3.3 Mechanics: Moments

EDEXCEL 9MA0 · Calculator allowed · about 320 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 State the SI unit of a moment.

(Total for Question 1 is 1 mark)

2 A force of 40 N acts at right angles to a lever at a point 0.5 m from the pivot. Find the moment of the force about the pivot.

(Total for Question 2 is 1 mark)

3 A force of 15 N acts at right angles to a bar at a point 3 m from a pivot. Find the moment of the force about the pivot.

(Total for Question 3 is 1 mark)

4 State the condition that must be satisfied by the moments of the forces acting on a rigid body for the body to be in rotational equilibrium.

(Total for Question 4 is 1 mark)

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5 A rod is described as non-uniform. State what this tells you about the position of its centre of mass, compared with a uniform rod of the same length.

(Total for Question 5 is 1 mark)

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6 A mechanic applies a force of 28 N at right angles to a car jack handle, at a point 1.5 m from the pivot. Find the moment of the force about the pivot.

(Total for Question 6 is 1 mark)

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7 A moment is the turning effect of a force about a point. State the two quantities whose product gives the magnitude of a moment.

(Total for Question 7 is 1 mark)

8 A force acting at right angles to a lever at a pivot produces a moment of 54 N m. Given that the force has magnitude 90 N, find the distance from the pivot to the point where the force acts.

(Total for Question 8 is 2 marks)

9 A moment of 45 N m is produced by a force acting at right angles to a lever, at a distance of 0.75 m from the pivot. Find the magnitude of the force.

(Total for Question 9 is 2 marks)

10 A uniform beam AB of length 6 m and weight 180 N rests horizontally in equilibrium on two smooth supports, one at each end. Find the reaction at each support.

(Total for Question 10 is 2 marks)

11 A non-uniform rod AB of length 2 m is held horizontal in equilibrium by two vertical strings, one attached at each end. The tension in the string at A is 30 N and the tension in the string at B is 50 N. Find the weight of the rod.

(Total for Question 11 is 2 marks)

12 A uniform strut AB rests horizontally on two smooth supports, one at each end. The strut has weight 130 N. Given that the reaction at end A is 55 N, find the reaction at end B.

(Total for Question 12 is 2 marks)

13 A force applied at right angles to a bicycle brake lever, at a distance of 0.08 m from the pivot, produces a moment of 3.6 N m. Find the magnitude of the force.

(Total for Question 13 is 2 marks)

14 A door is fitted with a single hinge mechanism. A cleaner pushes on the door with a force of 150 N applied at right angles to the door, at a point 0.8 m from the hinge.

(a) Calculate the moment of the 150 N force about the hinge, stating the appropriate units. **(2)**

(b) The hinge mechanism is rated to safely withstand a maximum moment of 100 N m before it risks damage. Determine, with a reason, whether the hinge is at risk of damage from this push. **(2)**

(Total for Question 14 is 4 marks)

15 A uniform beam AB has length 5 m and weight 240 N. The beam rests horizontally in equilibrium on two smooth supports, one at C, 0.8 m from A, and one at D, 1.2 m from B.

(a) By taking moments about C, show that the reaction at D, R_D, is 136 N. **(4)**

(b) Find the reaction at C, R_C. **(2)**

(c) State two assumptions made about the beam and its supports in this model, and explain briefly how each assumption is used in the calculation. **(4)**

(Total for Question 15 is 10 marks)

16 A gardener uses a wheelbarrow to move garden waste. When lifting the wheelbarrow, it is modelled as pivoting about the axle of its single front wheel. The load of garden waste has weight 250 N and acts at a horizontal distance of 0.4 m from the wheel axle. The gardener applies a vertical lifting force at the handles, which are a horizontal distance of 1 m from the wheel axle. The wheelbarrow's own frame is light, so its weight may be ignored.

(a) Calculate the moment of the 250 N load about the wheel axle, stating the appropriate units. **(2)**

(b) Find the minimum vertical lifting force the gardener must apply at the handles for the wheelbarrow to begin to lift off the ground (other than at the wheel). **(2)**

(Total for Question 16 is 4 marks)

17 A cyclist applies a force of 400 N to a bicycle pedal, at an angle of 60 degrees to the crank arm, which has length 0.17 m. The bottom bracket of the bicycle acts as a fixed pivot for the crank.

(a) Calculate the perpendicular distance from the pivot to the line of action of the 400 N force, giving your answer to 3 significant figures. **(2)**

(b) Hence calculate the moment (turning effect) of the force about the pivot, stating the appropriate units, giving your answer to 3 significant figures. **(2)**

(c) The cyclist repeats the push, this time applying a force of the same magnitude perpendicular to the crank (that is, at 90 degrees to the crank). Find the percentage increase in the moment compared to part (b), giving your answer to 3 significant figures. **(2)**

(Total for Question 17 is 6 marks)

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18 A uniform rod AB has length 4 m and weight 100 N. The rod rests horizontally in equilibrium on two smooth supports, one at A and one at a point 2.5 m from A. By taking moments about A, find the reaction at the support 2.5 m from A.

(Total for Question 18 is 2 marks)

19 A force of 80 N acts on a rod at an angle of 60 degrees to the rod, at a distance of 0.5 m from a pivot. Find the moment of the force about the pivot, giving your answer to 3 significant figures.

(Total for Question 19 is 2 marks)

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20 A force of 45 N acts on a bar at an angle of 40 degrees to the bar, at a distance of 0.8 m from a pivot. Find the moment of the force about the pivot, giving your answer to 3 significant figures.

(Total for Question 20 is 2 marks)

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21 A uniform rod is pivoted at its centre. A force of 60 N acts vertically downward at a distance of 0.9 m from the pivot on one side. Find the distance from the pivot at which a force of 45 N must act vertically downward on the other side, for the rod to remain in equilibrium.

(Total for Question 21 is 2 marks)

22 The table below shows three forces acting at right angles to a light rod that is free to rotate about a fixed pivot O.

Force	Perpendicular distance from O	Sense
18 N	0.4 m	clockwise
25 N	0.5 m	anticlockwise
30 N	0.3 m	clockwise

Find the magnitude of the resultant moment about O, and state its sense.

(Total for Question 22 is 2 marks)

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23 A garden gate is opened by a force of 55 N applied at an angle of 70 degrees to the gate, at a point 0.9 m from the hinge. Find the moment of the force about the hinge, giving your answer to 3 significant figures.

(Total for Question 23 is 2 marks)

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24 A decorator uses a screwdriver as a lever to prise open a paint tin lid. She applies a force of 40 N to the screwdriver at an angle of 25 degrees to the screwdriver, at a distance of 0.14 m from the pivot (the rim of the tin). Find the moment of the force about the pivot, giving your answer to 3 significant figures.

(Total for Question 24 is 2 marks)

25 A non-uniform rod AB has length 3.6 m. The rod is held in equilibrium in a horizontal position by two vertical strings, one attached at A and the other attached at a point 2.4 m from A. The tension in the string at A is 45 N and the tension in the other string is 75 N.

(a) Find the weight of the rod. **(2)**

(b) By taking moments about A, find the distance from A to the centre of mass of the rod. **(4)**

(c) State, without further calculation, whether the centre of mass lies closer to A or to the point where the 75 N string is attached. Justify your answer. **(2)**

(Total for Question 25 is 8 marks)

26 A see-saw is modelled as a uniform rod PQ of length 4 m and weight 150 N, resting on a smooth pivot at its midpoint. Chandrika, of weight 320 N, sits at P. Tafari, of weight 400 N, sits on the see-saw between the pivot and Q so that the see-saw is horizontal and in equilibrium.

(a) Explain why the weight of the see-saw does not appear in the moments equation about the pivot. **(1)**

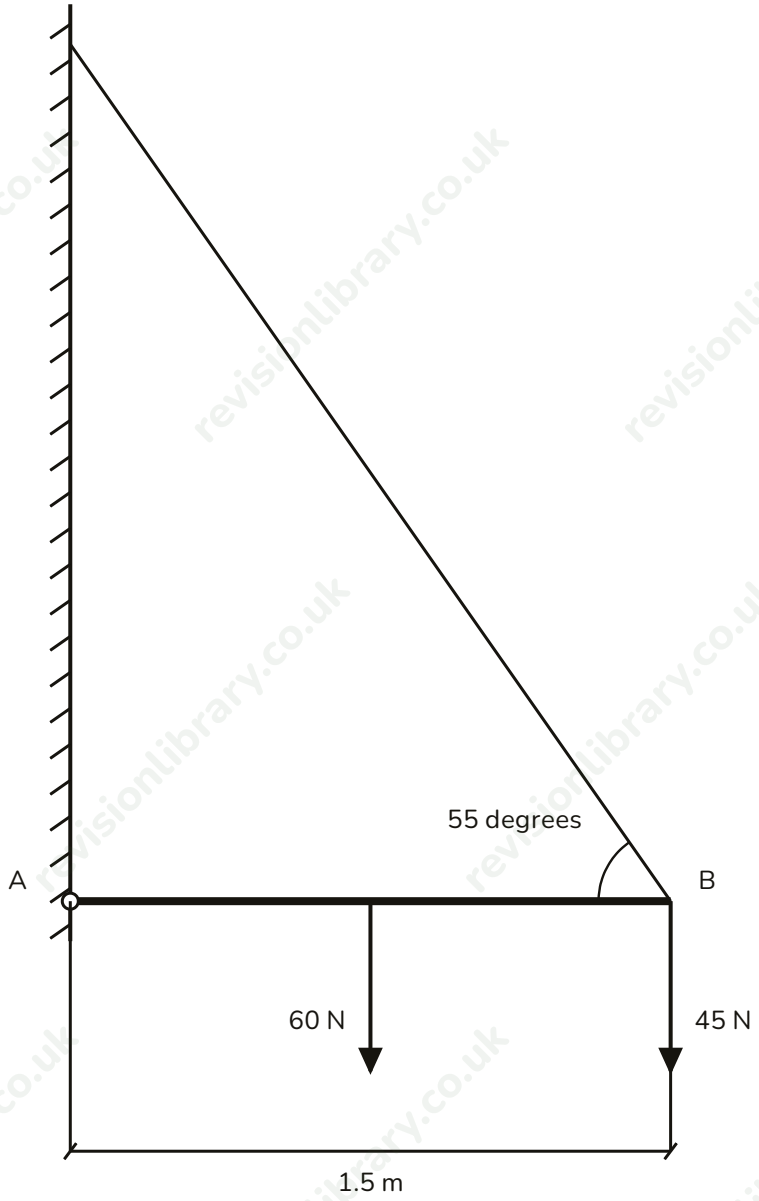
(b) Find the distance of Tafari from the pivot. **(4)**

(c) A third child, Nia, of weight 250 N, now also sits at Q itself, 2 m from the pivot, while Chandrika and Tafari stay in their original positions. Find the vertical force that must now be applied at P to restore equilibrium, stating whether it acts upward or downward. **(5)**

(Total for Question 26 is 10 marks)

27 A uniform rod AB has length 1.5 m and weight 60 N. The rod is smoothly hinged to a vertical wall at A and held in a horizontal position by a light inextensible string attached to the rod at B, making an angle of 55 degrees with the rod. A shop sign of weight 45 N hangs from a hook at B.

Diagram NOT accurately drawn



(a) By taking moments about A, find the tension in the string, giving your answer to 3 significant figures. **(4)**

(b) Find the magnitude of the vertical component of the force exerted on the rod by the hinge at A. **(3)**

(c) Given additionally that the horizontal component of the hinge force has magnitude 52.5 N, find the angle that the resultant hinge force makes with the rod. **(3)**

(Total for Question 27 is 10 marks)

28 A uniform beam AB has length 2 m and weight 50 N. The beam rests horizontally on a pivot at a point M, 0.6 m from A (M is not the midpoint of the beam). A vertical force of magnitude P is applied downward at A, and a counterweight of weight 80 N hangs from B. The beam remains horizontal and in equilibrium.

(a) Explain why the weight of the beam does contribute a moment about M in this situation, unlike in a case where the pivot is at the beam's midpoint. **(1)**

(b) By taking moments about M, find P. **(4)**

(c) Find the reaction at the pivot M. **(2)**

(d) The counterweight is repositioned so that P can be reduced to 150 N, with the beam's weight and the pivot position unchanged. Find the new distance of the counterweight from the pivot M. **(3)**

(Total for Question 28 is 10 marks)

29 A non-uniform curtain pole AB has length 2.4 m. It is suspended horizontally by two vertical wires, one attached at A and the other attached at a point 1.6 m from A. The centre of mass of the pole is 1.0 m from A. The tension in the wire at A is 36 N; let T newtons be the tension in the other wire.

(a) By resolving vertically and taking moments about A, form two equations connecting the weight of the pole, W, and T, and hence find the values of W and T. **(5)**

(b) Verify your answer to part (a) by taking moments about the point where the second wire is attached. **(2)**

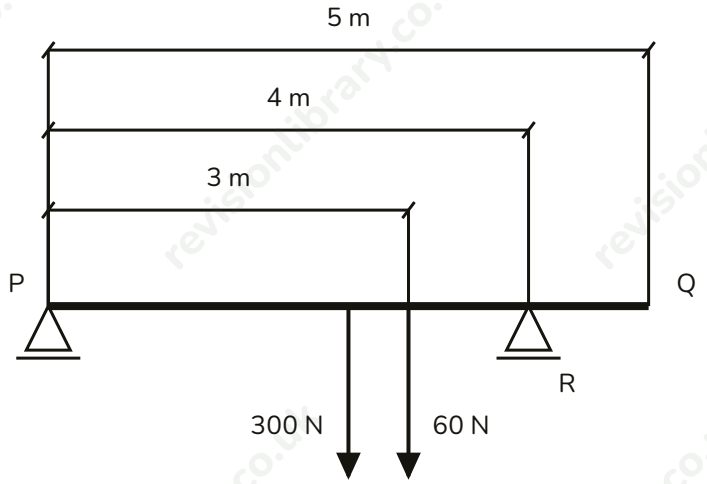
(c) State, giving a reason, whether the tension in the wire at A would be larger or smaller than 36 N if the pole were uniform instead of non-uniform (with the same length, weight and wire positions). **(2)**

(Total for Question 29 is 9 marks)

30 A uniform plank footbridge PQ over a garden pond has length 5 m and weight 300 N. It rests horizontally on two supports, one at P and one at R, where PR = 4 m (so R is 1 m from Q). A garden ornament of weight 60 N sits on the plank at a point 3 m from P. A student's attempt to find the reaction at R by taking moments about P is shown below:

Moments about P: R x 4 = 300 x 2.5 + 60 x 4
R = (750 + 240) / 4
R = 247.5 N

Diagram NOT accurately drawn



(a) Identify the error in the student's working, and state the correct distance that should have been used. **(2)**

(b) Hence find the correct value of R. **(3)**

(c) Find the reaction at P. **(2)**

(Total for Question 30 is 7 marks)

31 A mechanic applies a force of 70 N to the end of a wrench of length 0.3 m, at an angle of 55 degrees to the wrench, to loosen a bolt modelled as a fixed pivot at the other end of the wrench. Find the moment of the force about the pivot, giving your answer to 3 significant figures.

(Total for Question 31 is 3 marks)

32 A non-uniform beam AB has length 9 m and weight 240 N, acting at a point 4 m from A. The beam rests horizontally in equilibrium on two smooth supports, one at C, 1 m from A, and one at D, 7 m from A. By taking moments about C, find the reaction at D.

(Total for Question 32 is 3 marks)

33 A non-uniform rod AB has length 3 m and rests horizontally on a support at A. The rod is held horizontal by a light vertical string attached at B. The weight of the rod is 84 N and acts at a point 2 m from A. Show that the tension in the string at B is 56 N, and find the reaction at the support at A.

(Total for Question 33 is 3 marks)

34 A uniform ladder AB has length 5 m and weight 200 N. The ladder rests with end A on rough horizontal ground and end B against a smooth vertical wall, making an angle of 68 degrees with the horizontal ground. The ladder is on the point of slipping.

(a) State the four forces acting on the ladder, including the direction in which each acts. **(4)**

(b) By resolving vertically, find the normal reaction at A, N_A. **(2)**

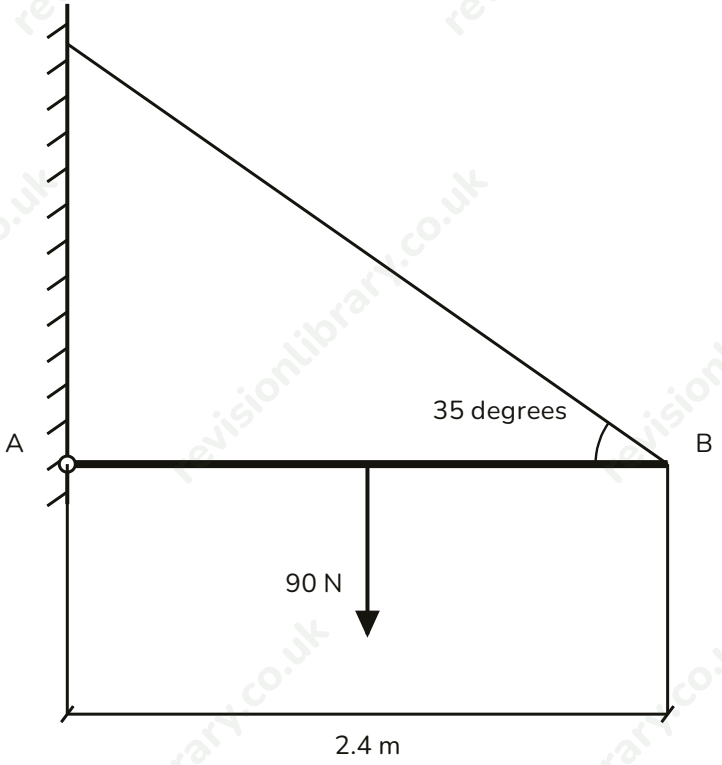
(c) By taking moments about A, find the normal reaction at the wall, N_B, giving your answer to 3 significant figures. **(4)**

(d) Given that the ladder is on the point of slipping, find the coefficient of friction between the ladder and the ground, giving your answer to 3 significant figures. **(4)**

(Total for Question 34 is 14 marks)

35 A uniform rod AB has length 2.4 m and weight 90 N. The rod is smoothly hinged to a vertical wall at A and held in a horizontal position by a light inextensible string attached to the rod at B and to a point on the wall above A, the string making an angle of 35 degrees with the rod.

Diagram NOT accurately drawn



(a) By taking moments about A, find the tension in the string, giving your answer to 3 significant figures. **(4)**

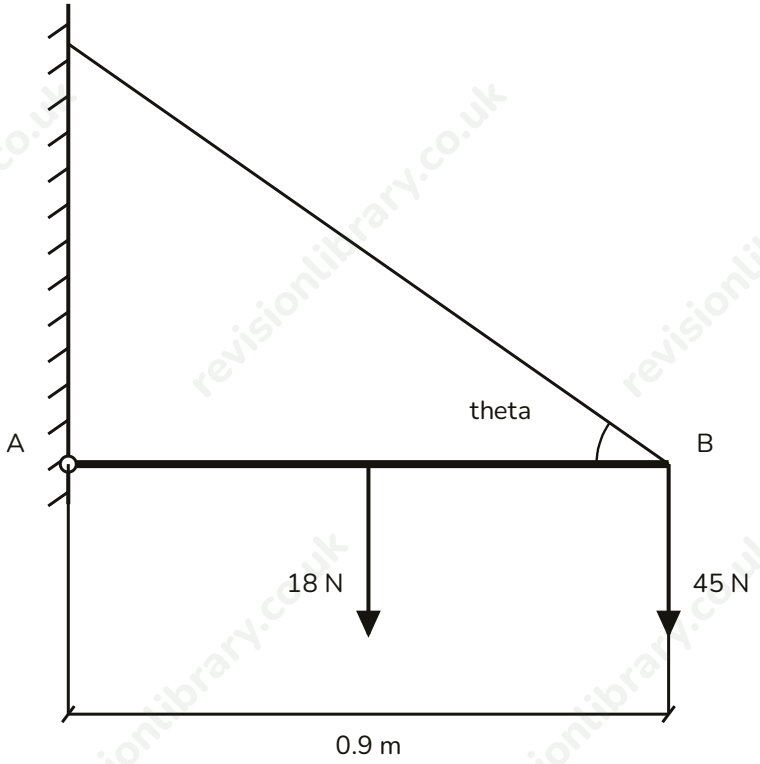
(b) Find the magnitude of the horizontal and vertical components of the force exerted on the rod by the hinge at A, stating the direction in which each acts. **(5)**

(c) Hence find the magnitude and direction of the resultant force exerted by the hinge on the rod. **(3)**

(Total for Question 35 is 12 marks)

36 A shop's hanging flower-basket bracket consists of a uniform rod AB of length 0.9 m and weight 18 N, smoothly hinged to the shop wall at A. The rod is held horizontal by a light string attached to the rod at B and to a point on the wall above A, the string making an angle θ with the rod. A basket of weight 45 N hangs from B. The string is rated to withstand a maximum tension of 150 N before it risks breaking.

Diagram NOT accurately drawn



(a) By taking moments about A, show that the tension in the string is given by $T = 54/\sin(\theta)$ (in newtons). **(4)**

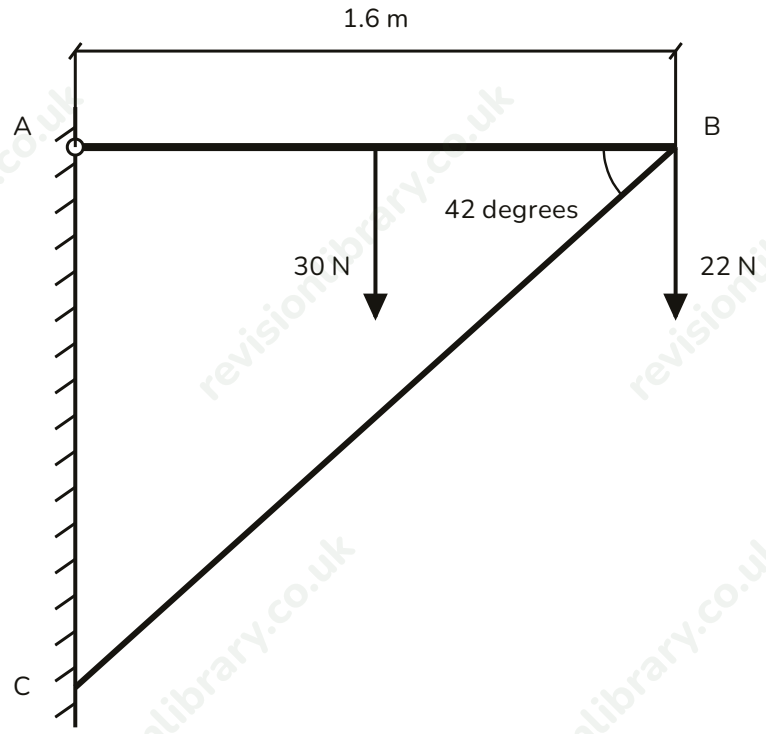
(b) Given that the string must not exceed its rated maximum tension of 150 N, find the minimum angle θ, to 1 decimal place, at which the string may be attached. **(3)**

(c) The basket is replaced with a heavier one of weight 70 N (the rod's weight is unchanged). Find the new minimum safe angle θ, to 1 decimal place. **(3)**

(Total for Question 36 is 10 marks)

37 A shop awning arm AB has length 1.6 m and weight 30 N. It is smoothly hinged to the shop wall at A and held horizontal by a rigid strut CB, where C is a fixed point on the wall vertically below A, the strut making an angle of 42 degrees with the arm. The strut exerts a force of magnitude S on the arm at B, directed along CB. A hanging sign of weight 22 N is attached to the arm at B.

Diagram NOT accurately drawn



(a) By taking moments about A, find the force S in the strut, giving your answer to 3 significant figures. **(4)**

(b) Find the magnitude of the horizontal and vertical components of the force exerted on the arm by the hinge at A. **(4)**

(c) Hence find the magnitude of the resultant force exerted by the hinge on the arm, and the angle this resultant makes with the arm, both to 3 significant figures. **(3)**

(Total for Question 37 is 11 marks)

38 A garden centre displays a uniform wooden potting bench, modelled as a rigid plank AB of length 2.4 m and weight 180 N, resting on two smooth support posts, one at C, 0.3 m from A, and one at D, 1.8 m from A. A decorative stone urn of weight 40 N sits at end A. A sack of compost of weight w newtons is placed at end B. The bench remains horizontal and rests only on the two posts; it will tip about D (lifting off the post at C) if w becomes too large.

(a) Explain briefly why, as w increases, the bench would tip about D rather than about C. **(1)**

(b) By taking moments about C, show that $R_D = 100 + 1.4w$ (in newtons), where R_D is the reaction at post D. **(4)**

(c) Find, in terms of w, an expression for R_C. **(2)**

(d) Find the maximum value of w for which the bench remains in equilibrium without tipping. **(3)**

(Total for Question 38 is 10 marks)

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39 A uniform rod AB has length 2 m and weight 70 N. The rod is smoothly hinged to a vertical wall at A and held in a horizontal position by a light inextensible string attached to the rod at B, making an angle of 50 degrees with the rod. Find the tension in the string, giving your answer to 3 significant figures, and find the magnitude of the vertical component of the force exerted on the rod by the hinge at A.

(Total for Question 39 is 4 marks)

40 A uniform beam AB has length 8 m and weight 200 N. The beam is held horizontally in equilibrium by two vertical cables attached to the beam at points C and D, where AC = 2 m and AD = 7 m. A load of weight 120 N is attached to the beam at point E, where AE = 4 m. By taking moments about C, find the tension in the cable at D, and hence find the tension in the cable at C.

(Total for Question 40 is 4 marks)

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41 A non-uniform rod AB has length 2.5 m and weight 90 N, acting at a point 1.5 m from A. The rod is smoothly hinged to a vertical wall at A and held in a horizontal position by a light inextensible string attached to the rod at B, making an angle of 40 degrees with the rod.

(a) Find the tension in the string, giving your answer to 3 significant figures. **(2)**

(b) Find the magnitude of the resultant force exerted on the rod by the hinge at A, giving your answer to 3 significant figures. **(2)**

(Total for Question 41 is 4 marks)

42 A uniform ladder AB has length 6 m and weight 250 N. The foot A rests on rough horizontal ground, where the coefficient of friction between the ladder and the ground is 0.3, and the top B rests against a smooth vertical wall. The ladder makes an angle of 60 degrees with the ground. Priya, of weight 700 N, climbs up the ladder. Let d metres be the distance she has climbed up the ladder, measured from A.

(a) Write down the normal reaction at A, N_A. **(1)**

(b) By taking moments about A, show that the normal reaction at the wall, N_B, is given by $N_B = (750 + 700d) / (6 \tan(60 \text{ degrees}))$ newtons. **(4)**

(c) Find the maximum distance d that Priya can climb up the ladder before it slips. **(5)**

(Total for Question 42 is 10 marks)

43 A uniform rod AC of length 2L and weight W is smoothly hinged to a vertical wall at A. The rod is held horizontal by a light inextensible string attached to the rod at C and to a point on the wall above A, the string making an angle θ with the rod.

(a) By taking moments about A, show that the tension in the string is $T = W / (2 \sin(\theta))$. **(4)**

(b) Show that the magnitude of the force exerted on the rod by the hinge at A is also equal to T. **(5)**

(c) Given that $\theta = 40$ degrees and $W = 150$ N, find the magnitude of the force exerted by the hinge on the rod, giving your answer to 3 significant figures. **(3)**

(Total for Question 43 is 12 marks)

44 A uniform ladder AB has length 4 m and weight 180 N. The foot A rests on rough horizontal ground, where the coefficient of friction is 0.28, and the ladder is on the point of slipping at A. The top B rests against a wall which is also rough, at an angle of 60 degrees to the ground.

(a) By resolving horizontally, and using the fact that the ladder is on the point of slipping at A (so the friction there is limiting, $F_A = 0.28\ N_A$), show that $N_B = 0.28\ N_A$, where N_B is the normal reaction at the wall. **(3)**

(b) By resolving vertically, write down an equation connecting N_A, F_B (the friction force at the wall) and the weight of the ladder. **(2)**

(c) By taking moments about A, and using your equations from parts (a) and (b), find the values of N_A, N_B and F_B, giving each to 3 significant figures. **(4)**

(d) Hence find the minimum coefficient of friction between the ladder and the wall required for the ladder to remain in equilibrium in this position, giving your answer to 3 significant figures. **(2)**

(Total for Question 44 is 11 marks)

45 A uniform ladder AB has length 5 m and weight 180 N. The foot A rests on rough horizontal ground, where the coefficient of friction between the ladder and the ground is 0.4, and the top B rests against a smooth vertical wall. The ladder makes an angle of 59 degrees with the horizontal ground. A decorator of weight w newtons climbs the ladder to a point 4 m from A.

(a) By resolving vertically, find N_A, the normal reaction at the ground, in terms of w. **(2)**

(b) By taking moments about A, show that the normal reaction at the wall is given by $N_B = 54.1 + 0.481w$, to 3 significant figures, where N_B is in newtons. **(4)**

(c) Given that the ladder is on the point of slipping when w takes its maximum possible value, find this maximum value of w, giving your answer to 3 significant figures. **(4)**

(d) Comment on whether it would be safe for a decorator of weight 800 N to climb to this same point on the ladder. **(2)**

(Total for Question 45 is 12 marks)

46 A uniform ladder of length L and weight W rests with one end A on rough horizontal ground, where the coefficient of friction is μ, and the other end B against a smooth vertical wall. The ladder makes an angle θ with the horizontal ground, and is on the point of slipping.

(a) Show that $\mu = 1/(2\tan(\theta))$. **(5)**

(b) Explain what happens to the minimum coefficient of friction required as θ approaches 90 degrees, and as θ approaches 0 degrees, and explain briefly why each result makes physical sense. **(3)**

(c) A particular ladder requires a minimum coefficient of friction of 0.35 to remain in equilibrium against a smooth wall. Find, to 1 decimal place, the smallest angle θ at which this ladder can safely be placed. **(3)**

(Total for Question 46 is 11 marks)

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47 A uniform ladder AB has length 4 m and weight 160 N. The ladder rests with end A on rough horizontal ground and end B against a smooth vertical wall, making an angle of 65 degrees with the horizontal ground. The ladder is in equilibrium and on the point of slipping. By taking moments about A, find the normal reaction at the wall, N_B, and hence find the coefficient of friction between the ladder and the ground. Give both answers to 3 significant figures.

(Total for Question 47 is 4 marks)

48 A uniform ladder AB has length 4 m and weight 150 N. The ladder rests with end A on rough horizontal ground, where the coefficient of friction between the ladder and the ground is 0.3, and end B against a smooth vertical wall, making an angle of 60 degrees with the horizontal ground. A person of weight 700 N climbs up the ladder.

(a) Given that the ladder is on the point of slipping, find the normal reaction exerted on the ladder by the wall. **(2)**

(b) By taking moments about A, find the distance up the ladder, measured from A, that the person can climb before the ladder is on the point of slipping, giving your answer to 3 significant figures. **(3)**

(Total for Question 48 is 5 marks)

Mark scheme · 3.3 Mechanics: Moments

Question 1

newton metre (N m) **B1**

Answer: newton metre (N m)

Question 2

20 cao **B1**

Answer: 20 N m

Question 3

45 cao **B1**

Answer: 45 N m

Question 4

the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point (oe) **B1**

Answer: The sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point.

Question 5

the centre of mass need not be at the midpoint of the rod (unlike a uniform rod, whose centre of mass is always at its midpoint) oe **B1**

Answer: For a non-uniform rod, the centre of mass need not be at the midpoint (unlike a uniform rod, where it always is).

Question 6

42 cao **B1**

Answer: 42 N m

Question 7

the magnitude of the force and the perpendicular distance from the point to the line of action of the force **B1**

Answer: The magnitude of the force and the perpendicular distance from the pivot to the force's line of action.

Question 8

d = moment / force = 54/90 **M1**

0.6 m cao **A1**

Answer: 0.6 m

Question 9

F = moment / d = 45/0.75 **M1**

60 N cao **A1**

Answer: 60 N

Question 10

by symmetry, or by taking moments, each reaction supports half the weight **M1**

90 N at each support **A1**

Answer: 90 N at each support

Question 11

resolve vertically: weight = sum of the two tensions **M1**

80 N cao **A1**

Answer: 80 N

Question 12

resolve vertically: $R_A + R_B$ = weight, so R_B = 130 - 55 **M1**

75 N cao **A1**

Answer: 75 N

Question 13

F = moment / d = 3.6 / 0.08 **M1**

45 N cao **A1**

Answer: 45 N

Question 14

(a) moment = 150 x 0.8 oe **M1**

(a) 120 N m cao (units required) **A1**

(a) Answer: 120 N m

(b) correct conclusion: yes, the hinge is at risk **B1**

(b) valid reason: 120 N m (from part a) exceeds the rated maximum of 100 N m, ft their (a) **B1**

(b) Answer: Yes, at risk, since 120 N m > 100 N m

Question 15

(a) moments equation about C attempted, with all three terms present **M1**

(a) correct unsimplified equation: R_D x 3.0 = 240 x 1.7 oe **A1**

(a) rearranges to make R_D the subject **dM1**

(a) R_D = 136 N cso **A1**

(a) Answer: R_D = 136 N

(b) R_C = 240 - 136 (vertical equilibrium), ft their (a) **M1**

(b) R_C = 104 N cao **A1**

(b) Answer: R_C = 104 N

(c) states one valid assumption: the beam is uniform, the beam is rigid, or the supports are smooth (accept other valid modelling assumptions) **B1**

(c) explains how that assumption is used: uniformity places the weight at the midpoint; rigidity preserves the moment arms; smooth supports give no frictional forces (or a correct explanation of their valid assumption) **B1**

(c) states a second distinct valid assumption **B1**

(c) correctly explains how the second assumption is used in the calculation **B1**

(c) Answer: e.g. uniform (weight acts at midpoint); rigid (moment arms remain fixed because the beam does not bend)

Question 16

(a) moment = 250 x 0.4 oe **M1**

(a) 100 N m cao (units required) **A1**

(a) Answer: 100 N m

(b) takes moments about the wheel: F x 1 = 250 x 0.4 oe **M1**

(b) F = 100 N cao **A1**

(b) Answer: 100 N

Question 17

(a) d = 0.17 x sin(60 degrees) oe **M1**

(a) awrt 0.147 (m) **A1**

(a) Answer: 0.147 m (3 s.f.)

(b) moment = 400 x their (a), or 400 x 0.17 x sin(60 degrees) **M1**

(b) awrt 58.9 N m (units required), ft their (a) **A1**

(b) Answer: 58.9 N m (3 s.f.)

(c) new moment = 400 x 0.17 (= 68 N m) **M1**

(c) awrt 15.5% increase, ft their (b) **A1**

(c) Answer: 15.5% increase (3 s.f.)

Question 18

take moments about A: R x 2.5 = 100 x 2 (weight acts at the midpoint, 2 m from A) **M1**

80 N cao **A1**

Answer: 80 N

Question 19

moment = 80 x 0.5 x sin60 **M1**

awrt 34.6 N m **A1**

Answer: 34.6 N m (3 s.f.)

Question 20

moment = 45 x 0.8 x sin40 **M1**

awrt 23.1 N m **A1**

Answer: 23.1 N m (3 s.f.)

Question 21

take moments about the pivot: 60 x 0.9 = 45 x d **M1**

1.2 m cao **A1**

Answer: 1.2 m

Question 22

total clockwise moment = (18 x 0.4) + (30 x 0.3) = 16.2; total anticlockwise moment = 25 x 0.5 = 12.5 **M1**

3.7 N m clockwise cao **A1**

Answer: 3.7 N m, clockwise

Question 23

moment = 55 x 0.9 x sin70 **M1**

awrt 46.5 N m **A1**

Answer: 46.5 N m (3 s.f.)

Question 24

moment = 40 x 0.14 x sin25 **M1**

awrt 2.37 N m **A1**

Answer: 2.37 N m (3 s.f.)

Question 25

(a) W = 45 + 75 (vertical equilibrium) **M1**

(a) W = 120 N cao **A1**

(a) Answer: 120 N

(b) moments equation about A attempted, with correct terms **M1**

(b) correct unsimplified equation: 75 x 2.4 = 120 x d oe **A1**

(b) rearranges to make d the subject **dM1**

(b) d = 1.5 m cao **A1**

(b) Answer: 1.5 m from A

(c) correct statement: closer to the point where the 75 N string is attached **B1**

(c) valid reason: the larger tension supports more of the weight, so the centre of mass lies nearer to that string, oe **B1**

(c) Answer: Closer to the 75 N string

Question 26

(a) the see-saw is uniform and the pivot is at its midpoint (centre of mass), so the weight acts through the pivot and has zero perpendicular distance, giving zero moment **B1**

(a) Answer: Weight acts through the pivot, so it has zero moment about it

(b) moments equation about the pivot: 320 x 2 = 400 x d oe **M1**

(b) 640 = 400d oe **A1**

(b) rearranges to make d the subject **dM1**

(b) d = 1.6 m cao **A1**

(b) Answer: 1.6 m from the pivot

(c) total moment on Tafari and Nia's side: 400 x 1.6 + 250 x 2 **M1**

(c) 1140 (N m) cao **A1**

(c) equation for extra force F at P: 320 x 2 + F x 2 = 1140 oe **M1**

(c) F = 250 N cao **A1**

(c) direction: downward (ft their F), since the Tafari/Nia side now has the greater moment and P's side must be increased to compensate **B1**

(c) Answer: 250 N, applied downward at P

Question 27

(a) moments equation about A with all three terms: T sin(55 degrees) x 1.5 = 60 x 0.75 + 45 x 1.5 oe **M1**

(a) correct unsimplified equation **A1**

(a) rearranges to make T the subject **dM1**

(a) awrt 91.6 N **A1**

(a) Answer: 91.6 N (3 s.f.)

(b) vertical equilibrium: Y + T sin(55 degrees) = 60 + 45 oe **M1**

(b) substitutes T sin(55 degrees) = 75 (exact, from part a) **A1**

(b) Y = 30 N cao **A1**

(b) Answer: 30 N

(c) tan(angle) = Y / X, using their Y and the given 52.5 N **M1**

(c) tan(angle) = 30 / 52.5 oe **A1**

(c) awrt 29.7 degrees **A1**

(c) Answer: 29.7 degrees (3 s.f.)

Question 28

(a) because M is not the midpoint (centre of mass) of the beam, so the weight's line of action does not pass through M, giving a nonzero perpendicular distance and hence a nonzero moment about M **B1**

(a) Answer: The weight's line of action does not pass through M, since M is not the centre of mass, so it has a nonzero moment about M

(b) identifies the distance of each force from M (P at 0.6 m from M; the beam's weight at 0.4 m from M; the counterweight at 1.4 m from M) **M1**

(b) correct unsimplified equation: P x 0.6 = 50 x 0.4 + 80 x 1.4 oe **A1**

(b) rearranges to make P the subject **dM1**

(b) P = 220 N cao **A1**

(b) Answer: P = 220 N

(c) R = P + 50 + 80 (vertical equilibrium), ft their P **M1**

(c) R = 350 N cao **A1**

(c) Answer: 350 N

(d) new moments equation: 150 x 0.6 = 50 x 0.4 + 80 x d **M1**

(d) 90 = 20 + 80d oe **A1**

(d) d = 0.875 m cao **A1**

(d) Answer: 0.875 m from M

Question 29

(a) vertical equilibrium equation formed: 36 + T = W oe **M1**

(a) moments equation about A formed: T x 1.6 = W x 1.0 oe **M1**

(a) eliminates one variable (e.g. substitutes W = 36 + T) and solves **dM1**

(a) T = 60 N cao **A1**

(a) W = 96 N cao **A1**

(a) Answer: W = 96 N, T = 60 N

(b) alternative moments equation formed: 36 x 1.6 = W x (1.6 - 1.0), using their W **M1**

(b) both sides equal 57.6 (N m), confirming the values found in part (a) **A1**

(b) Answer: Consistent: both sides equal 57.6 N m

(c) correct statement: smaller **B1**

(c) valid reason: a uniform pole's centre of mass would be at its midpoint (1.2 m from A), further from A than the given 1.0 m, so more weight is supported by the wire nearer the centre of mass, leaving less tension needed at A **B1**

(c) Answer: Smaller than 36 N

Question 30

(a) identifies that the student used 4 m (the distance PR) as the lever arm for the ornament's weight, instead of the ornament's own distance from P **B1**

(a) states the correct distance is 3 m (the ornament's actual position from P), so the correct moment term is 60 x 3, not 60 x 4 **B1**

(a) Answer: The ornament's distance from P should be 3 m, not 4 m

(b) correct moments equation: R x 4 = 300 x 2.5 + 60 x 3 **M1**

(b) R x 4 = 930 oe **A1**

(b) R = 232.5 N cao **A1**

(b) Answer: R = 232.5 N

(c) R_P = 300 + 60 - 232.5 (vertical equilibrium), ft their (b) **M1**

(c) R_P = 127.5 N cao **A1**

(c) Answer: 127.5 N

Question 31

perpendicular distance = 0.3 sin55 **M1**

moment = 70 x (0.3 sin55) **M1**

awrt 17.2 N m **A1**

Answer: 17.2 N m (3 s.f.)

Question 32

identify distances from C: the weight acts 3 m from C; D is 6 m from C **M1**

take moments about C: R_D x 6 = 240 x 3 **M1**

120 N cao **A1**

Answer: 120 N

Question 33

take moments about A: T x 3 = 84 x 2 **M1**

cso: T = 168/3 = 56 N, as required **A1**

resolve vertically: R_A = 84 - 56 = 28 N cao **B1**

Answer: T = 56 N (shown); reaction at A = 28 N.

Question 34

(a) weight, 200 N, acting vertically downward at the midpoint of the ladder **B1**

(a) normal reaction at A, acting vertically upward **B1**

(a) friction at A, acting horizontally, directed toward the wall (to prevent the foot sliding away) **B1**

(a) normal reaction at B, acting horizontally, directed away from the wall **B1**

(a) Answer: Weight (down, midpoint); N_A (up, at A); friction (horizontal, toward wall, at A); N_B (horizontal, away from wall, at B)

(b) vertical equilibrium: $N_A = 200$ (no other vertical forces act) **M1**

(b) $N_A = 200$ N cao **A1**

(b) Answer: 200 N

(c) moments equation about A: N_B x 5 sin(68 degrees) = 200 x 2.5 cos(68 degrees) oe **M1**

(c) correct unsimplified equation **A1**

(c) rearranges to make N_B the subject, e.g. $N_B = 100 / \tan(68$ degrees) **dM1**

(c) awrt 40.4 N **A1**

(c) Answer: 40.4 N (3 s.f.)

(d) horizontal equilibrium: $F = N_B$, ft their N_B **M1**

(d) F = awrt 40.4 N (ft) **A1**

(d) limiting friction: $\mu = F / N_A$, ft their F and N_A **M1**

(d) awrt 0.202 **A1**

(d) Answer: $\mu = 0.202$ (3 s.f.)

Question 35

(a) moments equation about A: T sin(35 degrees) x 2.4 = 90 x 1.2 oe **M1**

(a) correct unsimplified equation **A1**

(a) rearranges to make T the subject **dM1**

(a) awrt 78.5 N **A1**

(a) Answer: 78.5 N (3 s.f.)

(b) horizontal equilibrium: X = T cos(35 degrees), ft their T **M1**

(b) awrt 64.3 N, directed away from the wall **A1**

(b) vertical equilibrium: Y = 90 - T sin(35 degrees) oe, ft their T **M1**

(b) Y = 45 N cao, directed upward **A1**

(b) both directions correctly stated (away from wall / upward) **B1**

(b) Answer: Horizontal: 64.3 N (away from wall); Vertical: 45 N (upward)

(c) magnitude = $\sqrt{X^2 + Y^2}$, using their X and Y **M1**

(c) awrt 78.5 N (ft) **A1**

(c) direction: 35.0 degrees above the horizontal (ft), e.g. from tan(angle) = Y/X **A1**

(c) Answer: 78.5 N, at 35.0 degrees above the horizontal

Question 36

(a) moments equation about A: T sin(θ) x 0.9 = 18 x 0.45 + 45 x 0.9 oe **M1**

(a) correct unsimplified equation **A1**

(a) divides by 0.9 and rearranges to make T the subject **dM1**

(a) T = 54/sin(θ) cso **A1**

(a) Answer: T = 54/sin(θ)

(b) sets $T \leq 150$ and forms the inequality $54/\sin(\theta) \leq 150$ **M1**

(b) $\sin(\theta) \geq 0.36$ oe **A1**

(b) $\theta \geq 21.1$ degrees (1 d.p.) cao **A1**

(b) Answer: 21.1 degrees (1 d.p.)

(c) new constant found: T sin(θ) x 0.9 = 18 x 0.45 + 70 x 0.9, i.e. T sin(θ) = 79 **M1**

(c) $\sin(\theta) \geq 79/150 = 0.5267$ oe **A1**

(c) $\theta \geq 31.8$ degrees (1 d.p.) cao **A1**

(c) Answer: 31.8 degrees (1 d.p.)

Question 37

(a) moments equation about A with all three terms: S sin(42 degrees) x 1.6 = 30 x 0.8 + 22 x 1.6 oe **M1**

(a) correct unsimplified equation **A1**

(a) rearranges to make S the subject **dM1**

(a) awrt 55.3 N **A1**

(a) Answer: 55.3 N (3 s.f.)

(b) vertical equilibrium: Y + S sin(42 degrees) = 30 + 22 oe **M1**

(b) substitutes S sin(42 degrees) = 37 (exact, from part a) to get Y = 15 N cao **A1**

(b) horizontal equilibrium: X = S cos(42 degrees), ft their S **M1**

(b) awrt 41.1 N **A1**

(b) Answer: Vertical: 15 N; Horizontal: 41.1 N (3 s.f.)

(c) magnitude = $\sqrt{X^2 + Y^2}$, using their X and Y **M1**

(c) awrt 43.7 N (ft) **A1**

(c) angle: awrt 20.1 degrees (ft), e.g. from tan(angle) = Y/X **A1**

(c) Answer: 43.7 N, at 20.1 degrees to the arm

Question 38

(a) because the sack (at B) and part of the plank's own weight act beyond D, so increasing w increases the tendency to rotate the bench about D, lifting end A (and the reaction at C) off post C **B1**

(a) Answer: The bench would tip about D, lifting off at C

(b) moments equation about C attempted, with all four terms present **M1**

(b) correct unsimplified equation: R_D x 1.5 = 180 x 0.9 + w x 2.1 - 40 x 0.3 oe **A1**

(b) rearranges to make R_D the subject **dM1**

(b) $R_D = 100 + 1.4w$ cso **A1**

(b) Answer: $R_D = 100 + 1.4w$

(c) $R_C + R_D = 180 + 40 + w$ (vertical equilibrium), ft their R_D **M1**

(c) $R_C = 120 - 0.4w$ cao **A1**

(c) Answer: $R_C = 120 - 0.4w$

(d) sets $R_C \geq 0$ and solves: $120 - 0.4w \geq 0$ **M1**

(d) $w \leq 300$ **A1**

(d) maximum value of w is 300 N, ft their inequality **B1**

(d) Answer: 300 N

Question 39

take moments about A: T sin50 x 2 = 70 x 1 (weight at the midpoint, 1 m from A) **M1**

awrt 45.7 N **A1**

resolve vertically: hinge vertical component + T sin50 = 70 **M1**

35 N cao **A1**

Answer: T = 45.7 N (3 s.f.); vertical component of hinge force = 35 N

Question 40

take moments about C: T_D x 5 = 200 x 2 + 120 x 2 (beam's weight at 4 m from A is 2 m from C; load at E, 4 m from A, is also 2 m from C; D is 5 m from C) **M1**

128 N cao **A1**

resolve vertically: $T_C + T_D = 200 + 120$ **M1**

192 N cao **A1**

Answer: T_D = 128 N, T_C = 192 N

Question 41

(a) take moments about A: T sin40 x 2.5 = 90 x 1.5 **M1**

(a) awrt 84.0 N **A1**

(a) Answer: 84.0 N (3 s.f.)

(b) resolve to find the components of the hinge force: horizontal = T cos40 (awrt 64.4 N ft), vertical = 90 - T sin40 = 36 N; combine using resultant = $\sqrt{\text{horizontal}^2 + \text{vertical}^2}$ **M1**

(b) awrt 73.7 N **A1**

(b) Answer: 73.7 N (3 s.f.)

Question 42

(a) N_A = 950 N (vertical equilibrium: 250 + 700) **B1**

(a) Answer: 950 N

(b) moments equation about A with all three terms present: N_B x 6 sin(60 degrees) = 250 x 3 cos(60 degrees) + 700 x d cos(60 degrees) oe **M1**

(b) correct unsimplified equation **A1**

(b) rearranges to make N_B the subject **dM1**

(b) N_B = (750 + 700d) / (6 tan(60 degrees)) cso **A1**

(b) Answer: N_B = (750 + 700d) / (6 tan(60 degrees))

(c) limiting friction: F = 0.3 x 950, using their N_A **M1**

(c) F = 285 N cao **A1**

(c) horizontal equilibrium F = N_B, set equal to their expression from (b) **M1**

(c) rearranges to make d the subject **dM1**

(c) awrt 3.16 m **A1**

(c) Answer: d = 3.16 m (3 s.f.)

Question 43

(a) moments equation about A: T sin(θ) x 2L = W x L oe **M1**

(a) correct unsimplified equation **A1**

(a) divides both sides by 2L and rearranges **dM1**

(a) T = W / (2 sin(θ)) cso **A1**

(a) Answer: T = W / (2 sin(θ))

(b) horizontal equilibrium: X = T cos(θ) **M1**

(b) correct expression for X **A1**

(b) vertical equilibrium: Y = W - T sin(θ) **M1**

(b) uses T sin(θ) = W/2 from part (a) to get Y = W/2 = T sin(θ) **A1**

(b) magnitude = $\sqrt{X^2 + Y^2}$ = $\sqrt{(T \cos(\&\theta;))^2 + (T \sin(\&\theta;))^2}$ = T cso **A1**

(b) Answer: Hinge force magnitude = T

(c) substitutes into T = W / (2 sin(θ)): T = 150 / (2 sin(40 degrees)) **M1**

(c) awrt 117 N **A1**

(c) correctly states the hinge force equals this value of T, using the result of part (b) **B1**

(c) Answer: 117 N (3 s.f.)

Question 44

(a) horizontal equilibrium: $N_B = F_A$ **M1**

(a) F_A = 0.28 N_A stated (limiting friction) **A1**

(a) N_B = 0.28 N_A cso **A1**

(a) Answer: N_B = 0.28 N_A

(b) vertical equilibrium attempted **M1**

(b) $N_A + F_B$ = 180 oe **A1**

(b) Answer: $N_A + F_B$ = 180

(c) moments equation about A formed: N_B x 4 sin(60 degrees) + F_B x 4 cos(60 degrees) = 180 x 2 cos(60 degrees) oe **M1**

(c) correct unsimplified equation, e.g. N_B tan(60 degrees) + F_B = 90 (after dividing by 4 cos(60 degrees)) **A1**

(c) substitutes N_B = 0.28 N_A and F_B = 180 - N_A, and solves for N_A **dM1**

(c) N_A = awrt 175 N, N_B = awrt 48.9 N, F_B = awrt 5.25 N (ft) **A1**

(c) Answer: N_A = 175 N, N_B = 48.9 N, F_B = 5.25 N (all 3 s.f.)

(d) μ = F_B / N_B (using their F_B and N_B) **M1**

(d) awrt 0.107 **A1**

(d) Answer: 0.107 (3 s.f.)

Question 45

(a) vertical equilibrium: $N_A = 180 + w$ (the wall is smooth, so its reaction is horizontal) **M1**

(a) $N_A = 180 + w$ oe **A1**

(a) Answer: $N_A = 180 + w$

(b) moments equation about A with all three terms: N_B x 5 sin(59 degrees) = 180 x 2.5 cos(59 degrees) + w x 4 cos(59 degrees) oe **M1**

(b) correct unsimplified equation **A1**

(b) divides by 5 sin(59 degrees) and rearranges **dM1**

(b) $N_B = 54.1 + 0.481w$ cso (awrt) **A1**

(b) Answer: $N_B = 54.1 + 0.481w$

(c) limiting friction: $F_A = 0.4 \times N_A = 0.4(180 + w)$ **M1**

(c) horizontal equilibrium $F_A = N_B$, set equal to the expression from part (b), using unrounded coefficients for accuracy **M1**

(c) rearranges to make w the subject **dM1**

(c) awrt 222 (N) **A1**

(c) Answer: w = 222 N (3 s.f.)

(d) no, it would not be safe / the ladder would slip **B1**

(d) valid reason: 800 N exceeds the maximum weight of 222 N found in part (c), ft their (c) **B1**

(d) Answer: No, unsafe, since 800 N > 222 N

Question 46

(a) vertical equilibrium: $N_A = W$ **M1**

(a) moments equation about A: N_B x L sin(θ) = W x (L/2) cos(θ) oe **M1**

(a) rearranges to $N_B = (W/2)/\tan(\theta)$ (or equivalent unsimplified form) **A1**

(a) horizontal equilibrium with limiting friction: $F_A = N_B$, and $F_A = \mu W$ (using $N_A = W$) **M1**

(a) $\mu = 1/(2 \tan(\theta))$ cso **A1**

(a) Answer: $\mu = 1/(2 \tan(\theta))$

(b) as θ tends to 90 degrees, tan(θ) tends to infinity, so μ tends to 0 (almost no friction is needed for a nearly vertical ladder) **B1**

(b) as θ tends to 0 degrees from above, tan(θ) tends to 0, so the minimum μ grows without bound; at every fixed positive angle below 90 degrees the required μ is finite **B1**

(b) physical justification: a nearly vertical ladder has its weight acting almost directly above the foot, so little friction is needed; as the ladder becomes flatter, the wall reaction has a shrinking moment arm about the foot, requiring a larger wall reaction and hence more friction **B1**

(b) Answer: μ tends to 0 as θ tends to 90 degrees (the weight acts almost directly above the foot); μ grows without bound as θ tends to 0 degrees from above (the wall reaction has a shrinking moment arm, so more friction is required). At every fixed positive angle below 90 degrees, the minimum μ is finite.

(c) rearranges $\mu = 1/(2 \tan(\theta))$ to make θ the subject: $\tan(\theta) = 1/(2 \times 0.35)$ **M1**

(c) $\tan(\theta) = 1.42857$ oe **A1**

(c) θ = 55.0 degrees (1 d.p.) cao **A1**

(c) Answer: 55.0 degrees (1 d.p.)

Question 47

take moments about A: N_B x 4 sin65 = 160 x 2 cos65 (weight at the midpoint) **M1**

awrt 37.3 N **A1**

resolve vertically to find $N_A = 160$ N, resolve horizontally to find $F = N_B$ (ft), then $\mu = F/N_A$ **M1**

awrt 0.233 **A1**

Answer: $N_B = 37.3$ N (3 s.f.); μ = 0.233 (3 s.f.)

Question 48

(a) resolve vertically to find the normal reaction from the ground: N = 150 + 700 = 850; on the point of slipping, friction F = 0.3N; resolve horizontally: wall reaction = F **M1**

(a) 255 N cao **A1**

(a) Answer: 255 N

(b) take moments about A: (wall reaction) x 4 sin60 = 150 x 2 cos60 + 700 x x cos60 (the ladder's weight acts at its midpoint, 2 m from A; the person is a distance x from A) **M1**

(b) substitute the wall reaction found in part (a) and rearrange to make x the subject **dM1**

(b) awrt 2.10 m **A1**

(b) Answer: 2.10 m (3 s.f.)