2.5 Further Statistics: Discrete Distributions Depth: 10-minute Pack

Total Marks

Calculator allowed · about 10 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key facts: discrete distributions and probability generating functions

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For X ~ Po(lambda), P(X = x) = e^(-lambda) * lambda^x / x!, for x = 0, 1, 2, ..., with E(X) = lambda and Var(X) = lambda. If X ~ Po(lambda1) and Y ~ Po(lambda2) are independent, then X + Y ~ Po(lambda1 + lambda2).

For X ~ Geo(p), the number of independent trials up to and including the first success (write q = 1 - p), P(X = x) = q^(x-1) * p, for x = 1, 2, 3, ..., with E(X) = 1/p and Var(X) = q/p^2. The geometric distribution has the memoryless property: P(X > m+n | X > m) = P(X > n) for positive integers m and n.

For X ~ NB(r, p), the number of independent trials up to and including the rth success, P(X = x) = C(x-1, r-1) * p^r * q^(x-r), for x = r, r+1, r+2, ..., with E(X) = r/p and Var(X) = rq/p^2. X can be written as the sum of r independent Geo(p) random variables.

The probability generating function of a discrete random variable X taking non-negative integer values is G_X(t) = E(t^X) = sum of P(X = x) * t^x. It satisfies G_X(1) = 1, E(X) = G_X'(1), and Var(X) = G_X''(1) + G_X'(1) - [G_X'(1)]^2. If X and Y are independent random variables, then G_(X+Y)(t) = G_X(t) * G_Y(t).

1 The discrete random variable Y has the probability distribution shown in the table below.

Y	1	2	3	4
P(Y = y)	0.1	0.4	0.3	0.2

(a) Find the probability generating function $G_Y(t)$. **(2)**

(b) Use $G_Y(t)$ to find E(Y) and Var(Y). **(5)**

(c) Verify your value of E(Y) by direct calculation from the probability distribution. **(2)**

(Total for Question 1 is 9 marks)

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Mark scheme · 2.5 Further Statistics: Discrete Distributions Depth: 10-minute Pack

Question 1

(a) $G_Y(t)$ = sum of $P(Y=y)*t^y$, using the table	**M1**
(a) $G_Y(t) = 0.1t + 0.4t^2 + 0.3t^3 + 0.2t^4$, oe	**A1**
(a) Answer: $G_Y(t) = 0.1t + 0.4t^2 + 0.3t^3 + 0.2t^4$	
(b) differentiates: $G_Y'(t) = 0.1 + 0.8t + 0.9t^2 + 0.8t^3$	**M1**
(b) $E(Y) = G_Y'(1) = 2.6$, cao	**A1**
(b) differentiates again: $G_Y''(t) = 0.8 + 1.8t + 2.4t^2$, so $G_Y''(1) = 5.0$	**M1**
(b) uses $Var(Y) = G_Y''(1) + G_Y'(1) - [G_Y'(1)]^2$, oe	**M1**
(b) Var(Y) = 0.84, cao	**A1**
(b) Answer: E(Y) = 2.6, Var(Y) = 0.84	
(c) uses E(Y) = sum of y*P(Y=y) = 1(0.1)+2(0.4)+3(0.3)+4(0.2)	**M1**
(c) = 2.6, matching part (b), cao	**A1**
(c) Answer: E(Y) = 0.1+0.8+0.9+0.8 = 2.6, matching part (b).	

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