2.1 Further Statistics: Discrete Random Variables and Expectation: 30-minute Pack

AQA 7367 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key facts: discrete random variables and expectation

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A discrete random variable X takes a countable set of values, each with an associated probability P(X = x), where the probabilities sum to 1. The expectation (mean) is E(X) = sum of x P(X = x), and the variance is Var(X) = E(X^2) - [E(X)]^2, where E(X^2) = sum of x^2 P(X = x). For constants a and b, expectation and variance obey the rules E(aX + b) = aE(X) + b and Var(aX + b) = a^2 Var(X); more generally, E(g(X)) = sum of g(x) P(X = x) for any function g. A discrete uniform distribution on the integers 1 to n, where every value is equally likely, has the standard results E(X) = (n + 1)/2 and Var(X) = (n^2 - 1)/12. If X1 and X2 are independent random variables, then E(X1 + X2) = E(X1) + E(X2) and Var(X1 + X2) = Var(X1) + Var(X2); by contrast, doubling a single variable (2X) multiplies its variance by 2^2 = 4, since scaling one variable and summing two independent copies are different operations. These results underpin every calculation in this worksheet.

1 At a youth club in Leicester, a fair spinner is numbered 1, 2, 3, 4, 5, with each number equally likely. Let X be the number the spinner lands on, so X has a discrete uniform distribution on {1, 2, 3, 4, 5}. In a game, a player wins Y pounds, where Y = 10X - 5.

(a) Using the standard results E(X) = (n + 1)/2 and Var(X) = $(n^2 - 1)/12$ for a discrete uniform distribution on {1, 2, ..., n}, state E(X) and Var(X) for this spinner. **(2)**

(b) Find E(Y). **(2)**

(c) Find Var(Y). **(2)**

(d) Find the standard deviation of Y, giving your answer to 3 significant figures. **(2)**

(Total for Question 1 is 8 marks)

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2 The discrete random variable X has probability distribution $P(X = x) = kx^2$ for x = 1, 2, 3, 4 and 0 otherwise, where k is a constant.

(a) Show that k = 1/30. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find E(3X + 5) and Var(3X + 5). **(4)**

(Total for Question 2 is 11 marks)

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3 A discrete random variable X takes values 0, 1, 2, 3 with probability distribution:

x: 0, 1, 2, 3

P(X = x): a, b, 0.3, 2b

Given that E(X) = 1.3,

(a) form two equations in a and b, and hence find the values of a and b. **(5)**

(b) Hence find Var(X). **(4)**

(Total for Question 3 is 9 marks)

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Mark scheme · 2.1 Further Statistics: Discrete Random Variables and Expectation: 30-minute Pack

Question 1

(a) E(X) = 3 **B1**

(a) Var(X) = 2 (= 24/12) **B1**

(a) Answer: E(X) = 3, Var(X) = 2

(b) E(Y) = 10E(X) - 5 oe **M1**

(b) E(Y) = 25 cao **A1**

(b) Answer: 25

(c) Var(Y) = 10^2 Var(X) oe **M1**

(c) Var(Y) = 200 cao **A1**

(c) Answer: 200

(d) sd(Y) = $\sqrt{\text{Var(Y)}} = \sqrt{200}$ **M1**

(d) awrt 14.1 **A1**

(d) Answer: 14.1 (3 s.f.)

Question 2

(a) uses the sum of all probabilities = 1 **M1**

(a) forms the sum P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) in terms of k **M1**

(a) k(30) = 1, so k = 1/30 (cso) **A1**

(a) Answer: k = 1/30

(b) E(X) = sum of x P(X = x) **M1**

(b) E(X) = 10/3 **A1**

(b) Var(X) = $E(X^2) - [E(X)]^2$ **M1**

(b) $E(X^2)$ = 59/5, Var(X) = 31/45 **A1**

(b) Answer: E(X) = 10/3, Var(X) = 31/45

(c) E(aX + b) = aE(X) + b **M1**

(c) 15 **A1**

(c) Var(aX + b) = a^2 Var(X) **M1**

(c) 31/5 **A1**

(c) Answer: E = 15, Var = 31/5

Question 3

(a) forms an equation from the sum of probabilities = 1: a + b + 0.3 + 2b = 1, leading to a + 3b = 0.7 oe **M1**

(a) forms an equation from E(X) = 1.3: 0(a) + 1(b) + 2(0.3) + 3(2b) = 1.3, leading to 7b + 0.6 = 1.3 oe **M1**

(a) solves the two equations simultaneously (dependent on both equations being formed) **dM1**

(a) b = 0.1 **A1**

(a) a = 0.4 cao **A1**

(a) Answer: a = 0.4, b = 0.1

(b) $E(X^2)$ = 0(0.4) + 1(0.1) + 4(0.3) + 9(0.2) oe **M1**

(b) $E(X^2)$ = 3.1 **A1**

(b) Var(X) = $E(X^2) - [E(X)]^2 = 3.1 - 1.3^2$ (dependent on their $E(X^2)$) **dM1**

(b) Var(X) = 1.41 cao **A1**

(b) Answer: 1.41

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