4.1 Permutations and Combinations: Bridge and Extension

Total Marks

Calculator allowed · about 55 minutes

Name: _____ Date: ____ / ____ / _____

This is extension material beyond standalone Edexcel 9MA0 requirements. The narrower links are binomial coefficients, binomial expansion and binomial distributions. Answer all 20 questions and show your counting method. Give exact answers; equivalent exact forms are accepted. nPr counts ordered selections and nCr counts unordered selections, without replacement. Use each question's stated assumptions.

1 The same five distinct books A, B, C, D and E are placed in a straight row, each used once. How many orders have A and B separated by at least one book?

(Total for Question 1 is 3 marks)

2 Four identical red counters and three identical blue counters fill a straight row of seven positions. How many different colour patterns have no two blue counters adjacent? All counters are used.

(Total for Question 2 is 3 marks)

3 A panel of 4 is selected without replacement from 4 distinct artists and 5 distinct scientists, with disjoint categories. Order does not matter. How many panels contain at least one person from each category?

(Total for Question 3 is 3 marks)

4 Three people are selected without replacement from seven distinct people including Isha and Leon. Order does not matter. Isha and Leon must either both be selected or both be absent. How many selections satisfy this condition?

(Total for Question 4 is 3 marks)

5 In expanding $(2 + x)^6$, the six bracket positions are distinguishable. Explain how choosing the positions that supply x gives the coefficient of x^3, and find that coefficient.

(Total for Question 5 is 3 marks)

6 A bag contains 5 individually labelled red balls and 3 individually labelled blue balls. Three balls are drawn uniformly without replacement. Each unordered set of three labelled balls is equally likely; draw order is ignored. Find the probability of exactly two red balls.

(Total for Question 6 is 3 marks)

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7 A string is selected uniformly from all distinct arrangements of BALLOON, using every letter once. Identical Ls and identical Os are not individually labelled. Given that the two Os are adjacent, find the probability that the two Ls are also adjacent.

(Total for Question 7 is 4 marks)

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8 A badge has one of 3 distinct shapes and one of 4 distinct colours. Every shape can have every colour. How many different badges are possible?

(Total for Question 8 is 2 marks)

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9 A four-digit integer is selected uniformly from those formed using {0, 1, 2, 3, 4, 5}, without repeating a digit and with no leading zero. Given that the integer is even, find the probability that it exceeds 3000. Justify the counts in both numerator and denominator.

(Total for Question 9 is 5 marks)

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10 A code is an ordered string of 3 digits from {0, 1, 2, 3}. Digits may repeat, and a leading zero is allowed. How many codes are possible?

(Total for Question 10 is 2 marks)

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11 How many three-digit integers can be formed from {0, 1, 2, 3, 4}, without repeating a digit? A leading zero is not allowed.

(Total for Question 11 is 3 marks)

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12 Five different labelled folders are put in a straight row, each used once. How many orders are possible?

(Total for Question 12 is 2 marks)

13 A group of 3 is chosen from 7 distinct volunteers, without replacement. Order within the group does not matter. Express the count using nCr and evaluate it.

(Total for Question 13 is 3 marks)

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14 How many distinct five-letter strings use all the letters of LEVEL once each? The two Ls are identical and the two Es are identical. Order matters.

(Total for Question 14 is 3 marks)

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15 Five distinct books A, B, C, D and E are placed in a straight row, each used once. How many orders have A and B next to each other, in either order?

(Total for Question 15 is 3 marks)

Mark scheme · 4.1 Permutations and Combinations: Bridge and Extension

Question 1

Count all 5! orders.	**M1**
Subtract 2 x 4! adjacent orders, or insert A and B into separate gaps around C, D, E.	**M1**
72 orders; accept 5! - 2 x 4!.	**A1**

Answer: 72

Question 2

Identify 5 gaps around the red counters.	**M1**
Choose 3 distinct gaps, one blue in each.	**M1**
10 patterns; accept 5C3.	**A1**

Answer: 10

Question 3

Count all panels as 9C4.	**M1**
Remove all-artist and all-scientist panels, 4C4 + 5C4.	**M1**
120 panels; accept an equivalent sum of mixed-category cases.	**A1**

Answer: 120

Question 4

Count both present as 5C1.	**M1**
Count both absent as 5C3 and add the disjoint cases.	**M1**
15 selections; accept 5C1 + 5C3.	**A1**

Answer: 15

Question 5

Choose 3 of the 6 positions, giving 6C3.	**M1**
Multiply by 2^3 from the remaining positions.	**M1**
Coefficient 160; accept 6C3 x 2^3 with the positional explanation.	**A1**

Answer: 6C3 x 2^3 = 160

Question 6

Use denominator 8C3 = 56.	**M1**
Count favourable sets as 5C2 x 3C1 = 30.	**M1**
15/28; accept (5C2 x 3C1)/8C3 or any equivalent exact form.	**A1**

Answer: 15/28

Question 7

Count strings with OO adjacent as 6!/2! = 360.	**M1**
Count strings with both pairs adjacent as 5! = 120.	**M1**
Divide the joint count by the conditioned count.	**M1**
1/3; accept 5!/(6!/2!) or any equivalent exact form.	**A1**

Answer: 1/3

Question 8

Multiply 3 by 4.	**M1**
12 badges.	**A1**

Answer: 12

Question 9

Split the even denominator by last digit zero or non-zero.	**M1**
Denominator 60 + 96 = 156.	**A1**
Split the numerator by first digit 3 or 5 versus 4.	**M1**
Numerator 72 + 24 = 96.	**A1**
8/13; accept 96/156 or any equivalent exact form.	**A1**

Answer: 8/13

Question 10

Use 4 choices independently for each position.	**M1**
4^3 = 64; accept equivalent exact forms.	**A1**

Answer: 64

Question 11

Use 4 choices for the first digit.	**M1**
Use 4 then 3 choices for the next digits.	**M1**
48 integers; accept 4 x 4 x 3.	**A1**

Answer: 48

Question 12

Use 5!.	**M1**
120 orders; accept 5!.	**A1**

Answer: 120

Question 13

State 7C3.	**B1**
Divide the ordered count by 3!, or use the factorial formula.	**M1**
35 groups.	**A1**

Answer: 7C3 = 35

Question 14

Start from 5! labelled arrangements.	**M1**
Divide by 2! for each repeated pair.	**M1**
30 strings; accept 5!/(2!2!).	**A1**

Answer: 30

Question 15

Treat A and B as one block, giving 4! orders.	**M1**
Include both AB and BA.	**M1**
48 orders; accept 2 x 4!.	**A1**

Answer: 48