2.2 Further Statistics: Poisson and Geometric Distributions: 1-hour Pack

AQA 7367 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Buses arrive at a stop in central Manchester singly and at random, independently of each other, at a constant average rate of 2.5 buses per 15-minute interval.

(a) Find the probability that exactly 3 buses arrive during a 15-minute interval. **(2)**

(b) Find the probability that at least one bus arrives during a 6-minute interval. **(3)**

(c) Using the additive property of the Poisson distribution, find the probability that exactly 5 buses arrive during a 30-minute interval. **(3)**

(d) During the evening rush hour, a passenger notices that buses tend to arrive in clusters, with long gaps followed by several buses arriving close together. State, with a reason, whether the Poisson model above remains appropriate during rush hour. **(2)**

(Total for Question 1 is 10 marks)

---

2 A quality-control officer, Amara, at an electronics plant in Coventry tests resistors from a production line one at a time. Each resistor is independently faulty with probability 0.08. Let X be the number of resistors tested up to and including the first faulty one found.

(a) State the distribution of X. **(1)**

(b) Find P(X = 6). **(2)**

(c) Find P(X ≤ 10). **(2)**

(d) State E(X), and interpret its value in the context of this scenario. **(2)**

(e) Find the least number of resistors, n, such that P(X ≤ n) > 0.9. **(4)**

(Total for Question 2 is 11 marks)

© Revision Library | revisionlibrary.co.uk

3 A factory in Nottingham claims that the number of faulty items it produces per day follows a Poisson distribution with mean 2.3.

(a) Using this model, find $P(X \geq 6)$. **(2)**

(b) Over a 30-day production period, the factory records that on 2 of the 30 days there were 6 or more faulty items. Compare this observed proportion with the probability found in part (a), and comment on whether the claimed Po(2.3) model seems consistent with the data. **(3)**

(Total for Question 3 is 5 marks)

---

4 The random variable X has distribution B(100, 0.03).

(a) State two conditions under which a Poisson distribution is a suitable approximation to a binomial distribution, and give the parameter of the approximating distribution. **(2)**

(b) Use the binomial distribution to find $P(X = 4)$, and use the Poisson approximation to find the same probability, each to 4 significant figures. **(4)**

(c) Find the percentage error in using the Poisson approximation here, to 2 significant figures. **(2)**

(Total for Question 4 is 8 marks)

© Revision Library | revisionlibrary.co.uk

5 The number of calls to a helpline per minute may be modelled by a Poisson distribution with mean 2. Let X be the number in one such interval, so X ~ Po(2).

(a) Find P(X = 1) and P(X ≥ 1), each to 3 significant figures. **(4)**

(b) State the mean and variance of X, and explain why they are equal in a Poisson model. **(2)**

(c) A second independent count Y has distribution Po(1.5). Find P(X + Y = 1), to 3 significant figures. **(3)**

(Total for Question 5 is 9 marks)

---

6 A factory produces lightbulbs. Each lightbulb is defective independently of all others, with probability 0.004. A batch of 500 lightbulbs is inspected, and X is the number of defective lightbulbs in the batch.

(a) State the exact distribution of X. **(1)**

(b) Justify why X may be well approximated by a Poisson distribution, and state the parameter of the approximating distribution. **(2)**

(c) Using the Poisson approximation, find P(X = 3). **(2)**

(d) Calculate the exact binomial probability P(X = 3) to 4 significant figures, and comment on the accuracy of the Poisson approximation used in part (c). **(3)**

(Total for Question 6 is 8 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 2.2 Further Statistics: Poisson and Geometric Distributions: 1-hour Pack

Question 1

(a) correct use of $P(X=3) = e^{-2.5} * 2.5^3 / 3!$ with $X \sim Po(2.5)$ **M1**

(a) awrt 0.214 **A1**

(a) Answer: P(X = 3) = awrt 0.214

(b) scales the rate to a 6-minute interval, $\lambda = 2.5*(6/15) = 1.0$ **M1**

(b) uses $P(X \geq 1) = 1 - P(X=0)$ **M1**

(b) awrt 0.632 **A1**

(b) Answer: P(X ≥ 1) = awrt 0.632

(c) recognises the number of buses in two independent, consecutive 15-minute periods sum to give Po(2.5+2.5) = Po(5.0) by the additive property **M1**

(c) correct use of $P(X=5) = e^{-5} * 5^5 / 5!$ with $\lambda = 5.0$ **M1**

(c) awrt 0.175 **A1**

(c) Answer: P(X = 5) = awrt 0.175

(d) states no, the Poisson model is no longer appropriate **B1**

(d) valid reason, e.g. buses arriving in clusters suggests arrivals are no longer independent of each other and/or the rate is not constant throughout rush hour, violating the conditions required for a Poisson model (oe) **B1**

(d) Answer: No; clustering suggests arrivals are not independent and/or the rate is not constant during rush hour, so the Poisson conditions no longer hold.

Question 2

(a) $X \sim Geo(0.08)$, cao **B1**

(a) Answer: X ~ Geo(0.08)

(b) correct use of $P(X=6) = (0.92)^5 * 0.08$ **M1**

(b) awrt 0.0527 **A1**

(b) Answer: P(X = 6) = awrt 0.0527

(c) uses $P(X \leq 10) = 1 - (0.92)^{10}$ **M1**

(c) awrt 0.566 **A1**

(c) Answer: P(X ≤ 10) = awrt 0.566

(d) E(X) = 1/0.08 = 12.5 **B1**

(d) valid interpretation, e.g. on average, Amara needs to test 12.5 resistors before finding the first faulty one (oe) **B1**

(d) Answer: E(X) = 12.5; on average, 12.5 resistors need to be tested before the first faulty one is found.

(e) sets up the inequality $1 - (0.92)^n > 0.9$ **M1**

(e) rearranges to $(0.92)^n < 0.1$ **M1**

(e) takes logarithms and solves to $n > \ln(0.1)/\ln(0.92)$, awrt 27.6 **dM1**

(e) checks n=27 gives $P(X \leq 27)$ = awrt 0.895 (fails) and n=28 gives $P(X \leq 28)$ = awrt 0.903 (satisfies); least n = 28, cao **A1**

(e) Answer: Least n = 28

Question 3

(a) uses $P(X \geq 6) = 1 - P(X \leq 5)$ with $X \sim Po(2.3)$ **M1**

(a) awrt 0.0300 **A1**

(a) Answer: P(X ≥ 6) = awrt 0.0300

(b) computes the observed proportion of days with 6 or more faults, 2/30 = awrt 0.0667 **M1**

(b) compares with the model's probability, awrt 0.0300, noting the observed proportion is more than double the modelled probability **A1**

(b) valid conclusion, e.g. the data suggest the Po(2.3) model may understate the true rate of high-fault days, so the model may not be fully consistent with the data (e.g. the true mean could be higher, or faults may not occur independently/at a constant rate) (oe) **B1**

(b) Answer: Observed proportion = 2/30 = awrt 0.0667, more than double the modelled probability of awrt 0.0300; this suggests the Po(2.3) model may not be fully consistent with the data.

Question 4

(a) n is large and p is small **B1**

(a) X is approximately Po(100 x 0.03) = Po(3) **B1**

(a) Answer: Po(3)

(b) $100C4 \times 0.03^4 \times 0.97^{96}$ **M1**

(b) 0.1706 **A1**

(b) $e^{-3}\ 3^4/4!$ **M1**

(b) 0.1680 **A1**

(b) Answer: binomial 0.1706, Poisson 0.1680

(c) |approximate - exact|/exact x 100 **M1**

(c) 1.5% **A1**

(c) Answer: 1.5%

Question 5

(a) $P(X = 1) = e^{-2}\ 2/1!$ **M1**

(a) 0.271 **A1**

(a) $P(X \geq 1) = 1 - P(X \leq 0)$ **M1**

(a) 0.865 **A1**

(a) Answer: 0.271; 0.865

(b) E(X) = Var(X) = 2 **B1**

(b) for a Poisson distribution the variance equals the mean **B1**

(b) Answer: E(X) = Var(X) = 2

(c) X + Y ~ Po(2 + 1.5) = Po(3.5) **M1**

(c) $P = e^{-3.5}\ 3.5/1!$ **A1**

(c) 0.106 **A1**

(c) Answer: 0.106

© Revision Library | revisionlibrary.co.uk

Question 6

(a) X ~ B(500, 0.004), cao **B1**

(a) Answer: X ~ B(500, 0.004)

(b) n is large and p is small (so that np is of moderate size), making the Poisson approximation appropriate (oe) **B1**

(b) parameter $\lambda = np = 500 * 0.004 = 2$ **B1**

(b) Answer: n = 500 is large and p = 0.004 is small, so X is approximately Po(2), with λ = np = 2.

(c) correct use of $P(X=3) = e^{-2} * 2^3 / 3!$ with $\lambda = 2$ **M1**

(c) awrt 0.180 **A1**

(c) Answer: P(X = 3) = awrt 0.180 (Poisson approximation)

(d) correct use of $P(X=3) = C(500,3) * 0.004^3 * 0.996^{497}$ **M1**

(d) awrt 0.1808 **A1**

(d) valid comment, e.g. the Poisson value (0.1804) and exact binomial value (0.1808) differ by less than 0.001, so the approximation is reasonably accurate here since n is large and p is small (oe) **B1**

(d) Answer: Exact P(X=3) = awrt 0.1808; the Poisson approximation (0.1804) is close, differing by less than 0.001, so it is a reasonably accurate approximation.

© Revision Library | revisionlibrary.co.uk