string at a constant angle of 40 degrees to the downward vertical through O (a conical pendulum). Take $g = 9.8\ m/s^2$.



L = 0.5 m

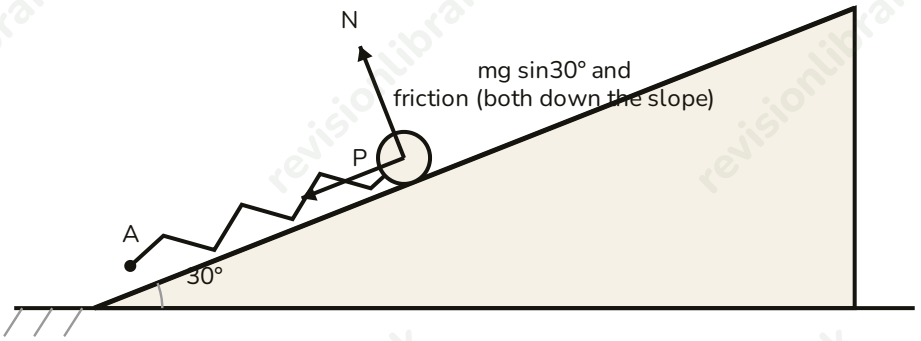
(a) By resolving vertically, show that the tension in the string is given by T = mg/cos(40 degrees), and find the value of T, giving your answer to 3 significant figures. **(3)**

(b) Find the radius r of the circle in which P moves, giving your answer to 3 significant figures. **(2)**

(c) By considering the horizontal equation of motion of P, find the value of ω, and hence find the speed of P, giving each answer to 3 significant figures. **(4)**

(Total for Question 19 is 9 marks)

20 A particle P of mass 1.5 kg lies on a rough plane inclined at 30 degrees to the horizontal. The coefficient of friction between P and the plane is 0.2. A light spring of natural length 0.9 m and modulus of elasticity 90 N lies along a line of greatest slope of the plane, with its lower end fixed to a point A at the bottom of the plane. P is attached to the upper end of the spring and is held at rest at the point where the spring is compressed to a length of 0.6 m. P is then released from rest and moves up the plane. Take $g = 9.8$ m/s^2.



spring natural length 0.9 m; P held at compressed length 0.6 m

(a) Find the elastic potential energy (EPE) stored in the spring in P's initial position. **(3)**

(b) Find the work done against friction, and the work done against gravity, as P moves from its initial position to the point where the spring first reaches its natural length (a distance of 0.3 m up the slope). **(4)**

(c) Use the work-energy principle to find the speed of P at the instant the spring reaches its natural length, giving your answer to 3 significant figures, ft your answers to (a) and (b). **(3)**

(Total for Question 20 is 10 marks)

21 A particle P of mass 0.5 kg is attached to one end of a light elastic string of natural length 1.0 m. The other end of the string is attached to a fixed point O. P is held at O and released from rest, falling vertically under gravity. The modulus of elasticity of the string is 20 N. Take g = 9.8 m/s^2.

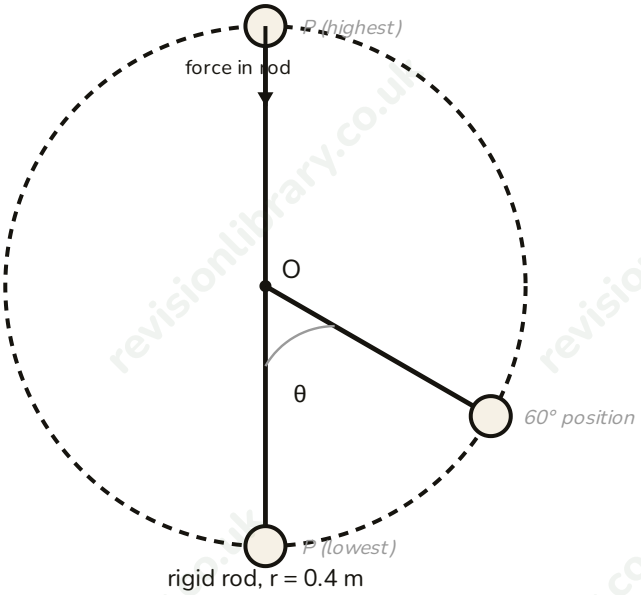
(a) Find the speed of P at the instant the string first becomes taut. **(3)**

(b) Given that P first comes to instantaneous rest when the string is stretched by a further extension of e metres beyond its natural length, use energy conservation to show that e satisfies $10*e^2 - 4.9*e - 4.9 = 0$, and hence find e, giving your answer to 3 significant figures. **(6)**

(c) Hence find the total distance fallen by P when it is first at instantaneous rest. **(2)**

(Total for Question 21 is 11 marks)

22 At an engineering workshop in Coventry, a technician called Dele is testing a model rotating-arm ride. A particle P of mass 0.25 kg is attached to one end of a light rigid rod OP of length 0.4 m; the other end of the rod is freely hinged at a fixed point O, and P moves in a complete vertical circle of radius 0.4 m about O. At the lowest point of the circle, P has speed 4 m/s. Take $g = 9.8$ m/s^2.



(a) Using the work-energy principle, find the speed of P at the point where OP makes an angle of 60 degrees with the downward vertical through O, giving your answer to 3 significant figures. **(4)**

(b) Find the speed of P at the highest point of the circle, giving your answer to 3 significant figures. **(3)**

(c) Find the magnitude of the force in the rod when P is at the highest point of the circle, and state whether the rod is in tension or in thrust (compression), ft your answer to (b). **(4)**

(Total for Question 22 is 11 marks)

Mark scheme · 3.6 Further Mechanics: Work, Energy and Elastic Strings Depth

Question 1

(a) finds the normal reaction R = mg = 49 N and the friction force F = μ*R = 0.25*49 = 12.25 N — **M1**

(a) work done against friction = 12.25*4 = 49 J, cao — **A1**

(a) Answer: 49 J

(b) finds the work done by the applied force: 30*4 = 120 J — **M1**

(b) sets up the work-energy equation: (1/2)*m*v^2 = work by force - work against friction, ft (a) — **M1**

(b) substitutes to get (1/2)(5)v^2 = 120 - 49 = 71 — **M1**

(b) v = awrt 5.33 m/s, cao — **A1**

(b) Answer: v = awrt 5.33 m/s

Question 2

(a) extension = 0.9 - 0.6 = 0.3 m — **B1**

(a) Answer: 0.3 m

(b) correct formula EPE = λ*x^2/(2*l) — **M1**

(b) correct substitution: 24*0.3^2/(2*0.6) — **M1**

(b) EPE = 1.8 J, cao — **A1**

(b) Answer: EPE = 1.8 J

(c) uses energy conservation: EPE = KE gained, since the table is smooth and no other horizontal force does work, ft (b) — **M1**

(c) substitutes: (1/2)(0.4)v^2 = 1.8 — **M1**

(c) v = 3 m/s, cao — **A1**

(c) Answer: v = 3 m/s

Question 3

(a) extension = 0.65 - 0.5 = 0.15 m — **B1**

(a) Answer: 0.15 m

(b) correct formula T = λ*x/l — **M1**

(b) correct substitution: 36*0.15/0.5, ft (a) — **M1**

(b) T = 10.8 N, cao — **A1**

(b) Answer: T = 10.8 N

(c) writes the equation of motion towards the centre: T = m*ω^2*r, ft (b) — **M1**

(c) rearranges to ω^2 = T/(m*r) — **M1**

(c) substitutes: ω^2 = 10.8/(0.6*0.65) = 27.7 (awrt) — **M1**

(c) ω = awrt 5.26 rad/s, cao — **A1**

(c) Answer: ω = awrt 5.26 rad/s

(d) uses time period = 2*π/ω, ft (c) — **M1**

(d) awrt 1.19 s — **A1**

(d) Answer: awrt 1.19 s

(e) uses v = ω*r, ft (c) — **M1**

(e) awrt 3.42 m/s — **A1**

(e) Answer: awrt 3.42 m/s

Question 4

(a) Hooke's law: T = λ x/l = mg — **M1**

(a) x = 0.5(9.8)(0.4)/40 = 0.0490 m — **A1**

(a) Answer: 0.0490 m

(b) EPE = λ x^2/(2l) — **M1**

(b) 40$(0.0490)^2$/(2 x 0.4) = 0.120 J — **A1**

(b) Answer: 0.120 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain — **M1**

(c) (1/2)mv^2 = λ[$(0.1990)^2$ - $(0.0490)^2$]/(2l) - mg(0.15) — **M1**

(c) v = 2.12 m/s — **A1**

(c) Answer: 2.12 m/s

Question 5

(a) conservation of energy (or v^2 = 2gh) — **M1**

(a) v = $\sqrt{2 \text{ x } 9.8 \text{ x } 1.5}$ = 5.42 m/s — **A1**

(a) Answer: 5.42 m/s

(b) loss of GPE over the whole descent = work done against the resistance — **M1**

(b) 2(9.8)(1.5 + 0.1) = R x 0.1 — **A1**

(b) R = 314 N — **A1**

(b) Answer: 314 N

Question 6

(a) friction = μ R = μ mg cos α — **M1**

(a) work = friction x distance — **M1**

(a) 43.4 J — **A1**

(a) Answer: 43.4 J

(b) KE gained = loss in GPE - work done against friction — **M1**

(b) (1/2)2v^2 = 2(9.8)(12)(5/13) - 43.422 — **A1**

(b) v = 6.86 m/s — **A1**

(b) Answer: 6.86 m/s

(c) work done against friction = initial KE — **M1**

(c) 0.3 x 2 x 9.8 x s = (1/2)2$(6.859)^2$ — **M1**

(c) s = 8.00 m — **A1**

(c) Answer: 8.00 m

Question 7

(a) Hooke's law: T = λ x/l = mg — **M1**

(a) x = 0.5(9.8)(0.4)/20 = 0.0980 m — **A1**

(a) Answer: 0.0980 m

(b) EPE = λ x^2/(2l) — **M1**

(b) 20$(0.0980)^2$/(2 x 0.4) = 0.240 J — **A1**

(b) Answer: 0.240 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain — **M1**

(c) (1/2)mv^2 = λ[$(0.1480)^2$ - $(0.0980)^2$]/(2l) - mg(0.05) — **M1**

(c) v = 0.500 m/s — **A1**

(c) Answer: 0.500 m/s

Question 8

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s **A1**

(a) Answer: 5.42 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $4(9.8)(1.5 + 0.1) = R \times 0.1$ **A1**

(b) $R = 627$ N **A1**

(b) Answer: 627 N

Question 9

(a) $T = \lambda x/l$ **M1**

(a) $T = 10 \times 0.3 = 3.00$ N **A1**

(a) $a = T/m = 6.00$ m/s^2 **A1**

(a) Answer: T = 3.00 N, a = 6.00 m/s^2

(b) EPE $= \lambda x^2/(2l)$ **M1**

(b) $10(0.3)^2/(2 \times 1) = 0.450$ J **A1**

(b) Answer: 0.450 J

(c) conservation of energy: EPE = KE **M1**

(c) $(1/2)(0.5)v^2 = 0.4500$ **A1**

(c) $v = 1.34$ m/s **A1**

(c) Answer: 1.34 m/s

Question 10

(a) friction $= \mu R = \mu mg \cos \alpha$ **M1**

(a) work = friction x distance **M1**

(a) 18.1 J **A1**

(a) Answer: 18.1 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) $(1/2)4v^2 = 4(9.8)(6)(12/13) - 18.092$ **A1**

(b) $v = 9.98$ m/s **A1**

(b) Answer: 9.98 m/s

(c) work done against friction = initial KE **M1**

(c) $0.3 \times 4 \times 9.8 \times s = (1/2)4(9.975)^2$ **M1**

(c) $s = 16.9$ m **A1**

(c) Answer: 16.9 m

Question 11

(a) $T = \lambda x/l$ **M1**

(a) $T = 30 \times 0.2/0.5 = 12.0$ N **A1**

(a) $a = T/m = 12.0$ m/s^2 **A1**

(a) Answer: T = 12.0 N, a = 12.0 m/s^2

(b) EPE $= \lambda x^2/(2l)$ **M1**

(b) $30(0.2)^2/(2 \times 0.5) = 1.20$ J **A1**

(b) Answer: 1.20 J

(c) conservation of energy: EPE = KE **M1**

(c) $(1/2)(1)v^2 = 1.2000$ **A1**

(c) $v = 1.55$ m/s **A1**

(c) Answer: 1.55 m/s

Question 12

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 2.5} = 7.00$ m/s **A1**

(a) Answer: 7.00 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $1(9.8)(2.5 + 0.05) = R \times 0.05$ **A1**

(b) $R = 500$ N **A1**

(b) Answer: 500 N

Question 13

(a) at maximum speed the driving force equals the resistance: $F = R$ **M1**

(a) uses $P = Fv$ **M1**

(a) $v = 40000/600 = 66.7$ m/s **A1**

(a) Answer: 66.7 m/s

(b) driving force = 40000/14 **M1**

(b) applies $F = ma$: driving force - resistance = ma **M1**

(b) $a = 1.5$ m/s^2 **A1**

(b) Answer: 1.5 m/s^2

(c) resolves along the slope: $F = R + mg \sin \alpha$ **M1**

(c) uses $P = Fv$ **M1**

(c) $v = 40000/(600 + 1500(9.8)/40) = 41.3$ m/s **A1**

(c) Answer: 41.3 m/s

Question 14

(a) conservation of energy (or $v^2 = 2gh$) **M1**

(a) $v = \sqrt{2 \times 9.8 \times 3} = 7.67$ m/s **A1**

(a) Answer: 7.67 m/s

(b) loss of GPE over the whole descent = work done against the resistance **M1**

(b) $1(9.8)(3 + 0.1) = R \times 0.1$ **A1**

(b) $R = 304$ N **A1**

(b) Answer: 304 N

Question 15

(a) Hooke's law: $T = \lambda x/l = mg$ **M1**

(a) $x = 0.5(9.8)(0.4)/50 = 0.0392$ m **A1**

(a) Answer: 0.0392 m

(b) EPE $= \lambda x^2/(2l)$ **M1**

(b) $50(0.0392)^2/(2 \times 0.4) = 0.0960$ J **A1**

(b) Answer: 0.0960 J

(c) energy: EPE at the lowest point = EPE at equilibrium + KE + GPE gain **M1**

(c) $(1/2)mv^2 = \lambda[(0.1392)^2 - (0.0392)^2]/(2l) - mg(0.1)$ **M1**

(c) $v = 1.58$ m/s **A1**

(c) Answer: 1.58 m/s

Question 16

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 30000/600 = 50.0 m/s **A1**

(a) Answer: 50.0 m/s

(b) driving force = 30000/10 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) a = 1.6 m/s^2 **A1**

(b) Answer: 1.6 m/s^2

(c) resolves along the slope: F = R + mg sin α **M1**

(c) uses P = Fv **M1**

(c) v = 30000/(600 + 1500(9.8)/40) = 31.0 m/s **A1**

(c) Answer: 31.0 m/s

Question 17

(a) friction = μ R = μ mg cos α **M1**

(a) work = friction x distance **M1**

(a) 14.5 J **A1**

(a) Answer: 14.5 J

(b) KE gained = loss in GPE - work done against friction **M1**

(b) (1/2)2v^2 = 2(9.8)(8)(5/13) - 14.474 **A1**

(b) v = 6.77 m/s **A1**

(b) Answer: 6.77 m/s

(c) work done against friction = initial KE **M1**

(c) 0.4 x 2 x 9.8 x s = (1/2)2(6.770)^2 **M1**

(c) s = 5.85 m **A1**

(c) Answer: 5.85 m

Question 18

(a) at maximum speed the driving force equals the resistance: F = R **M1**

(a) uses P = Fv **M1**

(a) v = 30000/500 = 60.0 m/s **A1**

(a) Answer: 60.0 m/s

(b) driving force = 30000/17 **M1**

(b) applies F = ma: driving force - resistance = ma **M1**

(b) a = 1.6 m/s^2 **A1**

(b) Answer: 1.6 m/s^2

(c) resolves along the slope: F = R + mg sin α **M1**

(c) uses P = Fv **M1**

(c) v = 30000/(500 + 800(9.8)/25) = 36.9 m/s **A1**

(c) Answer: 36.9 m/s

Question 19

(a) resolves vertically: T*cos(40) = mg **M1**

(a) rearranges to T = mg/cos(40), cso **A1**

(a) T = awrt 3.84 N **A1**

(a) Answer: T = mg/cos(40); T = awrt 3.84 N

(b) uses r = L*sin(40) with L = 0.5 m **M1**

(b) r = awrt 0.321 m **A1**

(b) Answer: r = awrt 0.321 m

(c) writes the horizontal equation of motion: T*sin(40) = m*ω^2*r, ft (a) and (b) **M1**

(c) rearranges and substitutes to find ω^2 = T*sin(40)/(m*r) **M1**

(c) ω = awrt 5.06 rad/s **A1**

(c) speed v = ω*r = awrt 1.63 m/s, ft their ω and r **A1**

(c) Answer: ω = awrt 5.06 rad/s; v = awrt 1.63 m/s

Question 20

(a) correct formula EPE = λ*x^2/(2*l) with x = 0.9 - 0.6 = 0.3 m **M1**

(a) correct substitution: 90*0.3^2/(2*0.9) **M1**

(a) EPE = 4.5 J, cao **A1**

(a) Answer: EPE = 4.5 J

(b) finds the normal reaction R = mg*cos(30) = awrt 12.7 N **M1**

(b) finds the friction force F = μ*R, then work done against friction = F*0.3 **M1**

(b) work done against friction = awrt 0.764 J **A1**

(b) work done against gravity = mg*0.3*sin(30) = 2.205 J **B1**

(b) Answer: Work against friction = awrt 0.764 J; work against gravity = 2.205 J

(c) sets up the energy equation: EPE = KE gained + work done against gravity + work done against friction, ft (a) and (b) **M1**

(c) substitutes: (1/2)(1.5)v^2 = 4.5 - 2.205 - 0.764 **M1**

(c) v = awrt 1.43 m/s, cao **A1**

(c) Answer: v = awrt 1.43 m/s

Question 21

(a) recognises P falls freely (the string is slack) for the first 1.0 m, and uses v^2 = u^2 + 2*g*s (or an energy method) with u = 0, s = 1.0 **M1**

(a) substitutes: v^2 = 2*9.8*1.0 = 19.6 **M1**

(a) v = awrt 4.43 m/s, cao **A1**

(a) Answer: v = awrt 4.43 m/s

(b) uses energy conservation between the release point and the lowest point, where both the initial and final kinetic energies are zero: loss in GPE = gain in EPE **M1**

(b) writes the total distance fallen as (1.0 + e), giving loss in GPE = mg*(1+e) = 4.9*(1+e) **M1**

(b) writes the EPE gained as λ*e^2/(2*l) = 20*e^2/2 = 10*e^2 **M1**

(b) forms the equation 4.9*(1+e) = 10*e^2, i.e. 10*e^2 - 4.9*e - 4.9 = 0, cso **A1**

(b) solves the quadratic using the quadratic formula, e = (4.9 ± $\sqrt{4.9^2 + 4*10*4.9}$)/20, and rejects the negative root **M1**

(b) e = awrt 0.987 m **A1**

(b) Answer: 10*e^2 - 4.9*e - 4.9 = 0; e = awrt 0.987 m

(c) total distance = 1.0 + e, ft (b) **M1**

(c) awrt 1.99 m **A1**

(c) Answer: awrt 1.99 m

Question 22

(a) finds the height risen above the lowest point: h = r*(1 - cos(60)) = 0.2 m **M1**

(a) uses energy conservation: $(1/2)*v^2 = (1/2)*v0^2 - g*h$, i.e. $v^2 = v0^2 - 2*g*h$ **M1**

(a) substitutes: $v^2 = 16 - 2*9.8*0.2 = 12.08$ **M1**

(a) v = awrt 3.48 m/s, cao **A1**

(a) Answer: v = awrt 3.48 m/s

(b) uses the height risen to the top, h = 2r = 0.8 m **M1**

(b) substitutes: $v_{top}^2 = 16 - 2*9.8*0.8 = 0.32$ **M1**

(b) v_{top} = awrt 0.566 m/s, cao **A1**

(b) Answer: v_{top} = awrt 0.566 m/s

(c) Newton's second law towards the centre (downward) at the top: $mg + F = m*v_{top}^2/r$, where F is the force in the rod taken as positive towards the centre (tension) **M1**

(c) substitutes: $F = m*v_{top}^2/r - mg = 0.25*0.32/0.4 - 0.25*9.8$, ft (b) **M1**

(c) F = -2.25 N, so the magnitude of the force in the rod is 2.25 N **A1**

(c) since F is negative (the weight alone exceeds the force needed for circular motion at this speed), the rod pushes P outward, so the rod is in thrust (compression), not tension **B1**

(c) Answer: 2.25 N of thrust (the rod is in compression)