2.2 Further Statistics: Poisson and Geometric Distributions: 30-minute Pack

AQA 7367 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A technician, Josef, attempts to fix a printer fault. Each attempt is independent of all others, and succeeds with probability 0.35. Let X be the number of attempts up to and including the first successful repair.

(a) State the distribution of X, including its parameter. **(1)**

(b) Find P(X = 3). **(2)**

(c) Find the probability that more than 4 attempts are needed before the fault is fixed. **(2)**

(d) Find P(X ≤ 6). **(2)**

(Total for Question 1 is 7 marks)

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2 At a fairground stall in Blackpool, Priya spins a wheel repeatedly. Each spin, independently of the others, results in a win with probability 0.2. Let Y be the number of spins up to and including her first win.

(a) State the formula for E(Y) for a geometric distribution with parameter p, and hence find E(Y). **(2)**

(b) State the formula for Var(Y) for a geometric distribution with parameter p, and hence find Var(Y) and the standard deviation of Y. **(3)**

(c) Interpret the value of E(Y) found in part (a) in the context of this scenario. **(1)**

(Total for Question 2 is 6 marks)

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3 A knitting machine in a textile factory in Leeds develops faults singly and at random, independently of each other, at a constant average rate of 0.6 faults per hour.

(a) Explain what is meant by saying that faults occur singly in this context. **(1)**

(b) Find the probability that the machine operates for an 8-hour shift with no faults. **(2)**

(c) The factory manager wants the probability of a shift with no faults to be less than 0.01. Find the least whole number of hours a shift can last so that this condition holds, showing sufficient working to justify your answer. **(4)**

(Total for Question 3 is 7 marks)

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4 A school's IT helpdesk receives emails singly and at random, independently of each other, at a constant average rate of 4.5 per hour. It also receives text message queries, independently of the emails, singly and at random at a constant average rate of 2.1 per hour.

(a) State, with a reason, the distribution of the total number of contacts (emails and text queries combined) received by the helpdesk in a randomly chosen hour. **(2)**

(b) Find the probability that the helpdesk receives exactly 8 contacts in a randomly chosen hour. **(2)**

(c) Find the probability that the helpdesk receives at least 10 contacts during a 2-hour period. **(3)**

(Total for Question 4 is 7 marks)

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Mark scheme · 2.2 Further Statistics: Poisson and Geometric Distributions: 30-minute Pack

Question 1

(a) X ~ Geo(0.35), cao **B1**

(a) Answer: X ~ Geo(0.35)

(b) correct use of $P(X=3) = (0.65)^2 * 0.35$ **M1**

(b) 0.147875, awrt 0.148 **A1**

(b) Answer: P(X = 3) = 0.147875, awrt 0.148

(c) recognises $P(X>4) = (0.65)^4$, i.e. the first 4 attempts all fail **M1**

(c) 0.17850625, awrt 0.179 **A1**

(c) Answer: P(X > 4) = 0.17850625, awrt 0.179

(d) uses $P(X\leq6) = 1 - (0.65)^6$ **M1**

(d) awrt 0.925 **A1**

(d) Answer: P(X ≤ 6) = awrt 0.925

Question 2

(a) E(Y) = 1/p **B1**

(a) E(Y) = 1/0.2 = 5, cao **A1**

(a) Answer: E(Y) = 5

(b) $Var(Y) = (1-p)/p^2$ **B1**

(b) substitutes p = 0.2: Var(Y) = 0.8/0.04 **M1**

(b) Var(Y) = 20, standard deviation = $\sqrt{20}$, awrt 4.47 **A1**

(b) Answer: Var(Y) = 20; standard deviation = $\sqrt{20}$ = awrt 4.47

(c) on average, Priya needs 5 spins to get her first win (oe) **B1**

(c) Answer: On average, 5 spins of the wheel are needed before the first win.

Question 3

(a) no two faults can occur at exactly the same instant (oe) **B1**

(a) Answer: Two (or more) faults cannot occur at the same instant in time.

(b) scales the rate to 8 hours, λ = 4.8, and attempts $P(X=0) = e^{-4.8}$ **M1**

(b) awrt 0.00823 **A1**

(b) Answer: P(X = 0) = awrt 0.00823

(c) sets up the inequality $e^{-0.6t} < 0.01$, where t is the shift length in hours **M1**

(c) takes logarithms and rearranges to t > ln(100)/0.6 **dM1**

(c) t > awrt 7.68 (hours) **A1**

(c) checks t = 7 gives P(X=0) = awrt 0.0150 (fails) and t = 8 gives P(X=0) = awrt 0.00823 (satisfies); least whole number of hours = 8, cao **A1**

(c) Answer: Least whole number of hours = 8

Question 4

(a) states total ~ Po(6.6) **B1**

(a) gives the reason: the sum of two independent Poisson random variables is itself Poisson, with parameter equal to the sum of the individual parameters (oe), i.e. the additive property **B1**

(a) Answer: Total contacts ~ Po(6.6), since the sum of two independent Poisson variables is Poisson with parameter 4.5 + 2.1 = 6.6.

(b) correct use of $P(X=8) = e^{-6.6} * 6.6^8 / 8!$ with λ = 6.6 **M1**

(b) awrt 0.121 **A1**

(b) Answer: P(X = 8) = awrt 0.121

(c) scales the parameter to a 2-hour period, λ = 13.2 **M1**

(c) uses $P(X\geq10) = 1 - P(X\leq9)$ **M1**

(c) awrt 0.847 **A1**

(c) Answer: P(X ≥ 10) = awrt 0.847

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