2.2 Further Statistics: Poisson and Geometric Distributions

AQA 7367 · Calculator allowed · about 220 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Customers arrive at the till of a small bakery in Falmouth singly and at random, independently of each other, at a constant average rate of 3.2 customers per 10-minute interval.

(a) Find the probability that exactly 5 customers arrive during a 10-minute interval. **(2)**

(b) Find the probability that at most 2 customers arrive during a 5-minute interval. **(3)**

(c) Find the probability that no customers arrive during a 2-minute period. **(2)**

(Total for Question 1 is 7 marks)

2 A technician, Josef, attempts to fix a printer fault. Each attempt is independent of all others, and succeeds with probability 0.35. Let X be the number of attempts up to and including the first successful repair.

(a) State the distribution of X, including its parameter. **(1)**

(b) Find P(X = 3). **(2)**

(c) Find the probability that more than 4 attempts are needed before the fault is fixed. **(2)**

(d) Find P(X ≤ 6). **(2)**

(Total for Question 2 is 7 marks)

3 At a fairground stall in Blackpool, Priya spins a wheel repeatedly. Each spin, independently of the others, results in a win with probability 0.2. Let Y be the number of spins up to and including her first win.

(a) State the formula for E(Y) for a geometric distribution with parameter p, and hence find E(Y). **(2)**

(b) State the formula for Var(Y) for a geometric distribution with parameter p, and hence find Var(Y) and the standard deviation of Y. **(3)**

(c) Interpret the value of E(Y) found in part (a) in the context of this scenario. **(1)**

(Total for Question 3 is 6 marks)

4 A knitting machine in a textile factory in Leeds develops faults singly and at random, independently of each other, at a constant average rate of 0.6 faults per hour.

(a) Explain what is meant by saying that faults occur singly in this context. **(1)**

(b) Find the probability that the machine operates for an 8-hour shift with no faults. **(2)**

(c) The factory manager wants the probability of a shift with no faults to be less than 0.01. Find the least whole number of hours a shift can last so that this condition holds, showing sufficient working to justify your answer. **(4)**

(Total for Question 4 is 7 marks)

5 A school's IT helpdesk receives emails singly and at random, independently of each other, at a constant average rate of 4.5 per hour. It also receives text message queries, independently of the emails, singly and at random at a constant average rate of 2.1 per hour.

(a) State, with a reason, the distribution of the total number of contacts (emails and text queries combined) received by the helpdesk in a randomly chosen hour. **(2)**

(b) Find the probability that the helpdesk receives exactly 8 contacts in a randomly chosen hour. **(2)**

(c) Find the probability that the helpdesk receives at least 10 contacts during a 2-hour period. **(3)**

(Total for Question 5 is 7 marks)

6 Buses arrive at a stop in central Manchester singly and at random, independently of each other, at a constant average rate of 2.5 buses per 15-minute interval.

(a) Find the probability that exactly 3 buses arrive during a 15-minute interval. **(2)**

(b) Find the probability that at least one bus arrives during a 6-minute interval. **(3)**

(c) Using the additive property of the Poisson distribution, find the probability that exactly 5 buses arrive during a 30-minute interval. **(3)**

(d) During the evening rush hour, a passenger notices that buses tend to arrive in clusters, with long gaps followed by several buses arriving close together. State, with a reason, whether the Poisson model above remains appropriate during rush hour. **(2)**

(Total for Question 6 is 10 marks)

7 A quality-control officer, Amara, at an electronics plant in Coventry tests resistors from a production line one at a time. Each resistor is independently faulty with probability 0.08. Let X be the number of resistors tested up to and including the first faulty one found.

(a) State the distribution of X. **(1)**

(b) Find P(X = 6). **(2)**

(c) Find P(X ≤ 10). **(2)**

(d) State E(X), and interpret its value in the context of this scenario. **(2)**

(e) Find the least number of resistors, n, such that $P(X \leq n) > 0.9$. **(4)**

(Total for Question 7 is 11 marks)

---

8 A factory in Nottingham claims that the number of faulty items it produces per day follows a Poisson distribution with mean 2.3.

(a) Using this model, find $P(X \geq 6)$. **(2)**

(b) Over a 30-day production period, the factory records that on 2 of the 30 days there were 6 or more faulty items. Compare this observed proportion with the probability found in part (a), and comment on whether the claimed Po(2.3) model seems consistent with the data. **(3)**

(Total for Question 8 is 5 marks)

9 The random variable X has distribution B(100, 0.03).

(a) State two conditions under which a Poisson distribution is a suitable approximation to a binomial distribution, and give the parameter of the approximating distribution. **(2)**

(b) Use the binomial distribution to find P(X = 4), and use the Poisson approximation to find the same probability, each to 4 significant figures. **(4)**

(c) Find the percentage error in using the Poisson approximation here, to 2 significant figures. **(2)**

(Total for Question 9 is 8 marks)

10 The number of calls to a helpline per minute may be modelled by a Poisson distribution with mean 2. Let X be the number in one such interval, so X ~ Po(2).

(a) Find P(X = 1) and P(X ≥ 1), each to 3 significant figures. **(4)**

(b) State the mean and variance of X, and explain why they are equal in a Poisson model. **(2)**

(c) A second independent count Y has distribution Po(1.5). Find P(X + Y = 1), to 3 significant figures. **(3)**

(Total for Question 10 is 9 marks)

11 The random variable X has distribution B(50, 0.01).

(a) State two conditions under which a Poisson distribution is a suitable approximation to a binomial distribution, and give the parameter of the approximating distribution. **(2)**

(b) Use the binomial distribution to find P(X = 5), and use the Poisson approximation to find the same probability, each to 4 significant figures. **(4)**

(c) Find the percentage error in using the Poisson approximation here, to 2 significant figures. **(2)**

(Total for Question 11 is 8 marks)

12 A player attempts a task repeatedly until the first success. Each attempt succeeds with probability 0.2, independently of the others. Let X be the number of the attempt on which the first success occurs, so X ~ Geo(0.2).

(a) Find P(X = 3) and P(X > 3), each to 3 significant figures. **(4)**

(b) Find E(X) and Var(X). **(4)**

(c) Find the least number of attempts n such that the probability of at least one success within n attempts exceeds 0.9. **(4)**

(Total for Question 12 is 12 marks)

13 The random variable X has distribution B(120, 0.01).

(a) State two conditions under which a Poisson distribution is a suitable approximation to a binomial distribution, and give the parameter of the approximating distribution. **(2)**

(b) Use the binomial distribution to find P(X = 5), and use the Poisson approximation to find the same probability, each to 4 significant figures. **(4)**

(c) Find the percentage error in using the Poisson approximation here, to 2 significant figures. **(2)**

(Total for Question 13 is 8 marks)

14 The number of emails Aisha receives per hour may be modelled by a Poisson distribution with mean 1.2. Let X be the number in one such interval, so X ~ Po(1.2).

(a) Find P(X = 1) and P(X ≥ 1), each to 3 significant figures. **(4)**

(b) State the mean and variance of X, and explain why they are equal in a Poisson model. **(2)**

(c) A second independent count Y has distribution Po(0.8). Find P(X + Y = 1), to 3 significant figures. **(3)**

(Total for Question 14 is 9 marks)

15 The number of flaws per metre in a roll of cloth may be modelled by a Poisson distribution with mean 3.5. Let X be the number in one such interval, so X ~ Po(3.5).

(a) Find P(X = 4) and P(X ≥ 4), each to 3 significant figures. **(4)**

(b) State the mean and variance of X, and explain why they are equal in a Poisson model. **(2)**

(c) A second independent count Y has distribution Po(1.5). Find P(X + Y = 4), to 3 significant figures. **(3)**

(Total for Question 15 is 9 marks)

16 The number of calls to a helpline per minute may be modelled by a Poisson distribution with mean 1.2. Let X be the number in one such interval, so $X \sim Po(1.2)$.

(a) Find $P(X = 1)$ and $P(X \geq 1)$, each to 3 significant figures. **(4)**

(b) State the mean and variance of X, and explain why they are equal in a Poisson model. **(2)**

(c) A second independent count Y has distribution Po(1). Find $P(X + Y = 1)$, to 3 significant figures. **(3)**

(Total for Question 16 is 9 marks)

17 A player attempts a task repeatedly until the first success. Each attempt succeeds with probability 0.1, independently of the others. Let X be the number of the attempt on which the first success occurs, so X ~ Geo(0.1).

(a) Find P(X = 3) and P(X > 3), each to 3 significant figures. **(4)**

(b) Find E(X) and Var(X). **(4)**

(c) Find the least number of attempts n such that the probability of at least one success within n attempts exceeds 0.9. **(4)**

(Total for Question 17 is 12 marks)

18 A player attempts a task repeatedly until the first success. Each attempt succeeds with probability 0.5, independently of the others. Let X be the number of the attempt on which the first success occurs, so X ~ Geo(0.5).

(a) Find P(X = 2) and P(X > 2), each to 3 significant figures. **(3)**

(b) Find E(X) and Var(X). **(4)**

(c) Find the least number of attempts n such that the probability of at least one success within n attempts exceeds 0.99. **(4)**

(Total for Question 18 is 11 marks)

19 A factory produces lightbulbs. Each lightbulb is defective independently of all others, with probability 0.004. A batch of 500 lightbulbs is inspected, and X is the number of defective lightbulbs in the batch.

(a) State the exact distribution of X. **(1)**

(b) Justify why X may be well approximated by a Poisson distribution, and state the parameter of the approximating distribution. **(2)**

(c) Using the Poisson approximation, find P(X = 3). **(2)**

(d) Calculate the exact binomial probability P(X = 3) to 4 significant figures, and comment on the accuracy of the Poisson approximation used in part (c). **(3)**

(Total for Question 19 is 8 marks)

20 Let X ~ Po(lambda1) and Y ~ Po(lambda2) be independent random variables.

(a) Show that X + Y ~ Po(lambda1 + lambda2). **(8)**

(Total for Question 20 is 8 marks)

Mark scheme · 2.2 Further Statistics: Poisson and Geometric Distributions

Question 1

(a) correct use of $P(X=5) = e^{-3.2} * 3.2^5 / 5!$ with $X \sim Po(3.2)$ **M1**

(a) awrt 0.114 **A1**

(a) Answer: P(X = 5) = awrt 0.114

(b) scales the rate to a 5-minute interval, $\lambda = 1.6$ **M1**

(b) attempts $P(X=0)+P(X=1)+P(X=2)$ with $\lambda = 1.6$ **M1**

(b) awrt 0.783 **A1**

(b) Answer: P(X ≤ 2) = awrt 0.783

(c) scales the rate to a 2-minute interval, $\lambda = 0.64$ **M1**

(c) awrt 0.527 **A1**

(c) Answer: P(X = 0) = awrt 0.527

Question 2

(a) $X \sim Geo(0.35)$, cao **B1**

(a) Answer: X ~ Geo(0.35)

(b) correct use of $P(X=3) = (0.65)^2 * 0.35$ **M1**

(b) 0.147875, awrt 0.148 **A1**

(b) Answer: P(X = 3) = 0.147875, awrt 0.148

(c) recognises $P(X>4) = (0.65)^4$, i.e. the first 4 attempts all fail **M1**

(c) 0.17850625, awrt 0.179 **A1**

(c) Answer: P(X > 4) = 0.17850625, awrt 0.179

(d) uses $P(X \leq 6) = 1 - (0.65)^6$ **M1**

(d) awrt 0.925 **A1**

(d) Answer: P(X ≤ 6) = awrt 0.925

Question 3

(a) $E(Y) = 1/p$ **B1**

(a) $E(Y) = 1/0.2 = 5$, cao **A1**

(a) Answer: E(Y) = 5

(b) $Var(Y) = (1-p)/p^2$ **B1**

(b) substitutes $p = 0.2$: $Var(Y) = 0.8/0.04$ **M1**

(b) $Var(Y) = 20$, standard deviation = $\sqrt{20}$, awrt 4.47 **A1**

(b) Answer: Var(Y) = 20; standard deviation = $\sqrt{20}$ = awrt 4.47

(c) on average, Priya needs 5 spins to get her first win (oe) **B1**

(c) Answer: On average, 5 spins of the wheel are needed before the first win.

Question 4

(a) no two faults can occur at exactly the same instant (oe) **B1**

(a) Answer: Two (or more) faults cannot occur at the same instant in time.

(b) scales the rate to 8 hours, $\lambda = 4.8$, and attempts $P(X=0) = e^{-4.8}$ **M1**

(b) awrt 0.00823 **A1**

(b) Answer: P(X = 0) = awrt 0.00823

(c) sets up the inequality $e^{-0.6t} < 0.01$, where t is the shift length in hours **M1**

(c) takes logarithms and rearranges to $t > \ln(100)/0.6$ **dM1**

(c) $t >$ awrt 7.68 (hours) **A1**

(c) checks $t = 7$ gives $P(X=0) =$ awrt 0.0150 (fails) and $t = 8$ gives $P(X=0) =$ awrt 0.00823 (satisfies); least whole number of hours = 8, cao **A1**

(c) Answer: Least whole number of hours = 8

Question 5

(a) states total ~ Po(6.6) **B1**

(a) gives the reason: the sum of two independent Poisson random variables is itself Poisson, with parameter equal to the sum of the individual parameters (oe), i.e. the additive property **B1**

(a) Answer: Total contacts ~ Po(6.6), since the sum of two independent Poisson variables is Poisson with parameter 4.5 + 2.1 = 6.6.

(b) correct use of $P(X=8) = e^{-6.6} * 6.6^8 / 8!$ with $\lambda = 6.6$ **M1**

(b) awrt 0.121 **A1**

(b) Answer: P(X = 8) = awrt 0.121

(c) scales the parameter to a 2-hour period, $\lambda = 13.2$ **M1**

(c) uses $P(X \geq 10) = 1 - P(X \leq 9)$ **M1**

(c) awrt 0.847 **A1**

(c) Answer: P(X ≥ 10) = awrt 0.847

Question 6

(a) correct use of $P(X=3) = e^{-2.5} * 2.5^3 / 3!$ with $X \sim Po(2.5)$ **M1**

(a) awrt 0.214 **A1**

(a) Answer: P(X = 3) = awrt 0.214

(b) scales the rate to a 6-minute interval, $\lambda = 2.5*(6/15) = 1.0$ **M1**

(b) uses $P(X \geq 1) = 1 - P(X=0)$ **M1**

(b) awrt 0.632 **A1**

(b) Answer: P(X ≥ 1) = awrt 0.632

(c) recognises the number of buses in two independent, consecutive 15-minute periods sum to give $Po(2.5+2.5) = Po(5.0)$ by the additive property **M1**

(c) correct use of $P(X=5) = e^{-5} * 5^5 / 5!$ with $\lambda = 5.0$ **M1**

(c) awrt 0.175 **A1**

(c) Answer: P(X = 5) = awrt 0.175

(d) states no, the Poisson model is no longer appropriate **B1**

(d) valid reason, e.g. buses arriving in clusters suggests arrivals are no longer independent of each other and/or the rate is not constant throughout rush hour, violating the conditions required for a Poisson model (oe) **B1**

(d) Answer: No; clustering suggests arrivals are not independent and/or the rate is not constant during rush hour, so the Poisson conditions no longer hold.

Question 7

(a) X ~ Geo(0.08), cao **B1**

(a) Answer: X ~ Geo(0.08)

(b) correct use of $P(X=6) = (0.92)^5 * 0.08$ **M1**

(b) awrt 0.0527 **A1**

(b) Answer: P(X = 6) = awrt 0.0527

(c) uses $P(X \leq 10) = 1 - (0.92)^{10}$ **M1**

(c) awrt 0.566 **A1**

(c) Answer: P(X ≤ 10) = awrt 0.566

(d) E(X) = 1/0.08 = 12.5 **B1**

(d) valid interpretation, e.g. on average, Amara needs to test 12.5 resistors before finding the first faulty one (oe) **B1**

(d) Answer: E(X) = 12.5; on average, 12.5 resistors need to be tested before the first faulty one is found.

(e) sets up the inequality $1 - (0.92)^n > 0.9$ **M1**

(e) rearranges to $(0.92)^n < 0.1$ **M1**

(e) takes logarithms and solves to n > ln(0.1)/ln(0.92), awrt 27.6 **dM1**

(e) checks n=27 gives P(X≤27) = awrt 0.895 (fails) and n=28 gives P(X≤28) = awrt 0.903 (satisfies); least n = 28, cao **A1**

(e) Answer: Least n = 28

Question 8

(a) uses P(X≥6) = 1 - P(X≤5) with X ~ Po(2.3) **M1**

(a) awrt 0.0300 **A1**

(a) Answer: P(X ≥ 6) = awrt 0.0300

(b) computes the observed proportion of days with 6 or more faults, 2/30 = awrt 0.0667 **M1**

(b) compares with the model's probability, awrt 0.0300, noting the observed proportion is more than double the modelled probability **A1**

(b) valid conclusion, e.g. the data suggest the Po(2.3) model may understate the true rate of high-fault days, so the model may not be fully consistent with the data (e.g. the true mean could be higher, or faults may not occur independently/at a constant rate) (oe) **B1**

(b) Answer: Observed proportion = 2/30 = awrt 0.0667, more than double the modelled probability of awrt 0.0300; this suggests the Po(2.3) model may not be fully consistent with the data.

Question 9

(a) n is large and p is small **B1**

(a) X is approximately Po(100 x 0.03) = Po(3) **B1**

(a) Answer: Po(3)

(b) $100C4 \times 0.03^4 \times 0.97^{96}$ **M1**

(b) 0.1706 **A1**

(b) $e^{-3}\ 3^4/4!$ **M1**

(b) 0.1680 **A1**

(b) Answer: binomial 0.1706, Poisson 0.1680

(c) |approximate - exact|/exact x 100 **M1**

(c) 1.5% **A1**

(c) Answer: 1.5%

Question 10

(a) $P(X = 1) = e^{-2}\ 2/1!$ **M1**

(a) 0.271 **A1**

(a) $P(X \geq 1) = 1 - P(X \leq 0)$ **M1**

(a) 0.865 **A1**

(a) Answer: 0.271; 0.865

(b) E(X) = Var(X) = 2 **B1**

(b) for a Poisson distribution the variance equals the mean **B1**

(b) Answer: E(X) = Var(X) = 2

(c) X + Y ~ Po(2 + 1.5) = Po(3.5) **M1**

(c) $P = e^{-3.5}\ 3.5/1!$ **A1**

(c) 0.106 **A1**

(c) Answer: 0.106

Question 11

(a) n is large and p is small **B1**

(a) X is approximately Po(50 x 0.01) = Po(0.5) **B1**

(a) Answer: Po(0.5)

(b) $50C5 \times 0.01^5 \times 0.99^{45}$ **M1**

(b) 0.0001348 **A1**

(b) $e^{-0.5}\ 0.5^5/5!$ **M1**

(b) 0.0001580 **A1**

(b) Answer: binomial 0.0001348, Poisson 0.0001580

(c) |approximate - exact|/exact x 100 **M1**

(c) 17% **A1**

(c) Answer: 17%

Question 12

(a) $P(X = 3) = (1 - 0.2)^2 \times 0.2$ **M1**

(a) 0.128 **A1**

(a) $P(X > 3) = (1 - 0.2)^3$ **M1**

(a) 0.512 **A1**

(a) Answer: 0.128; 0.512

(b) E(X) = 1/p **M1**

(b) 5 **A1**

(b) $Var(X) = (1 - p)/p^2$ **M1**

(b) 20.00 **A1**

(b) Answer: E(X) = 5.000, Var(X) = 20.00

(c) $P(X \leq n) = 1 - (1 - 0.2)^n > 0.9$ **M1**

(c) $(1 - 0.2)^n < 0.1$, and takes logs (changing the inequality) **M1**

(c) n > 10.319 **A1**

(c) n = 11 **A1**

(c) Answer: n = 11

Question 13

(a) n is large and p is small **B1**

(a) X is approximately Po(120 x 0.01) = Po(1.2) **B1**

(a) Answer: Po(1.2)

(b) $120C5 \times 0.01^5 \times 0.99^{115}$ **M1**

(b) 0.006000 **A1**

(b) $e^{-1.2}\ 1.2^5/5!$ **M1**

(b) 0.006246 **A1**

(b) Answer: binomial 0.006000, Poisson 0.006246

(c) |approximate - exact|/exact x 100 **M1**

(c) 4.1% **A1**

(c) Answer: 4.1%

Question 14

(a) $P(X = 1) = e^{-1.2}\ 1.2/1!$ **M1**

(a) 0.361 **A1**

(a) $P(X \geq 1) = 1 - P(X \leq 0)$ **M1**

(a) 0.699 **A1**

(a) Answer: 0.361; 0.699

(b) E(X) = Var(X) = 1.2 **B1**

(b) for a Poisson distribution the variance equals the mean **B1**

(b) Answer: E(X) = Var(X) = 1.2

(c) X + Y ~ Po(1.2 + 0.8) = Po(2) **M1**

(c) $P = e^{-2}\ 2/1!$ **A1**

(c) 0.271 **A1**

(c) Answer: 0.271

Question 15

(a) $P(X = 4) = e^{-3.5}\ 3.5^4/4!$ **M1**

(a) 0.189 **A1**

(a) $P(X \geq 4) = 1 - P(X \leq 3)$ **M1**

(a) 0.463 **A1**

(a) Answer: 0.189; 0.463

(b) E(X) = Var(X) = 3.5 **B1**

(b) for a Poisson distribution the variance equals the mean **B1**

(b) Answer: E(X) = Var(X) = 3.5

(c) X + Y ~ Po(3.5 + 1.5) = Po(5) **M1**

(c) $P = e^{-5}\ 5^4/4!$ **A1**

(c) 0.175 **A1**

(c) Answer: 0.175

Question 16

(a) $P(X = 1) = e^{-1.2}\ 1.2/1!$ **M1**

(a) 0.361 **A1**

(a) $P(X \geq 1) = 1 - P(X \leq 0)$ **M1**

(a) 0.699 **A1**

(a) Answer: 0.361; 0.699

(b) E(X) = Var(X) = 1.2 **B1**

(b) for a Poisson distribution the variance equals the mean **B1**

(b) Answer: E(X) = Var(X) = 1.2

(c) X + Y ~ Po(1.2 + 1) = Po(2.2) **M1**

(c) $P = e^{-2.2}\ 2.2/1!$ **A1**

(c) 0.244 **A1**

(c) Answer: 0.244

Question 17

(a) $P(X = 3) = (1 - 0.1)^2 \times 0.1$ **M1**

(a) 0.0810 **A1**

(a) $P(X > 3) = (1 - 0.1)^3$ **M1**

(a) 0.729 **A1**

(a) Answer: 0.0810; 0.729

(b) E(X) = 1/p **M1**

(b) 10 **A1**

(b) $Var(X) = (1 - p)/p^2$ **M1**

(b) 90.00 **A1**

(b) Answer: E(X) = 10.00, Var(X) = 90.00

(c) $P(X \leq n) = 1 - (1 - 0.1)^n > 0.9$ **M1**

(c) $(1 - 0.1)^n < 0.1$, and takes logs (changing the inequality) **M1**

(c) n > 21.854 **A1**

(c) n = 22 **A1**

(c) Answer: n = 22

Question 18

(a) $P(X = 2) = (1 - 0.5) \times 0.5$ **M1**

(a) $P(X > 2) = (1 - 0.5)^2$ **M1**

(a) 0.250 **A1**

(a) Answer: 0.250; 0.250

(b) E(X) = 1/p **M1**

(b) 2 **A1**

(b) $Var(X) = (1 - p)/p^2$ **M1**

(b) 2.000 **A1**

(b) Answer: E(X) = 2.000, Var(X) = 2.000

(c) $P(X \leq n) = 1 - (1 - 0.5)^n > 0.99$ **M1**

(c) $(1 - 0.5)^n < 0.01$, and takes logs (changing the inequality) **M1**

(c) n > 6.644 **A1**

(c) n = 7 **A1**

(c) Answer: n = 7

Question 19

(a) X ~ B(500, 0.004), cao **B1**

(a) Answer: X ~ B(500, 0.004)

(b) n is large and p is small (so that np is of moderate size), making the Poisson approximation appropriate (oe) **B1**

(b) parameter $\lambda = np = 500 * 0.004 = 2$ **B1**

(b) Answer: n = 500 is large and p = 0.004 is small, so X is approximately Po(2), with λ = np = 2.

(c) correct use of $P(X=3) = e^{-2} * 2^3 / 3!$ with $\lambda = 2$ **M1**

(c) awrt 0.180 **A1**

(c) Answer: P(X = 3) = awrt 0.180 (Poisson approximation)

(d) correct use of $P(X=3) = C(500,3) * 0.004^3 * 0.996^{497}$ **M1**

(d) awrt 0.1808 **A1**

(d) valid comment, e.g. the Poisson value (0.1804) and exact binomial value (0.1808) differ by less than 0.001, so the approximation is reasonably accurate here since n is large and p is small (oe) **B1**

(d) Answer: Exact P(X=3) = awrt 0.1808; the Poisson approximation (0.1804) is close, differing by less than 0.001, so it is a reasonably accurate approximation.

Question 20

(a) uses the convolution formula $P(X+Y=r) = \sum_{k=0}^{r} P(X=k)P(Y=r-k)$ **M1**

(a) substitutes the Poisson probability mass functions for X and Y **M1**

(a) combines the exponential terms to give $e^{-(\lambda_1+\lambda_2)} * \sum_{k=0}^{r} [\lambda_1^k * \lambda_2^{r-k} / (k!(r-k)!)]$ **A1**

(a) introduces r!/r! and identifies r!/(k!(r-k)!) as the binomial coefficient C(r,k) **M1**

(a) rewrites the sum as $(1/r!) * \sum_{k=0}^{r} C(r,k) * \lambda_1^k * \lambda_2^{r-k}$ **A1**

(a) applies the binomial theorem to recognise $\sum_{k=0}^{r} C(r,k) * \lambda_1^k * \lambda_2^{r-k} = (\lambda_1+\lambda_2)^r$ **M1**

(a) obtains $P(X+Y=r) = e^{-(\lambda_1+\lambda_2)} * (\lambda_1+\lambda_2)^r / r!$ **A1**

(a) identifies this as the probability mass function of $Po(\lambda_1+\lambda_2)$, so $X+Y \sim Po(\lambda_1+\lambda_2)$, cso **A1**

(a) Answer: $P(X+Y=r) = e^{-(\lambda_1+\lambda_2)} (\lambda_1+\lambda_2)^r / r!$, so $X+Y \sim Po(\lambda_1+\lambda_2)$ (shown).