3.1 Mechanics: Quantities, Units and Kinematics

EDEXCEL 9MA0 · Calculator allowed · about 340 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Convert a distance of 5.6 km into metres.

(Total for Question 1 is 1 mark)

2 Convert a time of 4 minutes 30 seconds into seconds.

(Total for Question 2 is 1 mark)

3 A particle moves at a constant speed of 15 m/s for 6 seconds. Find the distance it travels.

(Total for Question 3 is 1 mark)

4 State whether each of the following is a vector quantity or a scalar quantity: (a) speed, (b) displacement.

(Total for Question 4 is 1 mark)

5 State the SI unit of speed.

(Total for Question 5 is 1 mark)

6 Convert a time of 2 hours 15 minutes into seconds.

(Total for Question 6 is 1 mark)

7 A parcel moves at a constant speed of 0.8 m/s along a conveyor belt for 25 seconds. Find the distance it travels.

(Total for Question 7 is 1 mark)

8 A car accelerates uniformly from rest to a speed of 24 m/s in 15 seconds. It then travels at this constant speed for a further 20 seconds before decelerating uniformly to rest in 8 seconds. The car is modelled as a particle moving in a straight line.

(a) Find the acceleration of the car during the first 15 seconds. **(2)**

(b) Find the distance travelled by the car during the first 15 seconds. **(2)**

(c) Find the deceleration of the car during the final 8 seconds. **(2)**

(d) Calculate the total distance travelled by the car during the whole 43 second journey. **(3)**

(e) Hence find the average speed of the car for the whole journey, giving your answer to 3 significant figures. **(2)**

(Total for Question 8 is 11 marks)

9 A stone is released from rest at the top of a vertical cliff and hits the sea 3.2 s later. The stone is modelled as a particle moving freely under gravity, with $g = 9.8\ m/s^2$.

(a) Find the speed with which the stone hits the sea. **(2)**

(b) Find the height of the cliff above sea level. **(3)**

(c) State one modelling assumption, other than that the stone is a particle, that has been used in this question. **(1)**

(d) A second, identical stone is thrown vertically downwards from the same point on the cliff with initial speed 5 m/s. Using the height found in part (b), find how long this second stone takes to reach the sea, giving your answer to 3 significant figures. **(3)**

(Total for Question 9 is 9 marks)

10 A ball is thrown vertically upwards from a point 1.5 m above horizontal ground with speed 14 m/s. The ball is modelled as a particle moving freely under gravity, with $g = 9.8\ m/s^2$.

(a) Find the maximum height above the ground reached by the ball. **(3)**

(b) Find the time taken for the ball to return to the point from which it was thrown. **(2)**

(c) Find the total time from the instant of release until the ball hits the ground, giving your answer to 3 significant figures. **(4)**

(d) State, with a reason, the likely effect on the ball's actual maximum height, compared with the value found in part (a), if air resistance is not in fact negligible. **(1)**

(Total for Question 10 is 10 marks)

11 A ball is released from rest and falls freely under gravity, $g = 9.8\ m/s^2$. Find its speed after falling for 1 second.

(Total for Question 11 is 1 mark)

12 A particle has initial speed 3 m/s and accelerates uniformly at $2\ m/s^2$ for 9 seconds. Find its final speed.

(Total for Question 12 is 2 marks)

13 A cyclist accelerates uniformly from 2 m/s to 14 m/s in 4 seconds. Find the acceleration.

(Total for Question 13 is 2 marks)

14 A car decelerates uniformly from 24 m/s to rest in 6 seconds. Find the distance travelled while decelerating.

(Total for Question 14 is 2 marks)

15 A particle moves with initial speed 2 m/s and constant acceleration 3 m/s^2. Find the distance it travels in the first 4 seconds.

(Total for Question 15 is 2 marks)

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16 A particle accelerates uniformly from 5 m/s to 13 m/s while travelling 36 m. Find the acceleration.

(Total for Question 16 is 2 marks)

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17 A car is travelling in a straight line and brakes with constant deceleration 4 m/s^2, coming to rest after travelling a further 32 m. Find the speed of the car at the instant the brakes were applied.

(Total for Question 17 is 2 marks)

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18 A stone falls freely from rest under gravity, where g = 9.8 m/s^2. Find the time taken for the stone to reach a speed of 24.5 m/s.

(Total for Question 18 is 2 marks)

19 A skateboarder has initial speed 1.5 m/s and accelerates uniformly at 0.6 m/s^2 for 12 seconds. Find her final speed.

(Total for Question 19 is 2 marks)

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20 A delivery van slows down uniformly from 18 m/s to 6 m/s in 8 seconds. Find its deceleration.

(Total for Question 20 is 2 marks)

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21 A sledge decelerates uniformly from 9 m/s to rest in 3 seconds. Find the distance it travels while decelerating.

(Total for Question 21 is 2 marks)

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22 A shopping trolley starts with speed 1 m/s and accelerates uniformly at 0.5 m/s^2. Find the distance it travels in the first 6 seconds.

(Total for Question 22 is 2 marks)

23 At time t = 0, a particle P has position vector (-6i - 5j) m relative to a fixed origin O. The particle moves with constant velocity (3i + 4j) m/s. Unit vectors i and j are due east and due north respectively.

(a) Find the position vector of P at time t = 3 seconds. **(2)**

(b) Find the speed of P. **(2)**

(c) Find the value of t at which P is due north of O. **(3)**

(d) Find the distance of P from O when t = 5, giving your answer to 3 significant figures. **(3)**

(Total for Question 23 is 10 marks)

24 A particle moves in a straight line with constant acceleration. It passes through a point A with speed 4 m/s and, 6 seconds later, passes through a point B with speed 19 m/s. The distance AB is d metres.

(a) Find the acceleration of the particle. **(2)**

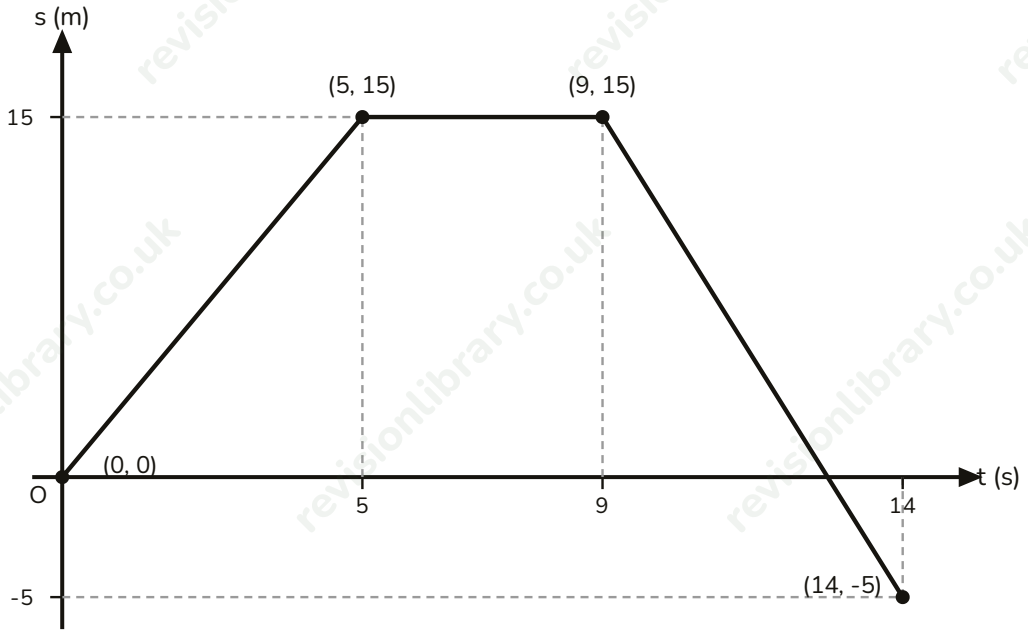
(b) Show that d = 69. **(3)**

(c) The particle continues to move with the same constant acceleration. Find the speed of the particle 2 seconds after it passes through B. **(2)**

(d) Find the total distance travelled by the particle from A until 2 seconds after it passes through B. **(3)**

(Total for Question 24 is 10 marks)

25 A remote-controlled toy car moves along a straight track. Its displacement s metres from a fixed point O at time t seconds is shown in the displacement-time graph, which consists of three straight line segments joining the points (0, 0), (5, 15), (9, 15) and (14, -5).



(a) State what feature of a displacement-time graph represents the velocity of the car. **(1)**

(b) Find the velocity of the car during the interval $0 \leq t \leq 5$. **(2)**

(c) Describe the motion of the car during the interval $5 \leq t \leq 9$. **(1)**

(d) Find the velocity of the car during the interval $9 \leq t \leq 14$, and state what the sign of your answer tells you about its direction of motion. **(3)**

(e) Find the value of t at which the car passes through O. **(3)**

(f) Find the total distance travelled by the car during the interval $0 \leq t \leq 14$. **(3)**

(Total for Question 25 is 13 marks)

26 A particle P has position vector $(2\mathbf{i} - 5\mathbf{j})$ m at time t = 0 and moves with constant velocity $(4\mathbf{i} + 3\mathbf{j})$ m/s. Find the position vector of P at time t = 3 seconds.

(Total for Question 26 is 2 marks)

27 A particle moves with velocity $\mathbf{v} = (5\mathbf{i} - 12\mathbf{j})$ m/s. Find the speed of the particle.

(Total for Question 27 is 2 marks)

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28 The displacement of a particle at time t seconds is $x = 4t^2 - 3t$ metres. Find an expression for the velocity v m/s at time t.

(Total for Question 28 is 1 mark)

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29 A particle P has position vector $(3\mathbf{i} + \mathbf{j})$ m at time $t = 0$ and moves with constant velocity $(-2\mathbf{i} + 5\mathbf{j})$ m/s. Find the value of t at which the position vector of P has zero i-component (that is, the instant at which P crosses the y-axis).

(Total for Question 29 is 3 marks)

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30 In the equation $s = ut + 1/2\ a\ t^2$, s is measured in metres, u in m/s, t in seconds and a in m/s^2. Show that the term $1/2\ a\ t^2$ has units of metres.

(Total for Question 30 is 2 marks)

31 A particle P moves along the x-axis. At time t seconds ($t \geq 0$), the displacement of P from a fixed point O is x metres, where $x = t^3 - 6t^2 + 9t$.

(a) Find an expression for the velocity v m/s of P at time t. **(2)**

(b) Find an expression for the acceleration a m/s^2 of P at time t. **(1)**

(c) Find the values of t for which P is instantaneously at rest. **(3)**

(d) Find the acceleration of P at each of the times found in part (c), and hence determine which of these times corresponds to a local maximum displacement. **(3)**

(e) Find the total distance travelled by P in the interval $0 \leq t \leq 3$. **(4)**

(Total for Question 31 is 13 marks)

32 A particle moves in a straight line. At time t seconds, the acceleration of the particle is a m/s^2, where a = 6 - 2t. When t = 0, the particle is at a fixed origin O and is moving with velocity 1 m/s in the positive direction.

(a) Find an expression for v, the velocity of the particle, in terms of t. **(3)**

(b) Find an expression for x, the displacement of the particle from O, in terms of t. **(3)**

(c) Find the maximum velocity of the particle and the value of t at which it occurs. **(3)**

(d) Find, to 3 significant figures, the value of t (other than t = 0) at which the particle returns to O. **(4)**

(Total for Question 32 is 13 marks)

33 At time t = 0, car A passes a fixed point O travelling in a straight line with speed 8 m/s and constant acceleration 1.5 m/s^2. At the same instant, car B is 10 m ahead of O on the same straight line, travelling in the same direction as A with constant speed 12 m/s (zero acceleration). Both cars are modelled as particles.

(a) Write down an expression, in terms of t, for the distance of A from O at time t. **(1)**

(b) Write down an expression, in terms of t, for the distance of B from O at time t. **(1)**

(c) Show that, at the time when A catches up with B, $3t^2 - 16t - 40 = 0$. **(3)**

(d) Hence find the time at which A catches up with B, giving your answer to 3 significant figures. **(3)**

(e) Find the distance of A from O at the time found in part (d). **(2)**

(Total for Question 33 is 10 marks)

34 Two cyclists, A and B, start 250 m apart on a straight, flat cycle path and move directly towards each other, setting off at the same instant. Cyclist A moves at a constant speed of 6 m/s. Cyclist B starts from rest and moves with constant acceleration 0.4 m/s^2. Let t seconds be the time after they set off.

(a) Write down, in terms of t, an expression for the distance travelled by cyclist A, and an expression for the distance travelled by cyclist B. **(2)**

(b) Show that, at the instant the cyclists meet, $t^2 + 30t - 1250 = 0$. **(2)**

(c) Hence find the time at which the cyclists meet, giving your answer to 3 significant figures. **(3)**

(d) Find the distance of the meeting point from cyclist A's starting point. **(2)**

(e) State one limitation of modelling the two cyclists as particles in this context. **(1)**

(Total for Question 34 is 10 marks)

35 A delivery robot starts from rest at one end of a straight corridor and accelerates uniformly at 0.5 m/s^2 for T seconds, reaching a maximum speed of V m/s. It then decelerates uniformly at 1 m/s^2, coming to rest exactly at the far end of the corridor, a distance of 54 m from its starting point.

(a) Show that T = 12. **(4)**

(b) Find the robot's maximum speed, V. **(2)**

(c) Find the total time taken by the robot to travel the length of the corridor. **(2)**

(d) Sketch the velocity-time graph for the whole journey, marking the coordinates of the point at which the velocity is greatest. **(3)**

(e) Find the robot's average speed for the whole 54 m journey. **(2)**

(Total for Question 35 is 13 marks)

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36 A particle moves in a straight line with constant acceleration. It passes point A with speed 4 m/s. Nine seconds later it passes point B with speed 22 m/s. Find the distance AB.

(Total for Question 36 is 3 marks)

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37 A ball is thrown vertically downwards from the top of a cliff with initial speed 6 m/s. It hits the sea 2.5 seconds later. Using $g = 9.8$ m/s^2, find the height of the cliff, giving your answer to 3 significant figures.

(Total for Question 37 is 3 marks)

38 A train travels along a straight track. It accelerates uniformly from rest to a speed of 20 m/s in 25 seconds, then travels at this constant speed for a further 40 seconds, before decelerating uniformly to rest in 10 seconds. Find the total distance travelled by the train.

(Total for Question 38 is 4 marks)

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39 A ball is thrown vertically upwards from ground level with initial speed 14 m/s and is modelled as a particle moving freely under gravity. Using $g = 9.8$ m/s^2, find the maximum height reached by the ball above the point of projection, and find the total time for which the ball is in the air before it returns to the point of projection, giving your final answer to 3 significant figures.

(Total for Question 39 is 4 marks)

40 A cyclist has velocity (3i + 4j) m/s relative to the ground, where i and j are unit vectors due east and due north respectively. A steady wind gives the cyclist an additional velocity of (-i + 2j) m/s relative to the ground. Find the cyclist's resultant velocity vector, and find her speed, giving your answer to 3 significant figures.

(Total for Question 40 is 4 marks)

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41 A particle moves so that its position vector at time t seconds is $r = (2t^2 - 3t)i + 4tj$ metres, where i and j are perpendicular unit vectors. Find the velocity vector of the particle at time t = 2 seconds, and find its speed at this instant, giving your answer to 3 significant figures.

(Total for Question 41 is 4 marks)

42 A particle P moves in a plane so that at time t seconds its velocity v m/s is given by $\mathbf{v} = (2t - 3)\mathbf{i} + (4 - t)\mathbf{j}$. When $t = 0$, the position vector of P relative to a fixed origin O is $(5\mathbf{i} + 2\mathbf{j})$ m.

(a) Find the position vector of P at time t. **(4)**

(b) Show that the acceleration of P is constant, and find the magnitude of this acceleration. **(4)**

(c) Find the value of t at which P is moving parallel to the vector i. **(2)**

(d) Find the speed of P at the instant found in part (c). **(3)**

(Total for Question 42 is 13 marks)

43 A particle P moves in a plane. At time t seconds ($t \geq 0$), the position vector of P relative to a fixed origin O is $\mathbf{r} = (2t^2 - 8t)\mathbf{i} + ((1/3)t^3 - 2t)\mathbf{j}$ metres, where i and j are unit vectors due east and due north respectively.

(a) Find v, the velocity of P at time t. **(2)**

(b) Find the value of t at which P is moving parallel to j. **(2)**

(c) Find the exact value of t, where $t > 0$, at which P is moving parallel to i. **(2)**

(d) Find, in the form $\pi + q\mathbf{j}$, the acceleration of P at the instant found in part (b). **(2)**

(e) Find, to the nearest degree, the bearing on which P is travelling at the instant t = 0. **(3)**

(Total for Question 43 is 11 marks)

44 A particle moves in a straight line with constant acceleration, without reversing direction between D and F. It passes through three points D, E and F, in that order, at times t=0, t=T and t=2T respectively, where T is a positive constant. The speeds of the particle at D, E and F are d m/s, e m/s and f m/s respectively, where d, e, f > 0.

(a) Show that e = (d + f)/2. **(4)**

(b) Given that d=5 and f=17, find the value of e. **(1)**

(c) Given further that T=4, find the acceleration of the particle. **(2)**

(d) Find the distance DF. **(3)**

(Total for Question 44 is 10 marks)

45 A particle P moves along the x-axis. At time t seconds ($0 \leq t \leq 6$), the velocity of P is v m/s, where $v = 3t^2 - 16t + 20$.

(a) Find an expression for the acceleration, a m/s^2, of P at time t. **(2)**

(b) Find the values of t at which P is instantaneously at rest. **(3)**

(c) Given that P is at the point where $x = 5$ when $t = 0$, find an expression for the displacement x of P from the origin, in terms of t. **(3)**

(d) Find the total distance travelled by P during the interval $0 \leq t \leq 4$. **(4)**

(Total for Question 45 is 12 marks)

46 A particle P moves along the x-axis. At time t seconds ($t \geq 0$) its displacement from a fixed point O is x metres, where $x = t^3 - 9t^2 + 24t$. Find the two values of t at which P is instantaneously at rest, and find the acceleration of P at the larger of these two values of t.

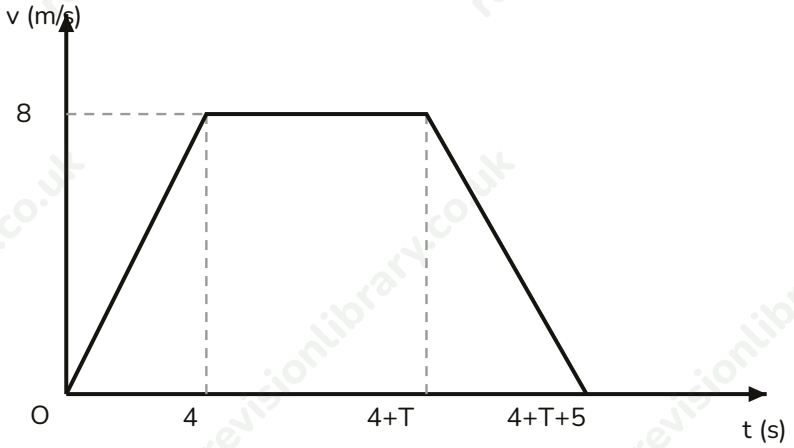
(Total for Question 46 is 4 marks)

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47 A particle P moves along the x-axis. At time t seconds its acceleration is $a = (6t - 4)$ m/s^2. When $t = 0$, the velocity of P is 3 m/s and the displacement of P from a fixed point O is 0. Find an expression for the velocity v m/s of P at time t, and find an expression for the displacement x metres of P from O at time t.

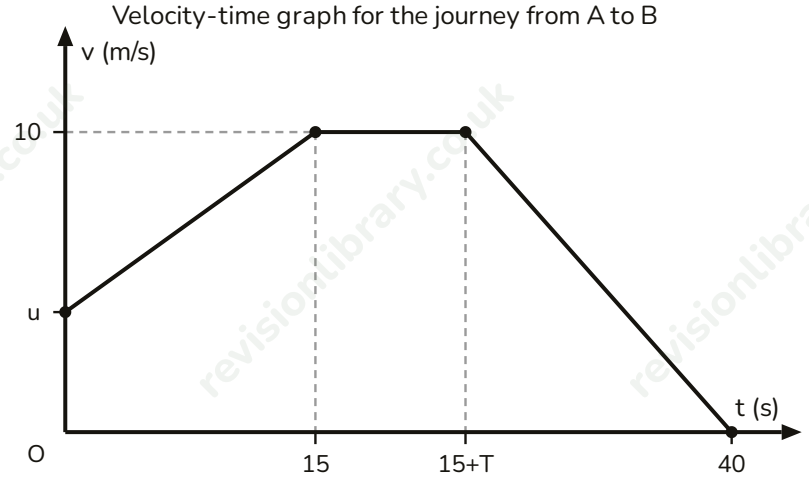
(Total for Question 47 is 4 marks)

48 A cyclist sets off from rest and accelerates uniformly to a speed of 8 m/s over the first 4 seconds of her journey. She then travels at this constant speed for a further T seconds, before decelerating uniformly to rest over the next 5 seconds. The velocity-time graph for her journey is shown below. Given that the total distance she travels is 100 m, find the value of T.



(Total for Question 48 is 4 marks)

49 A particle travels in a straight line from a point A to a point B, taking a total time of 40 seconds. The particle's velocity-time graph for the journey consists of three straight stages: Stage 1 ($0 \leq t \leq 15$): the particle accelerates uniformly from speed u m/s to speed 10 m/s, with constant acceleration 0.4 m/s^2. Stage 2 ($15 \leq t \leq 15+T$): the particle travels at a constant speed of 10 m/s for T seconds. Stage 3: the particle decelerates uniformly from 10 m/s to rest, arriving at B when t = 40. The total distance from A to B is 275 m.



(a) Find the value of u. **(2)**

(b) Find the distance travelled during Stage 1. **(2)**

(c) Show that T = 9. **(4)**

(d) Sketch the velocity-time graph for the whole journey, marking the values of t and v at each point where the gradient changes. **(3)**

(e) Find the deceleration during Stage 3. **(3)**

(Total for Question 49 is 14 marks)

50 Two particles, A and B, are 200 m apart on a straight horizontal road and move directly towards each other, starting at the same instant. Particle A starts with speed 4 m/s and accelerates at a constant 1.2 m/s^2 towards B. Particle B starts with speed 10 m/s and decelerates at a constant 0.6 m/s^2 towards A. Both particles are modelled as moving in a straight line until either they collide or B comes to rest, whichever happens first.

(a) Show that, provided the particles collide before B comes to rest, the time t seconds at which they meet satisfies $3t^2 + 140t - 2000 = 0$. **(4)**

(b) Solve the equation in part (a) to find the time at which the particles meet, giving your answer to 3 significant figures. **(3)**

(c) Find the time at which B would come to rest if it had not already collided with A, and use this to verify that the collision found in part (b) occurs while B is still moving. **(3)**

(d) Find the distance from A's starting point at which the collision occurs, giving your answer to 3 significant figures. **(2)**

(Total for Question 50 is 12 marks)

Mark scheme · 3.1 Mechanics: Quantities, Units and Kinematics

Question 1

5600 cao **B1**

Answer: 5600 m

Question 2

270 cao **B1**

Answer: 270 s

Question 3

90 cao **B1**

Answer: 90 m

Question 4

speed correctly identified as a scalar and displacement correctly identified as a vector, both required **B1**

Answer: Speed is a scalar quantity; displacement is a vector quantity.

Question 5

m/s (or m s^{-1}) oe **B1**

Answer: m/s

Question 6

8100 cao **B1**

Answer: 8100 s

Question 7

20 cao **B1**

Answer: 20 m

Question 8

(a) use a = (v - u)/t with v=24, u=0, t=15 **M1**

(a) 1.6 m/s^2 cao **A1**

(a) Answer: 1.6 m/s^2

(b) s = 1/2(u+v)t with u=0, v=24, t=15, or ft from (a) **M1**

(b) 180 m cao **A1**

(b) Answer: 180 m

(c) a = (0-24)/8 **M1**

(c) 3 m/s^2 (deceleration) cao **A1**

(c) Answer: 3 m/s^2

(d) distance at constant speed = 24 x 20 = 480 m **M1**

(d) distance decelerating = 1/2(24+0)(8) = 96 m **M1**

(d) 756 m cao (180+480+96) **A1**

(d) Answer: 756 m

(e) average speed = total distance / total time = 756/43 ft from (d) **M1**

(e) awrt 17.6 m/s **A1**

(e) Answer: 17.6 m/s (3 s.f.)

Question 9

(a) v = u + at with u=0, a=9.8, t=3.2 **M1**

(a) awrt 31.4 m/s **A1**

(a) Answer: 31.4 m/s (3 s.f.)

(b) s = ut + 1/2 a t^2 with u=0, a=9.8, t=3.2 **M1**

(b) s = 0.5*9.8*3.2^2 correctly evaluated **M1**

(b) awrt 50.2 m **A1**

(b) Answer: 50.2 m (3 s.f.)

(c) any valid assumption, e.g. air resistance is negligible / g is constant near the Earth's surface / the motion is vertical **B1**

(c) Answer: Air resistance on the stone is negligible (oe).

(d) form $4.9t^2 + 5t - 50.176 = 0$ (or equivalent using $s=ut+1/2at^2$) **M1**

(d) solve using the quadratic formula and reject the negative root **M1**

(d) awrt 2.73 s **A1**

(d) Answer: 2.73 s (3 s.f.)

Question 10

(a) use $v^2 = u^2 - 2gs$ with v=0, u=14, g=9.8 to find the height above the point of release **M1**

(a) s = 14^2/(2 x 9.8) correctly evaluated **M1**

(a) 11.5 m cao (height above release point of 10 m, plus the initial 1.5 m) **A1**

(a) Answer: 11.5 m

(b) t = 2u/g **M1**

(b) awrt 2.86 s **A1**

(b) Answer: 2.86 s (3 s.f.)

(c) form $4.9t^2 - 14t - 1.5 = 0$ (or equivalent using s = ut - 1/2 g t^2 with displacement -1.5 from the release point) **M1**

(c) rearrange correctly into a solvable quadratic **M1**

(c) apply the quadratic formula **M1**

(c) awrt 2.96 s, rejecting the negative root **A1**

(c) Answer: 2.96 s (3 s.f.)

(d) the actual maximum height would be less than 11.5 m, because air resistance would act against the ball's motion as it rises, decelerating it further (oe) **B1**

(d) Answer: The actual maximum height would be less than 11.5 m, because air resistance decelerates the ball further as it rises (oe).

Question 11

9.8 cao **B1**

Answer: 9.8 m/s

Question 12

use v = u + at with u=3, a=2, t=9 **M1**

21 m/s cao **A1**

Answer: 21 m/s

Question 13

a = (14-2)/4 **M1**

3 m/s^2 cao **A1**

Answer: 3 m/s^2

Question 14

s = 1/2(u+v)t with u=24, v=0, t=6 **M1**

72 m cao **A1**

Answer: 72 m

Question 15

$s = ut + 1/2\ a\ t^2$ with u=2, a=3, t=4 **M1**

32 m cao **A1**

Answer: 32 m

Question 16

use $v^2 = u^2 + 2as$ with u=5, v=13, s=36 **M1**

2 m/s^2 cao **A1**

Answer: 2 m/s^2

Question 17

use $v^2 = u^2 + 2as$ with v=0, a=-4, s=32 (or $v^2=u^2-2as$ with a=4) **M1**

16 m/s cao **A1**

Answer: 16 m/s

Question 18

use v = u + at (or t = v/g) with u=0, v=24.5, a=9.8 **M1**

2.5 s cao **A1**

Answer: 2.5 s

Question 19

use v = u + at with u=1.5, a=0.6, t=12 **M1**

8.7 m/s cao **A1**

Answer: 8.7 m/s

Question 20

a = (6-18)/8 **M1**

1.5 m/s^2 cao (deceleration) **A1**

Answer: 1.5 m/s^2

Question 21

s = 1/2(u+v)t with u=9, v=0, t=3 **M1**

13.5 m cao **A1**

Answer: 13.5 m

Question 22

$s = ut + 1/2\ a\ t^2$ with u=1, a=0.5, t=6 **M1**

15 m cao **A1**

Answer: 15 m

Question 23

(a) r = r0 + vt applied with t=3 **M1**

(a) (3i + 7j) m cao **A1**

(a) Answer: (3i + 7j) m

(b) speed = $\sqrt{3^2 + 4^2}$ **M1**

(b) 5 m/s cao **A1**

(b) Answer: 5 m/s

(c) set the i-component of position vector equal to 0: -6+3t=0 **M1**

(c) t = 2 (s) **A1**

(c) cao, with j-component positive confirming due north (not south) **A1**

(c) Answer: t = 2 s

(d) find position vector at t=5: (9i+15j) **M1**

(d) distance = $\sqrt{9^2+15^2}$ **M1**

(d) awrt 17.5 m **A1**

(d) Answer: 17.5 m (3 s.f.)

Question 24

(a) a = (19-4)/6 **M1**

(a) 2.5 m/s^2 cao **A1**

(a) Answer: 2.5 m/s^2

(b) use $s = ut + 1/2\ a\ t^2$ with u=4, a=2.5 (ft from (a)), t=6 **M1**

(b) s = 4*6 + 0.5*2.5*36 evaluated **M1**

(b) cso: d = 24+45 = 69 **A1**

(b) Answer: d = 69 m (shown)

(c) v = 19 + 2.5*2 **M1**

(c) 24 m/s cao **A1**

(c) Answer: 24 m/s

(d) total time from A = 6+2 = 8 s **M1**

(d) $s = 4*8 + 0.5*2.5*8^2$ (or 69 + distance B to final point) **M1**

(d) 112 m cao **A1**

(d) Answer: 112 m

Question 25

(a) the gradient (slope) of the graph **B1**

(a) Answer: The gradient of the displacement-time graph.

(b) gradient = (15-0)/(5-0) **M1**

(b) 3 m/s cao **A1**

(b) Answer: 3 m/s

(c) the car is stationary (at rest), remaining 15 m from O **B1**

(c) Answer: The car is stationary, 15 m from O.

(d) gradient = (-5-15)/(14-9) **M1**

(d) -4 m/s cao **A1**

(d) the negative sign shows the car is moving in the negative direction (back towards and past O) **B1**

(d) Answer: -4 m/s; the car is moving in the negative direction, back towards and through O.

(e) set up a linear equation for the third segment, e.g. s = 15 - 4(t-9), ft from (d) **M1**

(e) solve 15 - 4(t-9) = 0 **M1**

(e) t = 12.75 cao **A1**

(e) Answer: t = 12.75 s

(f) distance in the first segment = 15 (0 to 5), distance in the second segment = 0 (5 to 9) **M1**

(f) distance in the third segment = |(-5)-15| = 20 (9 to 14) **M1**

(f) 35 m cao, not the net displacement of -5 m **A1**

(f) Answer: 35 m

Question 26

r = r0 + vt applied with t=3 **M1**

(14i + 4j) m cao **A1**

Answer: (14i + 4j) m

Question 27

speed = $\sqrt{5^2 + 12^2}$ **M1**

13 m/s cao **A1**

Answer: 13 m/s

Question 28

v = 8t - 3 cao **B1**

Answer: v = 8t - 3

Question 29

form r(t) = (3-2t)i + (1+5t)j **M1**

set the i-component 3-2t = 0 and solve for t **M1**

t = 1.5 cao **A1**

Answer: t = 1.5 s

Question 30

units of a t^2 = (m s^{-2})(s^2) = m, with the s^2 terms shown cancelling **B1**

conclusion that 1/2 a t^2 therefore has units of metres, consistent with s cso **B1**

Answer: 1/2 a t^2 has units of metres, consistent with s.

Question 31

(a) differentiate x with respect to t **M1**

(a) v = $3t^2$ - 12t + 9 cao **A1**

(a) Answer: v = $3t^2$ - 12t + 9

(b) a = 6t - 12 (ft from (a)) **B1**

(b) Answer: a = 6t - 12

(c) set v=0: $3t^2$-12t+9=0, divide by 3 **M1**

(c) factorise (t-1)(t-3)=0 **M1**

(c) t=1 and t=3 cao **A1**

(c) Answer: t = 1 and t = 3

(d) a(1) = -6 m/s^2 **B1**

(d) a(3) = 6 m/s^2 **B1**

(d) t=1 gives a local maximum (a<0 means v changes from + to -, so x has a maximum) **A1**

(d) Answer: a(1)=-6, a(3)=6; t=1 gives the local maximum displacement.

(e) find x(0), x(1), x(3) **M1**

(e) x(0)=0, x(1)=4, x(3)=0 **A1**

(e) distance = |x(1)-x(0)| + |x(3)-x(1)| **M1**

(e) 8 m cao **A1**

(e) Answer: 8 m

Question 32

(a) integrate a with respect to t: v = 6t - t^2 + C **M1**

(a) correct unsimplified integral **A1**

(a) use v(0)=1 to find C=1, so v=6t-t^2+1 cao **A1**

(a) Answer: v = 6t - t^2 + 1

(b) integrate v with respect to t: x = $3t^2$ - t^3/3 + t + C2 **M1**

(b) correct unsimplified integral, ft from (a) **A1**

(b) use x(0)=0 to find C2=0, so x=$3t^2$-(t^3)/3+t cao **A1**

(b) Answer: x = $3t^2$ - (t^3)/3 + t

(c) set dv/dt = a = 0: 6-2t=0 **M1**

(c) t=3 **A1**

(c) v(3)=10 m/s cao **A1**

(c) Answer: Maximum velocity is 10 m/s, occurring at t = 3 s.

(d) set x=0 and factor out t: t(-t^2/3+3t+1)=0 **M1**

(d) form the quadratic t^2-9t-3=0 (multiplying through by -3) **M1**

(d) solve using the quadratic formula and reject the negative root **M1**

(d) awrt 9.32 s **A1**

(d) Answer: t = 9.32 s (3 s.f.)

Question 33

(a) $8t + 0.75t^2$ oe **B1**

(a) Answer: $8t + 0.75t^2$

(b) $10 + 12t$ **B1**

(b) Answer: $10 + 12t$

(c) set $8t+0.75t^2 = 10+12t$, ft from (a) and (b) **M1**

(c) rearrange to $0.75t^2-4t-10=0$ **M1**

(c) cso: multiply through by 4 to give $3t^2-16t-40=0$ **A1**

(c) Answer: $3t^2 - 16t - 40 = 0$ (shown)

(d) apply the quadratic formula to $3t^2-16t-40=0$ **M1**

(d) reject the negative root **A1**

(d) awrt 7.19 s **A1**

(d) Answer: t = 7.19 s (3 s.f.)

(e) substitute t=7.19 (ft) into $8t+0.75t^2$ or $10+12t$ **M1**

(e) awrt 96.3 m **A1**

(e) Answer: 96.3 m (3 s.f.)

Question 34

(a) A: $6t$ **B1**

(a) B: $0.2t^2$ **B1**

(a) Answer: A: $6t$; B: $0.2t^2$ (metres)

(b) $6t + 0.2t^2 = 250$, ft from (a) **M1**

(b) cso: multiply by 5 to obtain $t^2 + 30t - 1250 = 0$ **A1**

(b) Answer: $t^2 + 30t - 1250 = 0$ (shown)

(c) apply the quadratic formula to $t^2+30t-1250=0$ **M1**

(c) reject the negative root **A1**

(c) awrt 23.4 s **A1**

(c) Answer: t = 23.4 s (3 s.f.)

(d) substitute t=23.406 (ft) into $6t$ **M1**

(d) awrt 140 m **A1**

(d) Answer: 140 m (3 s.f.)

(e) any valid limitation, e.g. it ignores the length/size of the bicycles and riders, so in reality they could meet (or need to brake) slightly before the calculated point **B1**

(e) Answer: Modelling the cyclists as particles ignores the size of the bicycles and riders, so in practice they would meet slightly before the calculated point (oe).

Question 35

(a) $V = 0.5T$ **M1**

(a) distance in the acceleration phase = $1/2(0.5)(T^2) = 0.25T^2$ oe **M1**

(a) deceleration phase lasts $V/1 = 0.5T$ seconds and covers $1/2(V)(0.5T) = 0.125T^2$ oe; form the total-distance equation $0.25T^2+0.125T^2=54$, dependent on both previous M marks **M1**

(a) cso: $0.375T^2=54$ gives $T^2=144$, T=12 **A1**

(a) Answer: T = 12 (shown)

(b) V = 0.5(12), ft from (a) **M1**

(b) 6 m/s cao **A1**

(b) Answer: 6 m/s

(c) deceleration time = V/1 = 6, ft; total time = 12+6 **M1**

(c) 18 s cao **A1**

(c) Answer: 18 s

(d) correct overall shape: straight line rising from (0,0), straight line falling to the t-axis, both with positive velocity **B1**

(d) t-values 12 and 18 marked on the horizontal axis, ft **B1**

(d) v-value 6 marked on the vertical axis at t=12, ft **B1**

(d) Answer: Straight lines from (0,0) to (12,6), then (12,6) to (18,0).

(e) average speed = 54/18, ft from (c) **M1**

(e) 3 m/s cao **A1**

(e) Answer: 3 m/s

Question 36

a = (22-4)/9 **M1**

s = 1/2(u+v)t = 1/2(4+22)(9), or ft using $s = ut + 1/2\ a\ t^2$ **M1**

117 m cao **A1**

Answer: 117 m

Question 37

$s = ut + 1/2\ a\ t^2$ with u=6, a=9.8, t=2.5 **M1**

$s = 6*2.5 + 0.5*9.8*2.5^2$ correctly evaluated **M1**

awrt 45.6 m **A1**

Answer: 45.6 m (3 s.f.)

Question 38

distance in first stage = 1/2(0+20)(25) = 250 **M1**

distance at constant speed = 20*40 = 800 **M1**

distance decelerating = 1/2(20+0)(10) = 100 **M1**

1150 m cao (250+800+100) **A1**

Answer: 1150 m

Question 39

use $v^2 = u^2 - 2gs$ with v=0, u=14, g=9.8 to find the maximum height **M1**

10 m cao **A1**

use symmetry (T = 2u/g) or v = u - gt to find the total time of flight **M1**

awrt 2.86 s **A1**

Answer: Maximum height = 10 m; total time in the air = 2.86 s (3 s.f.)

Question 40

add the velocity components: (3-1)i + (4+2)j **M1**

(2i + 6j) m/s cao **A1**

speed = $\sqrt{2^2 + 6^2}$ **M1**

awrt 6.32 m/s **A1**

Answer: Resultant velocity = (2i + 6j) m/s; speed = 6.32 m/s (3 s.f.)

Question 41

differentiate: v = (4t-3)i + 4j **M1**

substitute t=2 **M1**

(5i + 4j) m/s cao **A1**

awrt 6.40 m/s **A1**

Answer: v(2) = (5i + 4j) m/s; speed = 6.40 m/s (3 s.f.)

Question 42

(a) integrate the i-component: t^2-3t+C1 **M1**

(a) use $r_i(0)$=5 to give i-component t^2-3t+5 **A1**

(a) integrate the j-component: 4t-0.5t^2+C2 **M1**

(a) use $r_j(0)$=2 to give j-component 4t-0.5t^2+2 cao **A1**

(a) Answer: r = (t^2 - 3t + 5)i + (4t - 0.5t^2 + 2)j

(b) a = dv/dt = 2i - j **B1**

(b) state a does not depend on t, so acceleration is constant **B1**

(b) magnitude = $\sqrt{2^2+(-1)^2}$ **M1**

(b) awrt 2.24 m/s^2 **A1**

(b) Answer: a = (2i - j) m/s^2, constant; magnitude 2.24 m/s^2 (3 s.f.)

(c) set the j-component of v equal to 0: 4-t=0 **M1**

(c) t=4 cao **A1**

(c) Answer: t = 4 s

(d) substitute t=4 (ft) into v to get 5i+0j **M1**

(d) correct components **A1**

(d) speed = 5 m/s cao **A1**

(d) Answer: 5 m/s

Question 43

(a) differentiate the i-component: 4t-8 **M1**

(a) differentiate the j-component: t^2-2, both correct **A1**

(a) Answer: v = (4t - 8)i + (t^2 - 2)j

(b) set the i-component of v to 0: 4t-8=0 **M1**

(b) t=2 cao **A1**

(b) Answer: t = 2 s

(c) set the j-component of v to 0: t^2-2=0 **M1**

(c) t=$\sqrt{2}$, rejecting the negative root **A1**

(c) Answer: t = $\sqrt{2}$ s

(d) differentiate v to find a(t)=4i+2tj **M1**

(d) substitute t=2, ft, to give 4i+4j **A1**

(d) Answer: a = 4i + 4j (m/s^2)

(e) find v(0) = -8i - 2j **M1**

(e) use right-angled trigonometry, e.g. the angle south of due west = arctan(2/8) **M1**

(e) awrt 256 degrees **A1**

(e) Answer: 256 degrees (nearest degree)

Question 44

(a) use v=u+at at E: e=d+aT **M1**

(a) use v=u+at at F: f=d+2aT **M1**

(a) eliminate a, e.g. substitute aT=e-d into the equation for f **M1**

(a) cso: rearrange to e=(d+f)/2 **A1**

(a) Answer: e = (d + f)/2 (shown)

(b) e=11, ft from (a) **B1**

(b) Answer: e = 11 m/s

(c) a=(e-d)/T, ft **M1**

(c) 1.5 m/s^2 cao **A1**

(c) Answer: 1.5 m/s^2

(d) use the average velocity over D to F, (d+f)/2, or an equivalent SUVAT method, ft **M1**

(d) multiply by the total time 2T, ft **M1**

(d) 88 m cao **A1**

(d) Answer: 88 m

Question 45

(a) differentiate v with respect to t **M1**

(a) a = 6t - 16 cao **A1**

(a) Answer: a = 6t - 16

(b) set v=0: 3t^2-16t+20=0 **M1**

(b) apply the quadratic formula (or factorise) **M1**

(b) t=2 and t=10/3 (awrt 3.33), both cao **A1**

(b) Answer: t = 2 and t = 10/3 (awrt 3.33)

(c) integrate v with respect to t: x = t^3-8t^2+20t+C **M1**

(c) correct unsimplified integral, ft **A1**

(c) use x(0)=5 to find C=5, giving x=t^3-8t^2+20t+5 cao **A1**

(c) Answer: x = t^3 - 8t^2 + 20t + 5

(d) evaluate x at the turning points found in (b) and at the interval endpoints: t=0, 2, 10/3, 4, ft **M1**

(d) x(0)=5, x(2)=21, x(10/3)=535/27 (awrt 19.8), x(4)=21 **A1**

(d) sum the magnitudes of the changes between consecutive values, since P changes direction at each turning point **M1**

(d) awrt 18.4 m **A1**

(d) Answer: 18.4 m (3 s.f.)

Question 46

differentiate x to find v = 3t^2 - 18t + 24 **M1**

set v=0 and solve, e.g. factorise 3(t-2)(t-4)=0 **M1**

t = 2 and t = 4 cao **A1**

a = 6t - 18, so a(4) = 6 m/s^2 cao **A1**

Answer: t = 2 s and t = 4 s; acceleration at t = 4 is 6 m/s^2

Question 47

integrate a to get v = 3t^2 - 4t + C **M1**

use v=3 at t=0 to find C=3, so v = 3t^2 - 4t + 3 cao **A1**

integrate v to get x = t^3 - 2t^2 + 3t + D **M1**

use x=0 at t=0 to find D=0, so x = t^3 - 2t^2 + 3t cao **A1**

Answer: v = 3t^2 - 4t + 3; x = t^3 - 2t^2 + 3t

Question 48

distance in the acceleration phase = 1/2 x 4 x 8 = 16	**M1**
distance in the deceleration phase = 1/2 x 5 x 8 = 20	**M1**
set up 16 + 8T + 20 = 100 (or equivalent)	**M1**
T = 8 cao	**A1**

Answer: T = 8 s

Question 49

(a) use v=u+at with v=10, a=0.4, t=15, rearranged for u	**M1**
(a) u=4 m/s cao	**A1**

(a) Answer: u = 4 m/s

(b) s = 1/2(u+v)t = 0.5*(4+10)*15, ft from (a)	**M1**
(b) 105 m cao	**A1**

(b) Answer: 105 m

(c) distance in Stage 2 = 10T	**M1**
(c) Stage 3 duration = 40-15-T = 25-T, distance = 1/2*10*(25-T) = 125-5T	**M1**
(c) form the total distance equation 105+10T+125-5T=275, dependent on both previous M marks	**dM1**
(c) cso: solve to give T=9	**A1**

(c) Answer: T = 9 (shown)

(d) correct overall shape: rising line, horizontal line, falling line, all with positive velocity	**B1**
(d) t-values 15, 24 (15+T with T=9, ft) and 40 marked on the horizontal axis	**B1**
(d) v-values 4, 10 and 0 marked on the vertical axis at the correct points	**B1**

(d) Answer: Straight lines from (0,4) to (15,10), (15,10) to (24,10), and (24,10) to (40,0).

(e) Stage 3 duration = 25-T = 25-9=16 s, ft from (c)	**M1**
(e) a = (0-10)/16	**M1**
(e) 0.625 m/s^2 cao	**A1**

(e) Answer: 0.625 m/s^2

Question 50

(a) distance travelled by A from its start: 4t+0.6t^2	**M1**
(a) distance travelled by B from its start: 10t-0.3t^2	**M1**
(a) set the sum of the two distances travelled equal to 200 and form an equation, dependent on both previous M marks	**M1**
(a) cso: rearrange and multiply by 10 to reach 3t^2+140t-2000=0	**A1**

(a) Answer: 3t^2 + 140t - 2000 = 0 (shown)

(b) apply the quadratic formula to 3t^2+140t-2000=0	**M1**
(b) reject the negative root	**A1**
(b) awrt 11.5 s	**A1**

(b) Answer: t = 11.5 s (3 s.f.)

(c) use v=u+at with v=0, u=10, a=-0.6, i.e. t=10/0.6	**M1**
(c) t=16.7 s (3 s.f.), or exact 50/3 s	**A1**
(c) since 11.5 (ft) < 16.7, the collision does occur before B comes to rest, so the equation in part (a) is valid	**B1**

(c) Answer: B would stop at t = 16.7 s (3 s.f.); since 11.5 s < 16.7 s, the collision does occur while B is still moving.

(d) substitute t=11.468 (ft) into 4t+0.6t^2, or the equivalent expression for B	**M1**
(d) awrt 125 m	**A1**

(d) Answer: 125 m (3 s.f.)