3.1 Mechanics: Quantities, Units and Kinematics: 2-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A car is travelling in a straight line and brakes with constant deceleration 4 m/s^2, coming to rest after travelling a further 32 m. Find the speed of the car at the instant the brakes were applied.

(Total for Question 1 is 2 marks)

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2 A stone falls freely from rest under gravity, where g = 9.8 m/s^2. Find the time taken for the stone to reach a speed of 24.5 m/s.

(Total for Question 2 is 2 marks)

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3 A skateboarder has initial speed 1.5 m/s and accelerates uniformly at 0.6 m/s^2 for 12 seconds. Find her final speed.

(Total for Question 3 is 2 marks)

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4 A delivery van slows down uniformly from 18 m/s to 6 m/s in 8 seconds. Find its deceleration.

(Total for Question 4 is 2 marks)

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5 A sledge decelerates uniformly from 9 m/s to rest in 3 seconds. Find the distance it travels while decelerating.

(Total for Question 5 is 2 marks)

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6 A shopping trolley starts with speed 1 m/s and accelerates uniformly at 0.5 m/s^2. Find the distance it travels in the first 6 seconds.

(Total for Question 6 is 2 marks)

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7 At time t = 0, a particle P has position vector (-6i - 5j) m relative to a fixed origin O. The particle moves with constant velocity (3i + 4j) m/s. Unit vectors i and j are due east and due north respectively.

(a) Find the position vector of P at time t = 3 seconds. **(2)**

(b) Find the speed of P. **(2)**

(c) Find the value of t at which P is due north of O. **(3)**

(d) Find the distance of P from O when t = 5, giving your answer to 3 significant figures. **(3)**

(Total for Question 7 is 10 marks)

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8 A particle P has position vector (2i - 5j) m at time t = 0 and moves with constant velocity (4i + 3j) m/s. Find the position vector of P at time t = 3 seconds.

(Total for Question 8 is 2 marks)

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9 A particle moves with velocity v = (5i - 12j) m/s. Find the speed of the particle.

(Total for Question 9 is 2 marks)

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10 The displacement of a particle at time t seconds is $x = 4t^2 - 3t$ metres. Find an expression for the velocity v m/s at time t.

(Total for Question 10 is 1 mark)

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11 A particle P has position vector (3i + j) m at time t = 0 and moves with constant velocity (-2i + 5j) m/s. Find the value of t at which the position vector of P has zero i-component (that is, the instant at which P crosses the y-axis).

(Total for Question 11 is 3 marks)

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12 A particle P moves along the x-axis. At time t seconds ($t \geq 0$), the displacement of P from a fixed point O is x metres, where $x = t^3 - 6t^2 + 9t$.

(a) Find an expression for the velocity v m/s of P at time t. **(2)**

(b) Find an expression for the acceleration a m/s^2 of P at time t. **(1)**

(c) Find the values of t for which P is instantaneously at rest. **(3)**

(d) Find the acceleration of P at each of the times found in part (c), and hence determine which of these times corresponds to a local maximum displacement. **(3)**

(e) Find the total distance travelled by P in the interval $0 \leq t \leq 3$. **(4)**

(Total for Question 12 is 13 marks)

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13 A particle moves in a straight line. At time t seconds, the acceleration of the particle is a m/s^2, where a = 6 - 2t. When t = 0, the particle is at a fixed origin O and is moving with velocity 1 m/s in the positive direction.

(a) Find an expression for v, the velocity of the particle, in terms of t. **(3)**

(b) Find an expression for x, the displacement of the particle from O, in terms of t. **(3)**

(c) Find the maximum velocity of the particle and the value of t at which it occurs. **(3)**

(d) Find, to 3 significant figures, the value of t (other than t = 0) at which the particle returns to O. **(4)**

(Total for Question 13 is 13 marks)

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14 At time t = 0, car A passes a fixed point O travelling in a straight line with speed 8 m/s and constant acceleration 1.5 m/s^2. At the same instant, car B is 10 m ahead of O on the same straight line, travelling in the same direction as A with constant speed 12 m/s (zero acceleration). Both cars are modelled as particles.

(a) Write down an expression, in terms of t, for the distance of A from O at time t. **(1)**

(b) Write down an expression, in terms of t, for the distance of B from O at time t. **(1)**

(c) Show that, at the time when A catches up with B, $3t^2 - 16t - 40 = 0$. **(3)**

(d) Hence find the time at which A catches up with B, giving your answer to 3 significant figures. **(3)**

(e) Find the distance of A from O at the time found in part (d). **(2)**

(Total for Question 14 is 10 marks)

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15 Two cyclists, A and B, start 250 m apart on a straight, flat cycle path and move directly towards each other, setting off at the same instant. Cyclist A moves at a constant speed of 6 m/s. Cyclist B starts from rest and moves with constant acceleration 0.4 m/s^2. Let t seconds be the time after they set off.

(a) Write down, in terms of t, an expression for the distance travelled by cyclist A, and an expression for the distance travelled by cyclist B. **(2)**

(b) Show that, at the instant the cyclists meet, $t^2 + 30t - 1250 = 0$. **(2)**

(c) Hence find the time at which the cyclists meet, giving your answer to 3 significant figures. **(3)**

(d) Find the distance of the meeting point from cyclist A's starting point. **(2)**

(e) State one limitation of modelling the two cyclists as particles in this context. **(1)**

(Total for Question 15 is 10 marks)

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16 A delivery robot starts from rest at one end of a straight corridor and accelerates uniformly at 0.5 m/s^2 for T seconds, reaching a maximum speed of V m/s. It then decelerates uniformly at 1 m/s^2, coming to rest exactly at the far end of the corridor, a distance of 54 m from its starting point.

(a) Show that T = 12. **(4)**

(b) Find the robot's maximum speed, V. **(2)**

(c) Find the total time taken by the robot to travel the length of the corridor. **(2)**

(d) Sketch the velocity-time graph for the whole journey, marking the coordinates of the point at which the velocity is greatest. **(3)**

(e) Find the robot's average speed for the whole 54 m journey. **(2)**

(Total for Question 16 is 13 marks)

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17 A ball is thrown vertically downwards from the top of a cliff with initial speed 6 m/s. It hits the sea 2.5 seconds later. Using g = 9.8 m/s^2, find the height of the cliff, giving your answer to 3 significant figures.

(Total for Question 17 is 3 marks)

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18 A train travels along a straight track. It accelerates uniformly from rest to a speed of 20 m/s in 25 seconds, then travels at this constant speed for a further 40 seconds, before decelerating uniformly to rest in 10 seconds. Find the total distance travelled by the train.

(Total for Question 18 is 4 marks)

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Mark scheme · 3.1 Mechanics: Quantities, Units and Kinematics: 2-hour Pack

Question 1

use $v^2 = u^2 + 2as$ with v=0, a=-4, s=32 (or $v^2=u^2-2as$ with a=4) **M1**

16 m/s cao **A1**

Answer: 16 m/s

Question 2

use v = u + at (or t = v/g) with u=0, v=24.5, a=9.8 **M1**

2.5 s cao **A1**

Answer: 2.5 s

Question 3

use v = u + at with u=1.5, a=0.6, t=12 **M1**

8.7 m/s cao **A1**

Answer: 8.7 m/s

Question 4

a = (6-18)/8 **M1**

1.5 m/s^2 cao (deceleration) **A1**

Answer: 1.5 m/s^2

Question 5

s = 1/2(u+v)t with u=9, v=0, t=3 **M1**

13.5 m cao **A1**

Answer: 13.5 m

Question 6

s = ut + 1/2 a t^2 with u=1, a=0.5, t=6 **M1**

15 m cao **A1**

Answer: 15 m

Question 7

(a) r = r0 + vt applied with t=3 **M1**

(a) (3i + 7j) m cao **A1**

(a) Answer: (3i + 7j) m

(b) speed = $\sqrt{3^2 + 4^2}$ **M1**

(b) 5 m/s cao **A1**

(b) Answer: 5 m/s

(c) set the i-component of position vector equal to 0: -6+3t=0 **M1**

(c) t = 2 (s) **A1**

(c) cao, with j-component positive confirming due north (not south) **A1**

(c) Answer: t = 2 s

(d) find position vector at t=5: (9i+15j) **M1**

(d) distance = $\sqrt{9^2+15^2}$ **M1**

(d) awrt 17.5 m **A1**

(d) Answer: 17.5 m (3 s.f.)

Question 8

r = r0 + vt applied with t=3 **M1**

(14i + 4j) m cao **A1**

Answer: (14i + 4j) m

Question 9

speed = $\sqrt{5^2 + 12^2}$ **M1**

13 m/s cao **A1**

Answer: 13 m/s

Question 10

v = 8t - 3 cao **B1**

Answer: v = 8t - 3

Question 11

form r(t) = (3-2t)i + (1+5t)j **M1**

set the i-component 3-2t = 0 and solve for t **M1**

t = 1.5 cao **A1**

Answer: t = 1.5 s

Question 12

(a) differentiate x with respect to t **M1**

(a) v = $3t^2$ - 12t + 9 cao **A1**

(a) Answer: v = $3t^2$ - 12t + 9

(b) a = 6t - 12 (ft from (a)) **B1**

(b) Answer: a = 6t - 12

(c) set v=0: $3t^2$-12t+9=0, divide by 3 **M1**

(c) factorise (t-1)(t-3)=0 **M1**

(c) t=1 and t=3 cao **A1**

(c) Answer: t = 1 and t = 3

(d) a(1) = -6 m/s^2 **B1**

(d) a(3) = 6 m/s^2 **B1**

(d) t=1 gives a local maximum (a<0 means v changes from + to -, so x has a maximum) **A1**

(d) Answer: a(1)=-6, a(3)=6; t=1 gives the local maximum displacement.

(e) find x(0), x(1), x(3) **M1**

(e) x(0)=0, x(1)=4, x(3)=0 **A1**

(e) distance = |x(1)-x(0)| + |x(3)-x(1)| **M1**

(e) 8 m cao **A1**

(e) Answer: 8 m

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Question 13

(a) integrate a with respect to t: $v = 6t - t^2 + C$ **M1**

(a) correct unsimplified integral **A1**

(a) use v(0)=1 to find C=1, so $v=6t-t^2+1$ cao **A1**

(a) Answer: $v = 6t - t^2 + 1$

(b) integrate v with respect to t: $x = 3t^2 - t^3/3 + t + C2$ **M1**

(b) correct unsimplified integral, ft from (a) **A1**

(b) use x(0)=0 to find C2=0, so $x=3t^2-(t^3)/3+t$ cao **A1**

(b) Answer: $x = 3t^2 - (t^3)/3 + t$

(c) set dv/dt = a = 0: 6-2t=0 **M1**

(c) t=3 **A1**

(c) v(3)=10 m/s cao **A1**

(c) Answer: Maximum velocity is 10 m/s, occurring at t = 3 s.

(d) set x=0 and factor out t: $t(-t^2/3+3t+1)=0$ **M1**

(d) form the quadratic $t^2-9t-3=0$ (multiplying through by -3) **M1**

(d) solve using the quadratic formula and reject the negative root **M1**

(d) awrt 9.32 s **A1**

(d) Answer: t = 9.32 s (3 s.f.)

Question 14

(a) $8t + 0.75t^2$ oe **B1**

(a) Answer: $8t + 0.75t^2$

(b) 10 + 12t **B1**

(b) Answer: 10 + 12t

(c) set $8t+0.75t^2 = 10+12t$, ft from (a) and (b) **M1**

(c) rearrange to $0.75t^2-4t-10=0$ **M1**

(c) cso: multiply through by 4 to give $3t^2-16t-40=0$ **A1**

(c) Answer: $3t^2 - 16t - 40 = 0$ (shown)

(d) apply the quadratic formula to $3t^2-16t-40=0$ **M1**

(d) reject the negative root **A1**

(d) awrt 7.19 s **A1**

(d) Answer: t = 7.19 s (3 s.f.)

(e) substitute t=7.19 (ft) into $8t+0.75t^2$ or 10+12t **M1**

(e) awrt 96.3 m **A1**

(e) Answer: 96.3 m (3 s.f.)

Question 15

(a) A: 6t **B1**

(a) B: $0.2t^2$ **B1**

(a) Answer: A: 6t; B: $0.2t^2$ (metres)

(b) $6t + 0.2t^2 = 250$, ft from (a) **M1**

(b) cso: multiply by 5 to obtain $t^2 + 30t - 1250 = 0$ **A1**

(b) Answer: $t^2 + 30t - 1250 = 0$ (shown)

(c) apply the quadratic formula to $t^2+30t-1250=0$ **M1**

(c) reject the negative root **A1**

(c) awrt 23.4 s **A1**

(c) Answer: t = 23.4 s (3 s.f.)

(d) substitute t=23.406 (ft) into 6t **M1**

(d) awrt 140 m **A1**

(d) Answer: 140 m (3 s.f.)

(e) any valid limitation, e.g. it ignores the length/size of the bicycles and riders, so in reality they could meet (or need to brake) slightly before the calculated point **B1**

(e) Answer: Modelling the cyclists as particles ignores the size of the bicycles and riders, so in practice they would meet slightly before the calculated point (oe).

Question 16

(a) V = 0.5T **M1**

(a) distance in the acceleration phase = $1/2(0.5)(T^2) = 0.25T^2$ oe **M1**

(a) deceleration phase lasts V/1 = 0.5T seconds and covers $1/2(V)(0.5T) = 0.125T^2$ oe; form the total-distance equation $0.25T^2+0.125T^2=54$, dependent on both previous M marks **M1**

(a) cso: $0.375T^2=54$ gives $T^2=144$, T=12 **A1**

(a) Answer: T = 12 (shown)

(b) V = 0.5(12), ft from (a) **M1**

(b) 6 m/s cao **A1**

(b) Answer: 6 m/s

(c) deceleration time = V/1 = 6, ft; total time = 12+6 **M1**

(c) 18 s cao **A1**

(c) Answer: 18 s

(d) correct overall shape: straight line rising from (0,0), straight line falling to the t-axis, both with positive velocity **B1**

(d) t-values 12 and 18 marked on the horizontal axis, ft **B1**

(d) v-value 6 marked on the vertical axis at t=12, ft **B1**

(d) Answer: Straight lines from (0,0) to (12,6), then (12,6) to (18,0).

(e) average speed = 54/18, ft from (c) **M1**

(e) 3 m/s cao **A1**

(e) Answer: 3 m/s

Question 17

$s = ut + 1/2\ a\ t^2$ with u=6, a=9.8, t=2.5 **M1**

$s = 6*2.5 + 0.5*9.8*2.5^2$ correctly evaluated **M1**

awrt 45.6 m **A1**

Answer: 45.6 m (3 s.f.)

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Question 18

distance in first stage = 1/2(0+20)(25) = 250	**M1**
distance at constant speed = 20*40 = 800	**M1**
distance decelerating = 1/2(20+0)(10) = 100	**M1**
1150 m cao (250+800+100)	**A1**

Answer: 1150 m

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