2.1 Further Statistics: Discrete Random Variables and Expectation

AQA 7367 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key facts: discrete random variables and expectation

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A discrete random variable X takes a countable set of values, each with an associated probability P(X = x), where the probabilities sum to 1. The expectation (mean) is E(X) = sum of x P(X = x), and the variance is Var(X) = E(X^2) - [E(X)]^2, where E(X^2) = sum of x^2 P(X = x). For constants a and b, expectation and variance obey the rules E(aX + b) = aE(X) + b and Var(aX + b) = a^2 Var(X); more generally, E(g(X)) = sum of g(x) P(X = x) for any function g. A discrete uniform distribution on the integers 1 to n, where every value is equally likely, has the standard results E(X) = (n + 1)/2 and Var(X) = (n^2 - 1)/12. If X1 and X2 are independent random variables, then E(X1 + X2) = E(X1) + E(X2) and Var(X1 + X2) = Var(X1) + Var(X2); by contrast, doubling a single variable (2X) multiplies its variance by 2^2 = 4, since scaling one variable and summing two independent copies are different operations. These results underpin every calculation in this worksheet.

1 A discrete random variable X has E(X) = 6 and Var(X) = 4. Find each of the following, stating the rule used in each case.

(a) E(2X) **(1)**

(b) E(X + 7) **(1)**

(c) Var(5X) **(1)**

(d) Var(X - 3) **(1)**

(e) E(4 - X) **(1)**

(f) Var(4 - X) **(1)**

(g) $E(X^2)$ **(2)**

(Total for Question 1 is 8 marks)

2 At a youth club in Leicester, a fair spinner is numbered 1, 2, 3, 4, 5, with each number equally likely. Let X be the number the spinner lands on, so X has a discrete uniform distribution on {1, 2, 3, 4, 5}. In a game, a player wins Y pounds, where $Y = 10X - 5$.

(a) Using the standard results $E(X) = (n + 1)/2$ and $Var(X) = (n^2 - 1)/12$ for a discrete uniform distribution on {1, 2, ..., n}, state E(X) and Var(X) for this spinner. **(2)**

(b) Find E(Y). **(2)**

(c) Find Var(Y). **(2)**

(d) Find the standard deviation of Y, giving your answer to 3 significant figures. **(2)**

(Total for Question 2 is 8 marks)

3 A discrete random variable X takes values 2, 4, 6 only, with $P(X = x) = kx$ for these values, where k is a constant.

(a) Show that $k = 1/12$. **(2)**

(b) Find E(X), giving your answer as an exact fraction. **(3)**

(c) Find Var(X), giving your answer as an exact fraction. **(4)**

(d) Hence find Var(6X - 2), giving your answer as an exact value. **(2)**

(Total for Question 3 is 11 marks)

4 The independent discrete random variables X and Y each take the values 1, 2 and 3. P(X = 1), P(X = 2), P(X = 3) are 1/3, 2/9, 4/9 respectively, and P(Y = 1), P(Y = 2), P(Y = 3) are 4/9, 4/9, 1/9 respectively.

(a) Find P(X = Y). **(2)**

(b) Find P(X > Y). **(2)**

(c) Find E(X + Y) and Var(X - Y). **(4)**

(Total for Question 4 is 8 marks)

5 The discrete random variable X has probability distribution $P(X = x) = kx^2$ for $x = 1, 2, 3, 4$ and 0 otherwise, where k is a constant.

(a) Show that $k = 1/30$. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find E(3X + 5) and Var(3X + 5). **(4)**

(Total for Question 5 is 11 marks)

6 The discrete random variable X has probability distribution $P(X = x) = k(5 - x)$ for $x = 1, 2, 3, 4$ and 0 otherwise, where k is a constant.

(a) Show that $k = 1/10$. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find E(4X + 5) and Var(4X + 5). **(4)**

(Total for Question 6 is 11 marks)

7 The independent discrete random variables X and Y each take the values 1, 2 and 3. P(X = 1), P(X = 2), P(X = 3) are 5/14, 3/7, 3/14 respectively, and P(Y = 1), P(Y = 2), P(Y = 3) are 4/11, 3/11, 4/11 respectively.

(a) Find P(X = Y). **(2)**

(b) Find P(X > Y). **(2)**

(c) Find E(X + Y) and Var(X - Y). **(4)**

(Total for Question 7 is 8 marks)

8 The independent discrete random variables X and Y each take the values 1, 2 and 3. P(X = 1), P(X = 2), P(X = 3) are 1/2, 5/12, 1/12 respectively, and P(Y = 1), P(Y = 2), P(Y = 3) are 3/11, 4/11, 4/11 respectively.

(a) Find P(X = Y). **(2)**

(b) Find P(X > Y). **(2)**

(c) Find E(X + Y) and Var(X - Y). **(4)**

(Total for Question 8 is 8 marks)

9 The discrete random variable X has probability distribution $P(X = x) = k(x + 1)$ for $x = 0, 1, 2, 3$ and 0 otherwise, where k is a constant.

(a) Show that $k = 1/10$. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find E(3X - 2) and Var(3X - 2). **(4)**

(Total for Question 9 is 11 marks)

10 The independent discrete random variables X and Y each take the values 1, 2 and 3. P(X = 1), P(X = 2), P(X = 3) are 3/14, 5/14, 3/7 respectively, and P(Y = 1), P(Y = 2), P(Y = 3) are 4/11, 4/11, 3/11 respectively.

(a) Find P(X = Y). **(2)**

(b) Find P(X > Y). **(2)**

(c) Find E(X + Y) and Var(X - Y). **(4)**

(Total for Question 10 is 8 marks)

11 The discrete random variable X has probability distribution $P(X = x) = k(x + 1)$ for $x = 0, 1, 2, 3$ and 0 otherwise, where k is a constant.

(a) Show that $k = 1/10$. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find E(4X + 2) and Var(4X + 2). **(4)**

(Total for Question 11 is 11 marks)

12 The independent discrete random variables X and Y each take the values 1, 2 and 3. P(X = 1), P(X = 2), P(X = 3) are 1/2, 1/4, 1/4 respectively, and P(Y = 1), P(Y = 2), P(Y = 3) are 2/7, 3/7, 2/7 respectively.

(a) Find P(X = Y). **(2)**

(b) Find P(X > Y). **(2)**

(c) Find E(X + Y) and Var(X - Y). **(4)**

(Total for Question 12 is 8 marks)

13 The discrete random variable X has probability distribution $P(X = x) = k(x + 1)$ for $x = 0, 1, 2, 3$ and 0 otherwise, where k is a constant.

(a) Show that $k = 1/10$. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find E(2X + 0) and Var(2X + 0). **(3)**

(Total for Question 13 is 10 marks)

14 At a school fete in Halifax, Priya runs a stall where a player's net winnings, X pounds, have the probability distribution:

x: -2, 0, 3, 5

P(X = x): 0.35, 0.25, 0.30, 0.10

(a) Find E(X). **(2)**

(b) Find Var(X). **(3)**

(c) Priya plays the game twice, independently; her net winnings on the two plays are X1 and X2, each with the same distribution as X. Find E(X1 + X2) and Var(X1 + X2). **(3)**

(d) Find $E(X^2 - 3X + 1)$. **(3)**

(Total for Question 14 is 11 marks)

15 A discrete random variable X takes values 0, 1, 2, 3 with probability distribution:

x: 0, 1, 2, 3

P(X = x): a, b, 0.3, 2b

Given that E(X) = 1.3,

(a) form two equations in a and b, and hence find the values of a and b. **(5)**

(b) Hence find Var(X). **(4)**

(Total for Question 15 is 9 marks)

16 At a school fete in Sheffield, a raffle uses tickets numbered 1 to 20, and the winning ticket, X, is drawn at random, so X has a discrete uniform distribution on {1, 2, ..., 20}.

(a) State E(X) and Var(X). **(2)**

(b) The prize money is Y = 5X + £10. Find E(Y) and Var(Y). **(4)**

(c) Two tickets are drawn independently, with replacement, giving ticket numbers X1 and X2. Let T = X1 + X2. Find E(T) and Var(T). **(4)**

(d) Find E(X1 - X2) and Var(X1 - X2). **(3)**

(Total for Question 16 is 13 marks)

17 A discrete random variable X takes only the values 0 and 1, with $P(X = 1) = p$ and $P(X = 0) = 1 - p$, where $0 < p < 1$.

(a) Show that $E(X) = p$. **(2)**

(b) Show that $Var(X) = p(1 - p)$. **(3)**

(Total for Question 17 is 5 marks)

18 A fairground 'Lucky Dip' bag at a fete in Cardiff contains tickets numbered 1 to 8, one of which is drawn at random. Let X be the number on the ticket drawn, so X has a discrete uniform distribution on {1, 2, ..., 8}. The prize money is modelled by $P = 3X^2 - 4X + 1$ (in pounds).

(a) State E(X) and Var(X), using the standard results for a discrete uniform distribution. **(2)**

(b) Find $E(X^2)$. **(3)**

(c) Hence find E(P). **(3)**

(d) The organiser wants $E(P) \leq 50$ to keep the game profitable for the fete. State, with a reason, whether the current prize structure meets this requirement. **(2)**

(Total for Question 18 is 10 marks)

19 Let X be a discrete random variable with mean $\mu = E(X)$. By definition, $Var(X) = E[(X - \mu)^2]$.

(a) Show that $Var(X) = E(X^2) - \mu^2$. **(4)**

(b) Hence show that $Var(aX + b) = a^2 Var(X)$, for constants a and b. **(4)**

(Total for Question 19 is 8 marks)

Mark scheme · 2.1 Further Statistics: Discrete Random Variables and Expectation

Question 1

(a) $E(2X) = 2E(X) = 12$ cao **B1**

(a) Answer: 12

(b) $E(X + 7) = E(X) + 7 = 13$ cao **B1**

(b) Answer: 13

(c) $Var(5X) = 5^2\ Var(X) = 100$ cao **B1**

(c) Answer: 100

(d) $Var(X - 3) = 1^2\ Var(X) = 4$ cao (subtracting a constant does not change the variance) **B1**

(d) Answer: 4

(e) $E(4 - X) = 4 - E(X) = -2$ cao **B1**

(e) Answer: -2

(f) $Var(4 - X) = (-1)^2\ Var(X) = 4$ cao **B1**

(f) Answer: 4

(g) rearranges $Var(X) = E(X^2) - [E(X)]^2$ to $E(X^2) = Var(X) + [E(X)]^2$ oe **M1**

(g) $E(X^2) = 4 + 36 = 40$ cao **A1**

(g) Answer: 40

Question 2

(a) $E(X) = 3$ **B1**

(a) $Var(X) = 2$ (= 24/12) **B1**

(a) Answer: E(X) = 3, Var(X) = 2

(b) $E(Y) = 10E(X) - 5$ oe **M1**

(b) $E(Y) = 25$ cao **A1**

(b) Answer: 25

(c) $Var(Y) = 10^2\ Var(X)$ oe **M1**

(c) $Var(Y) = 200$ cao **A1**

(c) Answer: 200

(d) $sd(Y) = \sqrt{Var(Y)} = \sqrt{200}$ **M1**

(d) awrt 14.1 **A1**

(d) Answer: 14.1 (3 s.f.)

Question 3

(a) $k(2 + 4 + 6) = 1$, i.e. $12k = 1$ oe **M1**

(a) $k = 1/12$ cso **A1**

(a) Answer: k = 1/12

(b) $E(X) = 2(2/12) + 4(4/12) + 6(6/12)$ oe **M1**

(b) $= (4 + 16 + 36)/12 = 56/12$ **A1**

(b) $= 14/3$ cao (exact fraction, simplified) **A1**

(b) Answer: E(X) = 14/3

(c) $E(X^2) = 4(2/12) + 16(4/12) + 36(6/12)$ oe **M1**

(c) $E(X^2) = (8 + 64 + 216)/12 = 288/12 = 24$ **A1**

(c) $Var(X) = E(X^2) - [E(X)]^2 = 24 - (14/3)^2$ (dependent on their $E(X^2)$ and $E(X)$) **dM1**

(c) $= 20/9$ cao **A1**

(c) Answer: Var(X) = 20/9

(d) $Var(6X - 2) = 6^2\ Var(X) = 36(20/9)$ oe **M1**

(d) $= 80$ cao **A1**

(d) Answer: 80

Question 4

(a) sum of $P(X = i)P(Y = i)$ over $i = 1, 2, 3$, using independence **M1**

(a) 8/27 **A1**

(a) Answer: 8/27

(b) adds $P(X = i)P(Y = j)$ over the cases $i > j$ **M1**

(b) 40/81 **A1**

(b) Answer: 40/81

(c) $E(X + Y) = E(X) + E(Y)$ **M1**

(c) 34/9 **A1**

(c) $Var(X - Y) = Var(X) + Var(Y)$ for independent variables **M1**

(c) 98/81 **A1**

(c) Answer: E(X + Y) = 34/9, Var(X - Y) = 98/81

Question 5

(a) uses the sum of all probabilities = 1 **M1**

(a) forms the sum $P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$ in terms of k **M1**

(a) $k(30) = 1$, so $k = 1/30$ (cso) **A1**

(a) Answer: k = 1/30

(b) $E(X) =$ sum of $x\ P(X = x)$ **M1**

(b) $E(X) = 10/3$ **A1**

(b) $Var(X) = E(X^2) - [E(X)]^2$ **M1**

(b) $E(X^2) = 59/5$, $Var(X) = 31/45$ **A1**

(b) Answer: E(X) = 10/3, Var(X) = 31/45

(c) $E(aX + b) = aE(X) + b$ **M1**

(c) 15 **A1**

(c) $Var(aX + b) = a^2\ Var(X)$ **M1**

(c) 31/5 **A1**

(c) Answer: E = 15, Var = 31/5

Question 6

(a) uses the sum of all probabilities = 1 **M1**

(a) forms the sum $P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$ in terms of k **M1**

(a) $k(10) = 1$, so $k = 1/10$ (cso) **A1**

(a) Answer: k = 1/10

(b) $E(X) =$ sum of $x\ P(X = x)$ **M1**

(b) $E(X) = 2$ **A1**

(b) $Var(X) = E(X^2) - [E(X)]^2$ **M1**

(b) $E(X^2) = 5$, $Var(X) = 1$ **A1**

(b) Answer: E(X) = 2, Var(X) = 1

(c) $E(aX + b) = aE(X) + b$ **M1**

(c) 13 **A1**

(c) $Var(aX + b) = a^2\ Var(X)$ **M1**

(c) 16 **A1**

(c) Answer: E = 13, Var = 16

Question 7

(a) sum of $P(X = i)P(Y = i)$ over i = 1, 2, 3, using independence **M1**

(a) 25/77 **A1**

(a) Answer: 25/77

(b) adds $P(X = i)P(Y = j)$ over the cases i > j **M1**

(b) 45/154 **A1**

(b) Answer: 45/154

(c) E(X + Y) = E(X) + E(Y) **M1**

(c) 27/7 **A1**

(c) Var(X - Y) = Var(X) + Var(Y) for independent variables **M1**

(c) 689/539 **A1**

(c) Answer: E(X + Y) = 27/7, Var(X - Y) = 689/539

Question 8

(a) sum of $P(X = i)P(Y = i)$ over i = 1, 2, 3, using independence **M1**

(a) 7/22 **A1**

(a) Answer: 7/22

(b) adds $P(X = i)P(Y = j)$ over the cases i > j **M1**

(b) 1/6 **A1**

(b) Answer: 1/6

(c) E(X + Y) = E(X) + E(Y) **M1**

(c) 485/132 **A1**

(c) Var(X - Y) = Var(X) + Var(Y) for independent variables **M1**

(c) 18083/17424 **A1**

(c) Answer: E(X + Y) = 485/132, Var(X - Y) = 18083/17424

Question 9

(a) uses the sum of all probabilities = 1 **M1**

(a) forms the sum P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) in terms of k **M1**

(a) k(10) = 1, so k = 1/10 (cso) **A1**

(a) Answer: k = 1/10

(b) E(X) = sum of x P(X = x) **M1**

(b) E(X) = 2 **A1**

(b) $Var(X) = E(X^2) - [E(X)]^2$ **M1**

(b) $E(X^2) = 5$, Var(X) = 1 **A1**

(b) Answer: E(X) = 2, Var(X) = 1

(c) E(aX + b) = aE(X) + b **M1**

(c) 4 **A1**

(c) $Var(aX + b) = a^2 Var(X)$ **M1**

(c) 9 **A1**

(c) Answer: E = 4, Var = 9

Question 10

(a) sum of $P(X = i)P(Y = i)$ over i = 1, 2, 3, using independence **M1**

(a) 25/77 **A1**

(a) Answer: 25/77

(b) adds $P(X = i)P(Y = j)$ over the cases i > j **M1**

(b) 34/77 **A1**

(b) Answer: 34/77

(c) E(X + Y) = E(X) + E(Y) **M1**

(c) 635/154 **A1**

(c) Var(X - Y) = Var(X) + Var(Y) for independent variables **M1**

(c) 29053/23716 **A1**

(c) Answer: E(X + Y) = 635/154, Var(X - Y) = 29053/23716

Question 11

(a) uses the sum of all probabilities = 1 **M1**

(a) forms the sum P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) in terms of k **M1**

(a) k(10) = 1, so k = 1/10 (cso) **A1**

(a) Answer: k = 1/10

(b) E(X) = sum of x P(X = x) **M1**

(b) E(X) = 2 **A1**

(b) $Var(X) = E(X^2) - [E(X)]^2$ **M1**

(b) $E(X^2) = 5$, Var(X) = 1 **A1**

(b) Answer: E(X) = 2, Var(X) = 1

(c) E(aX + b) = aE(X) + b **M1**

(c) 10 **A1**

(c) $Var(aX + b) = a^2 Var(X)$ **M1**

(c) 16 **A1**

(c) Answer: E = 10, Var = 16

Question 12

(a) sum of $P(X = i)P(Y = i)$ over i = 1, 2, 3, using independence **M1**

(a) 9/28 **A1**

(a) Answer: 9/28

(b) adds $P(X = i)P(Y = j)$ over the cases i > j **M1**

(b) 1/4 **A1**

(b) Answer: 1/4

(c) E(X + Y) = E(X) + E(Y) **M1**

(c) 15/4 **A1**

(c) Var(X - Y) = Var(X) + Var(Y) for independent variables **M1**

(c) 141/112 **A1**

(c) Answer: E(X + Y) = 15/4, Var(X - Y) = 141/112

Question 13

(a) uses the sum of all probabilities = 1 **M1**

(a) forms the sum P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) in terms of k **M1**

(a) k(10) = 1, so k = 1/10 (cso) **A1**

(a) Answer: k = 1/10

(b) E(X) = sum of x P(X = x) **M1**

(b) E(X) = 2 **A1**

(b) Var(X) = $E(X^2) - [E(X)]^2$ **M1**

(b) $E(X^2) = 5$, Var(X) = 1 **A1**

(b) Answer: E(X) = 2, Var(X) = 1

(c) E(aX + b) = aE(X) + b **M1**

(c) Var(aX + b) = a^2 Var(X) **M1**

(c) 4 **A1**

(c) Answer: E = 4, Var = 4

Question 14

(a) sum x P(X = x) = -2(0.35) + 0(0.25) + 3(0.30) + 5(0.10) oe **M1**

(a) E(X) = 0.7 cao **A1**

(a) Answer: 0.7

(b) $E(X^2) = (-2)^2(0.35) + 0^2(0.25) + 3^2(0.30) + 5^2(0.10)$ oe **M1**

(b) $E(X^2) = 6.6$ **A1**

(b) Var(X) = $6.6 - 0.7^2 = 6.11$ cao **A1**

(b) Answer: 6.11

(c) E(X1 + X2) = E(X1) + E(X2) = 2(0.7) oe **M1**

(c) E(X1 + X2) = 1.4 **A1**

(c) since X1 and X2 are independent, Var(X1 + X2) = Var(X1) + Var(X2) = 2(6.11) = 12.22 ft **B1**

(c) Answer: E(X1 + X2) = 1.4, Var(X1 + X2) = 12.22

(d) $E(X^2 - 3X + 1) = E(X^2) - 3E(X) + 1$ oe **M1**

(d) substitutes their $E(X^2) = 6.6$ and E(X) = 0.7 **A1**

(d) = 5.5 cao **A1**

(d) Answer: 5.5

Question 15

(a) forms an equation from the sum of probabilities = 1: a + b + 0.3 + 2b = 1, leading to a + 3b = 0.7 oe **M1**

(a) forms an equation from E(X) = 1.3: 0(a) + 1(b) + 2(0.3) + 3(2b) = 1.3, leading to 7b + 0.6 = 1.3 oe **M1**

(a) solves the two equations simultaneously (dependent on both equations being formed) **dM1**

(a) b = 0.1 **A1**

(a) a = 0.4 cao **A1**

(a) Answer: a = 0.4, b = 0.1

(b) $E(X^2) = 0(0.4) + 1(0.1) + 4(0.3) + 9(0.2)$ oe **M1**

(b) $E(X^2) = 3.1$ **A1**

(b) Var(X) = $E(X^2) - [E(X)]^2 = 3.1 - 1.3^2$ (dependent on their $E(X^2)$) **dM1**

(b) Var(X) = 1.41 cao **A1**

(b) Answer: 1.41

Question 16

(a) E(X) = 10.5 **B1**

(a) Var(X) = 33.25 (= 399/12) **B1**

(a) Answer: E(X) = 10.5, Var(X) = 33.25

(b) E(Y) = 5E(X) + 10 oe **M1**

(b) E(Y) = 62.5 **A1**

(b) Var(Y) = 5^2 Var(X) oe **M1**

(b) Var(Y) = 831.25 cao **A1**

(b) Answer: E(Y) = 62.5, Var(Y) = 831.25

(c) E(T) = E(X1) + E(X2) = 2(10.5) oe **M1**

(c) E(T) = 21 **A1**

(c) since the draws are independent, Var(T) = Var(X1) + Var(X2) **B1**

(c) Var(T) = 66.5 cao (ft) **A1**

(c) Answer: E(T) = 21, Var(T) = 66.5

(d) E(X1 - X2) = E(X1) - E(X2) = 0 **B1**

(d) since X1 and X2 are independent, Var(X1 - X2) = Var(X1) + Var(X2) (variances still add for a difference) **M1**

(d) Var(X1 - X2) = 66.5 cao **A1**

(d) Answer: E(X1 - X2) = 0, Var(X1 - X2) = 66.5

Question 17

(a) E(X) = 0(1 - p) + 1(p) oe **M1**

(a) = p cso **A1**

(a) Answer: E(X) = p

(b) $E(X^2) = 0^2(1 - p) + 1^2(p) = p$ **M1**

(b) Var(X) = $E(X^2) - [E(X)]^2 = p - p^2$ **M1**

(b) = p(1 - p) cso (factorised form required) **A1**

(b) Answer: Var(X) = p(1 - p)

Question 18

(a) E(X) = 4.5 **B1**

(a) Var(X) = 5.25 (= 63/12) **B1**

(a) Answer: E(X) = 4.5, Var(X) = 5.25

(b) rearranges Var(X) = $E(X^2) - [E(X)]^2$ to $E(X^2)$ = Var(X) + $[E(X)]^2$ oe **M1**

(b) $E(X^2) = 5.25 + 4.5^2$ **A1**

(b) = 25.5 cao **A1**

(b) Answer: 25.5

(c) E(P) = $3E(X^2) - 4E(X) + 1$ oe **M1**

(c) substitutes 3(25.5) - 4(4.5) + 1 **A1**

(c) = 59.5 cao **A1**

(c) Answer: 59.5

(d) correctly states that the requirement is not met (No) **B1**

(d) gives a valid reason referencing the calculated value, e.g. since E(P) = 59.5 > 50 oe **B1**

(d) Answer: No; since E(P) = £59.5, which is greater than £50, the average prize paid out exceeds the organiser's target, so the game as currently structured is not sufficiently profitable.

Question 19

(a) expands $(X - \mu)^2 = X^2 - 2\mu X + \mu^2$ **M1**

(a) uses linearity of E, treating μ as a constant: $E[(X - \mu)^2] = E(X^2) - 2\mu E(X) + \mu^2$ **M1**

(a) substitutes $E(X) = \mu$ to give $E(X^2) - 2\mu^2 + \mu^2$ **A1**

(a) $Var(X) = E(X^2) - \mu^2$ cso **A1**

(a) Answer: $Var(X) = E(X^2) - \mu^2$ (shown)

(b) lets $Y = aX + b$, so $E(Y) = aE(X) + b = a\mu + b$ **M1**

(b) $Y - E(Y) = aX + b - (a\mu + b) = a(X - \mu)$ **M1**

(b) $Var(Y) = E[(Y - E(Y))^2] = E[a^2 (X - \mu)^2] = a^2 E[(X - \mu)^2]$ **A1**

(b) $= a^2 Var(X)$ cso **A1**

(b) Answer: $Var(aX + b) = a^2 Var(X)$ (shown)