2.1 Further Statistics: Discrete Random Variables and Expectation: 10-minute Pack

AQA 7367 · Calculator allowed · about 10 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key facts: discrete random variables and expectation

Original content written for Revision Library.

A discrete random variable X takes a countable set of values, each with an associated probability P(X = x), where the probabilities sum to 1. The expectation (mean) is E(X) = sum of x P(X = x), and the variance is Var(X) = E(X^2) - [E(X)]^2, where E(X^2) = sum of x^2 P(X = x). For constants a and b, expectation and variance obey the rules E(aX + b) = aE(X) + b and Var(aX + b) = a^2 Var(X); more generally, E(g(X)) = sum of g(x) P(X = x) for any function g. A discrete uniform distribution on the integers 1 to n, where every value is equally likely, has the standard results E(X) = (n + 1)/2 and Var(X) = (n^2 - 1)/12. If X1 and X2 are independent random variables, then E(X1 + X2) = E(X1) + E(X2) and Var(X1 + X2) = Var(X1) + Var(X2); by contrast, doubling a single variable (2X) multiplies its variance by 2^2 = 4, since scaling one variable and summing two independent copies are different operations. These results underpin every calculation in this worksheet.

1 A discrete random variable X has E(X) = 6 and Var(X) = 4. Find each of the following, stating the rule used in each case.

(a) E(2X) **(1)**

(b) E(X + 7) **(1)**

(c) Var(5X) **(1)**

(d) Var(X - 3) **(1)**

(e) E(4 - X) **(1)**

(f) Var(4 - X) **(1)**

(g) $E(X^2)$ **(2)**

(Total for Question 1 is 8 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 2.1 Further Statistics: Discrete Random Variables and Expectation: 10-minute Pack

Question 1

(a) $E(2X) = 2E(X) = 12$ cao **B1**

(a) Answer: 12

(b) $E(X + 7) = E(X) + 7 = 13$ cao **B1**

(b) Answer: 13

(c) $Var(5X) = 5^2 Var(X) = 100$ cao **B1**

(c) Answer: 100

(d) $Var(X - 3) = 1^2 Var(X) = 4$ cao (subtracting a constant does not change the variance) **B1**

(d) Answer: 4

(e) $E(4 - X) = 4 - E(X) = -2$ cao **B1**

(e) Answer: -2

(f) $Var(4 - X) = (-1)^2 Var(X) = 4$ cao **B1**

(f) Answer: 4

(g) rearranges $Var(X) = E(X^2) - [E(X)]^2$ to $E(X^2) = Var(X) + [E(X)]^2$ oe **M1**

(g) $E(X^2) = 4 + 36 = 40$ cao **A1**

(g) Answer: 40

© Revision Library | revisionlibrary.co.uk