2.4 Further Statistics: Hypothesis Testing, Type I and Type II Errors: 10-minute Pack

AQA 7367 · Calculator allowed · about 10 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Hypothesis Testing for a Poisson or Binomial Parameter

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For a hypothesis test on a Poisson mean lambda or a binomial probability p, a null hypothesis H0 states a fixed value of the parameter, and an alternative hypothesis H1 states that the parameter has increased, has decreased (a one-tailed test), or is simply different (a two-tailed test). A test statistic X, assumed to follow the distribution stated in H0, is compared against a critical region: the set of values of X for which H0 is rejected in favour of H1.

Because X is discrete, the probability of X falling in a proposed critical region can rarely be made to equal a chosen significance level exactly. The usual convention is to choose the largest critical region whose probability does not exceed the significance level requested. This probability is called the actual significance level of the test, and it is also equal to P(Type I error), the probability of wrongly rejecting a true H0.

A Type II error occurs when H0 is not rejected even though H1 is actually true. Its probability depends on the true value of the parameter, and is found by calculating the probability that X falls outside the critical region, using the true (alternative) value of the parameter in place of the value stated in H0. The power of a test is defined as 1 - P(Type II error): the probability of correctly rejecting a false H0. Power increases as the true parameter value moves further from the value stated in H0, and is also affected by the significance level chosen and the sample size used.

1 A fair coin-like device is claimed to land on 'success' with probability 0.15. A tester believes the probability of success is actually lower. Let X be the number of trials needed to get the first success, so under the claim X ~ Geo(0.15). The first success occurs on trial 16.

(a) State suitable hypotheses for the test, in terms of p. **(2)**

(b) Find P(X ≥ 16) assuming H0 is true, and test the tester's belief at the 10% level. **(3)**

(c) Find the critical region for the test at the 10% level, and the probability of a Type II error if the true value of p is 0.08, to 3 significant figures. **(4)**

(Total for Question 1 is 9 marks)

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Mark scheme · 2.4 Further Statistics: Hypothesis Testing, Type I and Type II Errors: 10-minute Pack

Question 1

(a) H0: $p = 0.15$ **B1**

(a) H1: $p < 0.15$ **B1**

(a) Answer: H0: $p = 0.15$, H1: $p < 0.15$

(b) $P(X \geq 16) = (1 - 0.15)^{16 - 1}$ **M1**

(b) 0.0874 **A1**

(b) reject H0 **A1**

(b) Answer: 0.0874; reject H0

(c) finds the smallest d with $(1 - 0.15)^{d - 1} \leq 0.1$ **M1**

(c) critical region $X \geq 16$ **A1**

(c) $P(X \leq 15 \mid p = 0.08) = 1 - (1 - 0.08)^{15}$ **M1**

(c) 0.714 **A1**

(c) Answer: $X \geq 16$; Type II error 0.714

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