3.4 Mechanics: Projectiles and Further Kinematics

EDEXCEL 9MA0 · Calculator allowed · about 220 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A particle is projected horizontally from the top of a diving board. State the horizontal component of the particle's acceleration while it is in the air.

(Total for Question 1 is 1 mark)

2 A particle is projected at an angle above horizontal ground and moves freely under gravity. State the vertical component of the particle's acceleration during the flight, using $g = 9.8\ m/s^2$.

(Total for Question 2 is 1 mark)

3 A particle is projected from horizontal ground and moves freely under gravity, following a projectile path. State the direction of the particle's velocity at the instant it reaches its greatest height.

(Total for Question 3 is 1 mark)

4 A particle is projected with speed 30 m/s at an angle of 40 degrees above the horizontal. Find the initial vertical component of its velocity, giving your answer to 3 significant figures.

(Total for Question 4 is 1 mark)

---

5 A particle is projected with speed 30 m/s at an angle of 40 degrees above the horizontal. Find the initial horizontal component of its velocity, giving your answer to 3 significant figures.

(Total for Question 5 is 1 mark)

---

6 A ball is thrown horizontally from a height and takes 2 seconds to hit the ground. Using $g = 9.8$ m/s^2, find the vertical component of its velocity at the instant it hits the ground.

(Total for Question 6 is 1 mark)

---

7 A particle is projected horizontally with speed 15 m/s from the top of a cliff. It takes 3 seconds to reach the sea. Find the horizontal distance travelled by the particle.

(Total for Question 7 is 2 marks)

8 A hang glider is launched from a hillside. The horizontal component of the hang glider's initial velocity is 12.4 m/s, at an angle of 37 degrees above the horizontal.

(a) Find the initial speed of the hang glider, giving your answer to 3 significant figures. **(1)**

(b) Find the initial vertical component of the hang glider's velocity, giving your answer to 3 significant figures. **(1)**

(Total for Question 8 is 2 marks)

---

9 A conker falls from rest from a window ledge 4.9 m above the ground. Using $g = 9.8$ m/s^2 and modelling the conker as a particle, find the speed at which it hits the ground.

(Total for Question 9 is 2 marks)

---

10 A paraglider launches horizontally from a hillside and lands 37.4 m (measured horizontally) from the launch point after 3.4 seconds of flight. Find the horizontal component of the paraglider's velocity.

(Total for Question 10 is 2 marks)

11 A particle moves with velocity $v = 4ti - 3j$ m/s. Find the acceleration vector of the particle.

(Total for Question 11 is 1 mark)

---

12 A particle moves in a straight line with velocity $v = 6t^2 - 2$ m/s at time t seconds. Find the acceleration of the particle when $t = 3$ seconds.

(Total for Question 12 is 2 marks)

---

13 A particle moves in a straight line with acceleration $a = 4t - 1$ m/s^2 at time t seconds. Given that the particle is at rest when $t = 0$, find an expression for the velocity v m/s at time t.

(Total for Question 13 is 2 marks)

---

14 A particle is projected at an angle above horizontal ground and reaches its greatest height 2.5 seconds after projection. Using $g = 9.8$ m/s^2 and the fact that the vertical velocity is zero at greatest height, find the initial vertical component of velocity.

(Total for Question 14 is 2 marks)

15 A particle P has velocity (2i - j) m/s at time t = 0 and moves with constant acceleration (1i + 2j) m/s^2. Find the velocity of P at time t = 4 seconds.

(Total for Question 15 is 2 marks)

---

16 A particle P has position vector (1i - 4j) m at time t = 0, velocity (3i + 0j) m/s at t = 0, and moves with constant acceleration (0i + 4j) m/s^2. Find the position vector of P at time t = 2 seconds.

(Total for Question 16 is 2 marks)

---

17 A particle has position vector $r = (t^2 - 3t)i + (2t + 5)j$ metres at time t seconds. Find an expression for the velocity of the particle at time t.

(Total for Question 17 is 2 marks)

---

18 A remote-controlled car P has velocity (4i - 9j) m/s at time t = 0 and moves with constant acceleration (-1i + 3j) m/s^2. Find the value of t at which the velocity of P is parallel to the vector i.

(Total for Question 18 is 2 marks)

19 A particle has position vector $\mathbf{r} = (2t^3 - 5t)\mathbf{i} + (4t^2 + 3)\mathbf{j}$ metres at time t seconds. Find the velocity of the particle when t = 2 seconds.

(Total for Question 19 is 2 marks)

---

20 A particle P has velocity $\mathbf{v} = (4t - 1)\mathbf{i} + 6\mathbf{j}$ m/s at time t seconds. When t = 0, P has position vector $(2\mathbf{i} + 5\mathbf{j})$ m. Find the position vector of P at time t seconds.

(Total for Question 20 is 2 marks)

---

21 A particle is projected horizontally with speed 18 m/s from the top of a vertical cliff 78.4 m above the sea, modelled as a horizontal plane. Using $g = 9.8$ m/s^2, find the horizontal distance from the base of the cliff to the point where the particle lands.

(Total for Question 21 is 3 marks)

22 A particle is projected from a point O on horizontal ground with speed 21 m/s at an angle of 40 degrees above the horizontal. Using $g = 9.8$ m/s^2, find the greatest height reached by the particle above O, giving your answer to 3 significant figures.

(Total for Question 22 is 3 marks)

23 A particle is projected from a point O on horizontal ground with speed 24 m/s at an angle of 35 degrees above the horizontal. The particle moves freely under gravity until it returns to the ground. Using $g = 9.8$ m/s^2, find the time of flight, and hence find the range of the particle. Give each answer to 3 significant figures.

(Total for Question 23 is 4 marks)

24 A particle P moves in a plane with acceleration $\mathbf{a} = (6t - 2)\mathbf{i} + 4\mathbf{j}$ m/s^2 at time t seconds ($t \geq 0$). When $t = 0$, P has velocity $(1\mathbf{i} - 3\mathbf{j})$ m/s and position vector $(0\mathbf{i} + 2\mathbf{j})$ m. Find the position vector of P at time $t = 2$ seconds.

(Total for Question 24 is 4 marks)

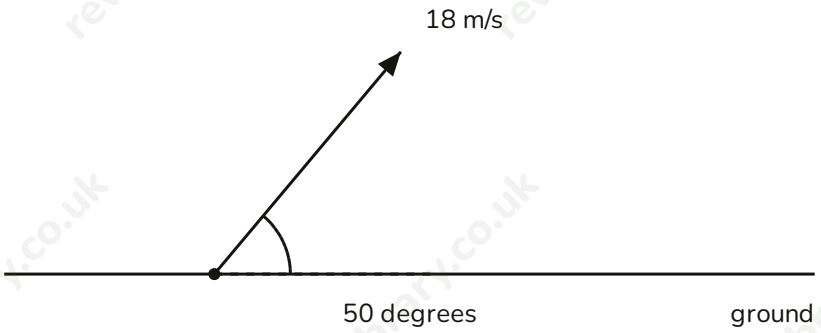
---

25 A particle is projected from horizontal ground with speed 24 m/s at an angle of 35 degrees above the horizontal. Using $g = 9.8$ m/s^2, find the speed of the particle at time $t = 1.5$ seconds after projection, giving your answer to 3 significant figures.

(Total for Question 25 is 3 marks)

26 A particle is projected from horizontal ground with speed 18 m/s at an angle of 50 degrees above the horizontal. Using $g = 9.8$ m/s^2, find the angle the particle's velocity makes with the horizontal at time $t = 1$ second after projection, giving your answer to 3 significant figures.

Diagram NOT accurately drawn



(Total for Question 26 is 3 marks)

27 A particle is projected from horizontal ground with speed 23 m/s at an angle of 44 degrees above the horizontal. Using $g = 9.8$ m/s^2, show that the greatest height reached by the particle is 13.0 m, correct to 3 significant figures.

(Total for Question 27 is 3 marks)

28 A firework is launched from a platform 12 m above horizontal ground with initial speed 15 m/s at an angle of 25 degrees above the horizontal. Using $g = 9.8$ m/s^2, find the time taken for the firework to reach the ground, giving your answer to 3 significant figures.

(Total for Question 28 is 3 marks)

29 At time $t = 0$, a particle A has position vector $(3\mathbf{i} + 4\mathbf{j})$ m and moves with constant velocity $(2\mathbf{i} - \mathbf{j})$ m/s. At the same time, a particle B has position vector $(-5\mathbf{i} - 2\mathbf{j})$ m and moves with constant velocity $(4\mathbf{i} + 3\mathbf{j})$ m/s. Find the distance between A and B when $t = 2$ seconds, and state, with a reason, whether A and B collide at this time.

(Total for Question 29 is 4 marks)

30 A particle P of mass 0.4 kg moves under the action of a single force F newtons, where $F = (3t + 2)i - 5j$, and t is the time in seconds ($t \geq 0$). When $t = 0$, P has velocity $(1i + 2j)$ m/s. Find the velocity of P at time $t = 3$ seconds.

(Total for Question 30 is 4 marks)

---

31 At time $t = 0$, a delivery drone A has position vector $(2i + 6j)$ m and moves with constant velocity $(3i - 1j)$ m/s. At the same time, a second drone B has position vector $(10i + 2j)$ m and moves with constant velocity $(-1i + 2j)$ m/s. Find the value of t at which the distance between A and B is least.

(Total for Question 31 is 3 marks)

32 A ball is thrown from a point on horizontal ground with speed 13 m/s and lands 15 m from the point of projection. Using the range formula $R = u^2 \sin(2*\theta) / g$ and taking $g = 9.8$ m/s^2, find the smaller possible angle of projection θ, giving your answer to 3 significant figures.

(Total for Question 32 is 2 marks)

---

33 A ball is thrown from a point 1.5 m above horizontal ground with speed 20 m/s at an angle of 15 degrees above the horizontal. Using $g = 9.8$ m/s^2:

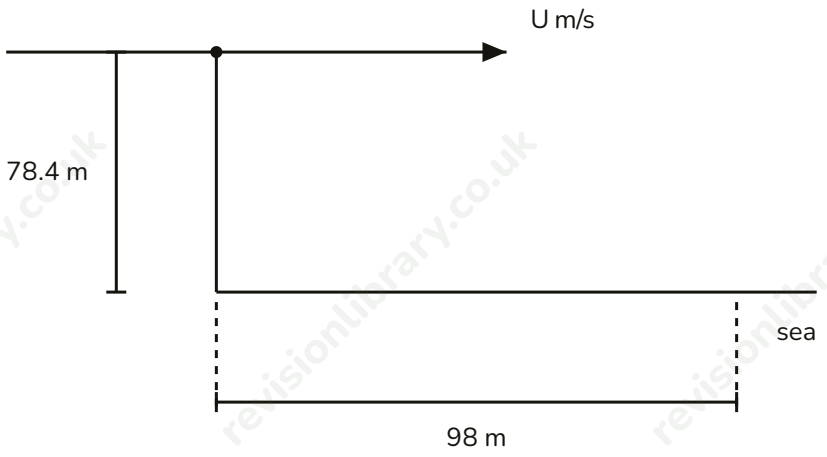
(a) Find the time taken for the ball to reach the ground, giving your answer to 3 significant figures. **(2)**

(b) Find the horizontal distance travelled by the ball before it lands, giving your answer to 3 significant figures. Use your unrounded value from part (a). **(2)**

(Total for Question 33 is 4 marks)

34 A particle is projected horizontally with speed U m/s from a point on top of a vertical cliff. The point of projection is 78.4 m above the sea, modelled as a horizontal plane, and the particle lands in the sea at a horizontal distance of 98 m from the base of the cliff. The particle is modelled as moving freely under gravity. Use $g = 9.8$ m/s^2.

Diagram NOT accurately drawn



(a) Find the time taken for the particle to reach the sea. **(3)**

(b) Find the value of U. **(2)**

(c) Find the speed of the particle immediately before it enters the sea, giving your answer to 3 significant figures. **(3)**

(d) Find the angle the velocity makes with the horizontal immediately before impact, to the nearest degree. **(2)**

(Total for Question 34 is 10 marks)

35 A particle is projected from a point O on horizontal ground with speed 20 m/s at an angle θ above the horizontal, where $0 < \theta < 90$. Relative to O, the particle passes through the point with horizontal distance 15 m and height 5 m above the ground. Use $g = 9.8$ m/s^2.

(a) State the equation of the trajectory of a projectile launched from the origin with speed u at angle θ to the horizontal, in terms of x, y, u, θ and g. **(2)**

(b) Given that $u = 20$ m/s and the particle passes through the point (15, 5), show that $441 \tan^2(\theta) - 2400 \tan(\theta) + 1241 = 0$. **(4)**

(c) Hence find the two possible values of θ, giving each to 1 decimal place. **(5)**

(Total for Question 35 is 11 marks)

36 A particle is projected from a point O with speed 40 m/s at an angle of 50 degrees above the horizontal. The point O lies on a straight slope which is inclined at 20 degrees to the horizontal and rises in the same vertical plane as the motion of the particle. The particle lands at the point A on the slope. Horizontal and vertical distances from O are denoted x and y respectively (both measured in metres). Use $g = 9.8$ m/s^2.

(a) Write down the equation of the slope in terms of x and y, and the equation of the trajectory of the particle in terms of x and y. **(3)**

(b) Hence find the horizontal distance travelled by the particle before it lands at A, giving your answer to 3 significant figures. **(5)**

(c) Find the distance OA. **(3)**

(d) Find the time of flight from O to A, giving your answer to 3 significant figures. **(3)**

(Total for Question 36 is 14 marks)

37 A stone is projected horizontally with speed 15 m/s from the top of a vertical cliff, at a point 19.6 m above horizontal ground. The motion is modelled as that of a particle moving freely under gravity. Take $g = 9.8$ m/s^2.

(a) Find the time taken to reach the ground. **(2)**

(b) Find the horizontal distance from the point of projection to the point where it lands. **(2)**

(c) Find the speed and direction of motion at the instant before it lands, to 3 significant figures and the nearest degree. **(4)**

(Total for Question 37 is 8 marks)

38 At time t = 0, a particle P has position vector (2i - 3j) m relative to a fixed origin O, and velocity (4i + j) m/s. P moves with constant acceleration (-i + 2j) m/s^2. Position vectors are measured in metres and time in seconds.

(a) Find the velocity of P at time t = 3 seconds. **(2)**

(b) Find the position vector of P at time t = 3 seconds. **(3)**

(c) Find the value of t for which P is moving parallel to the vector (i + j). **(3)**

(d) Find the speed of P at the instant found in part (c). **(2)**

(Total for Question 38 is 10 marks)

39 A particle is projected from a point O on horizontal ground with speed 28 m/s at an angle of 30 degrees above the horizontal. The particle moves freely under gravity until it returns to the horizontal ground. Use $g = 9.8$ m/s^2.

(a) Find the time taken for the particle to return to the ground. **(3)**

(b) Find the greatest height reached by the particle above the ground. **(3)**

(c) Find the horizontal distance travelled by the particle before it first returns to the ground (the range). **(3)**

(d) Show that, for a particle projected with speed u at angle θ above horizontal ground and landing at the same level, the range is given by $R = u^2 \sin(2\text{theta})/g$. **(4)**

(Total for Question 39 is 13 marks)

40 A particle P moves in a plane so that at time t seconds ($t \geq 0$), its velocity is $\mathbf{v} = (3t^2 - 6)\mathbf{i} + (4t - 2)\mathbf{j}$ m/s. When $t = 0$, P is at the point with position vector $(5\mathbf{i} + 2\mathbf{j})$ m relative to a fixed origin O.

(a) Find an expression for the acceleration of P at time t seconds, and hence find the acceleration when $t = 2$. **(3)**

(b) Find the position vector of P at time $t = 2$ seconds. **(4)**

(c) Show that P is moving parallel to the vector $\mathbf{i}$ when $t = 0.5$ seconds. **(2)**

(d) Find the speed of P at this instant. **(2)**

(Total for Question 40 is 11 marks)

41 A particle P moves along the positive x-axis. At time t seconds ($0 \leq t \leq 4$), the velocity of P is v m/s, where $v = 12t - 3t^2$. When $t = 0$, P is at the origin O.

(a) Find the acceleration of P when $t = 1$. **(2)**

(b) Find the value of t, other than $t = 0$, at which P is instantaneously at rest. **(2)**

(c) Find the distance travelled by P between $t = 0$ and $t = 4$. **(4)**

(d) Find the maximum speed of P for $0 \leq t \leq 4$. **(3)**

(Total for Question 41 is 11 marks)

42 A particle P of mass 0.5 kg moves under the action of a single force F newtons, where $F = (6t - 3)i + 4j$, and t is the time in seconds ($t \geq 0$). When $t = 0$, P has velocity $(2i - j)$ m/s.

(a) Using $F = ma$, find an expression for the acceleration of P at time t seconds. **(2)**

(b) Show that the velocity of P at time t seconds is given by $v = (6t^2 - 6t + 2)i + (8t - 1)j$. **(3)**

(c) Find the value of t at which P is moving parallel to the vector i. **(3)**

(d) Find the acceleration of P at this instant, and hence find the magnitude of the force acting on P at this time, giving your answer to 3 significant figures. **(3)**

(Total for Question 42 is 11 marks)

Mark scheme · 3.4 Mechanics: Projectiles and Further Kinematics

Question 1

$0\ m/s^2$ **B1**

Answer: $0\ m/s^2$

Question 2

$9.8\ m/s^2$ downwards oe (i.e. $-9.8\ m/s^2$ vertically) **B1**

Answer: $9.8\ m/s^2$ downwards (i.e. $-9.8\ m/s^2$ vertically)

Question 3

horizontal (parallel to the ground) oe **B1**

Answer: Horizontal (parallel to the ground)

Question 4

awrt 19.3 m/s **B1**

Answer: 19.3 m/s (3 s.f.)

Question 5

awrt 23.0 m/s **B1**

Answer: 23.0 m/s (3 s.f.)

Question 6

19.6 cao **B1**

Answer: 19.6 m/s

Question 7

horizontal distance = horizontal speed x time **M1**

45 m cao **A1**

Answer: 45 m

Question 8

(a) awrt 15.5 m/s **B1**

(a) Answer: 15.5 m/s (3 s.f.)

(b) awrt 9.34 m/s **B1**

(b) Answer: 9.34 m/s (3 s.f.)

Question 9

use $v^2 = u^2 + 2as$ (or $v^2 = 2gh$) with u=0, s=4.9 **M1**

9.8 m/s cao **A1**

Answer: 9.8 m/s

Question 10

horizontal speed = horizontal distance / time **M1**

11 m/s cao **A1**

Answer: 11 m/s

Question 11

4i cao **B1**

Answer: $4i\ m/s^2$ (i.e. 4i + 0j)

Question 12

differentiate: a = 12t, substitute t=3 **M1**

$36\ m/s^2$ cao **A1**

Answer: $36\ m/s^2$

Question 13

integrate a with respect to t: $v = 2t^2 - t + C$, use v(0)=0 to find C=0 **M1**

$v = 2t^2 - t$ cao **A1**

Answer: $v = 2t^2 - t$

Question 14

use 0 = u - gt with t=2.5, rearranged for u **M1**

24.5 m/s cao **A1**

Answer: 24.5 m/s

Question 15

v = u + at applied with t=4 **M1**

(6i + 7j) m/s cao **A1**

Answer: (6i + 7j) m/s

Question 16

$r = r0 + ut + 1/2\ a\ t^2$ applied with t=2 **M1**

(7i + 4j) m cao **A1**

Answer: (7i + 4j) m

Question 17

differentiate each component of r with respect to t **M1**

v = (2t - 3)i + 2j cao **A1**

Answer: v = (2t - 3)i + 2j

Question 18

form v = u + at and set the j-component equal to zero: -9 + 3t = 0 oe **M1**

t = 3 s cao **A1**

Answer: t = 3 s

Question 19

differentiate each component of r with respect to t: $v = (6t^2 - 5)i + 8tj$ **M1**

substitute t=2: v = (19i + 16j) m/s cao **A1**

Answer: v = (19i + 16j) m/s

Question 20

integrate v with respect to t, introducing a constant vector **M1**

use r(0) = (2i+5j) to find the constants, giving $r = (2t^2 - t + 2)i + (6t + 5)j$ cao **A1**

Answer: $r = (2t^2 - t + 2)i + (6t + 5)j$

Question 21

find time of flight using $s = 1/2\ g\ t^2$ with s=78.4 **M1**

t = 4 s, then horizontal distance = 18 x 4 **M1**

72 m cao **A1**

Answer: 72 m

Question 22

find initial vertical component: $u_y = 21 \sin 40$ **M1**

use $v^2 = u_y^2 - 2\ g\ h$ with v=0, rearranged for h **M1**

awrt 9.30 m **A1**

Answer: 9.30 m (3 s.f.)

Question 23

find $u_y = 24 \sin 35$, use time of flight = $2 u_y / g$ **M1**

awrt 2.81 s **A1**

find $u_x = 24 \cos 35$, range = u_x x time of flight (ft) **M1**

awrt 55.2 m **A1**

Answer: time of flight = 2.81 s; range = 55.2 m (both 3 s.f.)

Question 24

integrate a with respect to t to find v(t), introducing a constant vector **M1**

use v(0) = (1i-3j) to find the constant, giving $v = (3t^2-2t+1)i + (4t-3)j$ **A1**

integrate v(t) with respect to t to find r(t), introducing a constant vector, and use r(0) = (0i+2j) to find it **M1**

r(2) = (6i + 4j) m cao **A1**

Answer: r(2) = (6i + 4j) m

Question 25

find $u_x = 24\cos 35$ (unchanged throughout) and $u_y = 24\sin 35$ **M1**

find $v_y = u_y - g(1.5)$, then speed = $\sqrt{v_x^2 + v_y^2}$ **M1**

awrt 19.7 m/s **A1**

Answer: 19.7 m/s (3 s.f.)

Question 26

find $v_x = 18\cos 50$ (unchanged) and $v_y = 18\sin 50 - 9.8(1)$ **M1**

angle = $\arctan(v_y / v_x)$ **M1**

awrt 19.0 degrees (above the horizontal) **A1**

Answer: 19.0 degrees above the horizontal (3 s.f.)

Question 27

find $u_y = 23\sin 44$ (= awrt 16.0) **M1**

use $v^2 = u_y^2 - 2gh$ with v=0, rearranged for h **M1**

h = 13.0 m (3 s.f.) correctly shown, cso **A1**

Answer: Greatest height = 13.0 m (as required)

Question 28

find $u_y = 15\sin 25$ and form $-12 = u_y t - 4.9t^2$, rearranged to $4.9t^2 - 6.34t - 12 = 0$ oe **M1**

solve the quadratic using the quadratic formula, selecting the positive root **M1**

awrt 2.34 s **A1**

Answer: 2.34 s (3 s.f.)

Question 29

substitute t=2 into r = r0 + vt for both A and B **M1**

A at (7i + 2j) m, B at (3i + 4j) m **A1**

distance = magnitude of (position of A - position of B) **M1**

awrt 4.47 m; they do not collide since the distance between them is not zero **A1**

Answer: Distance = 4.47 m (3 s.f.); A and B do not collide at t = 2

Question 30

use F = ma to find a = F/m = (7.5t + 5)i - 12.5j **M1**

integrate a with respect to t to find v(t), introducing a constant vector **M1**

use v(0) = (1i + 2j) to find the constant, giving $v = (3.75t^2 + 5t + 1)i + (-12.5t + 2)j$ **A1**

v(3) = (49.75i - 35.5j) m/s cao **A1**

Answer: v(3) = (49.75i - 35.5j) m/s

Question 31

find the position vector of A relative to B as a function of t: (4t-8)i + (4-3t)j oe **M1**

form $(\text{distance})^2 = (4t-8)^2 + (4-3t)^2 = 25t^2 - 88t + 80$ and minimise (differentiate and set to 0, or use t=-b/2a) **M1**

t = 1.76 s cao **A1**

Answer: t = 1.76 s

Question 32

substitute into $R=u^2 \sin(2*\theta)/g$: $15 = 169 \sin(2*\theta)/9.8$, rearrange to $\sin(2*\theta) = 147/169$ (awrt 0.870), and solve for $2*\theta$ using arcsin (taking the value less than 90 degrees) **M1**

θ = awrt 30.2 degrees **A1**

Answer: θ = 30.2 degrees (3 s.f.)

Question 33

(a) find $u_y=20\sin 15$ and set up and solve $4.9t^2 - 5.18t - 1.5 = 0$ using the quadratic formula, selecting the positive root **M1**

(a) awrt 1.29 s **A1**

(a) Answer: 1.29 s (3 s.f.)

(b) horizontal distance = 20cos15 x t (ft their time from part (a)) **M1**

(b) awrt 25.0 m (ft) **A1**

(b) Answer: 25.0 m (3 s.f.)

Question 34

(a) use $s = ut + 0.5at^2$ vertically with u = 0, s = 78.4 (oe) **M1**

(a) rearrange to $t^2 = 16$ **M1**

(a) t = 4 (cao) **A1**

(a) Answer: 4 s

(b) U = 98/4 (oe) **M1**

(b) U = 24.5 m/s (cao) **A1**

(b) Answer: 24.5 m/s

(c) $v_y = gt = 9.8 \times 4 = 39.2$ (oe) **M1**

(c) speed = $\sqrt{24.5^2 + 39.2^2}$ **M1**

(c) awrt 46.2 m/s **A1**

(c) Answer: 46.2 m/s (3sf)

(d) $\tan(\theta) = 39.2/24.5 = 1.6$ (oe) **M1**

(d) θ = awrt 58 degrees **A1**

(d) Answer: 58 degrees below the horizontal

Question 35

(a) $y = x \tan(\theta)$ - ... term with correct $x \tan(\theta)$ component **B1**

(a) correct full trajectory equation $y = x \tan(\theta) - (g x^2 (1+\tan^2(\theta)))/(2u^2)$ (oe, e.g. using $\sec^2(\theta)$) **B1**

(a) Answer: $y = x \tan(\theta) - (g x^2 (1 + \tan^2(\theta))) / (2u^2)$

(b) substitute x=15, y=5, u=20, g=9.8 into trajectory equation **M1**

(b) use $1+\tan^2(\theta) = \sec^2(\theta)$ and simplify the coefficient $9.8x225/(2x400) = 2.75625$ (oe) **M1**

(b) rearrange into a quadratic in $\tan(\theta)$ and clear decimals **M1**

(b) $441 \tan^2(\theta) - 2400 \tan(\theta) + 1241 = 0$ (cso, or exact equivalent unsimplified form with correct working) **A1**

(b) Answer: $441 \tan^2(\theta) - 2400 \tan(\theta) + 1241 = 0$ (shown)

(c) use the quadratic formula to solve for $\tan(\theta)$ **M1**

(c) $\tan(\theta)$ = awrt 4.86 or 0.579 (both values, ft from (b)) **A1**

(c) use arctan to find θ for each value **M1**

(c) θ = awrt 78.4 degrees **A1**

(c) θ = awrt 30.1 degrees **A1**

(c) Answer: θ = awrt 30.1 degrees or θ = awrt 78.4 degrees

Question 36

(a) slope equation $y = x \tan(20)$ (oe) **B1**

(a) trajectory equation with θ=50, u=40 substituted: $y = x \tan(50) - (9.8x^2(1+\tan^2(50)))/(2x40^2)$ (oe) **B1**

(a) correctly simplified coefficient (accept unsimplified equivalent form) **B1**

(a) Answer: $y = x \tan(20)$; $y = x \tan(50) - (9.8 x^2 \sec^2(50)) / 3200$

(b) equate the two equations for y **M1**

(b) factor out x (x not equal to 0) to obtain a linear equation in x **M1**

(b) substitute numerical values of $\tan(20)$, $\tan(50)$ and $\sec^2(50)$ **M1**

(b) correct simplified linear equation, e.g. $0.0074121x = 0.827784$ (oe) **A1**

(b) x = awrt 112 m **A1**

(b) Answer: x = awrt 112 m

(c) OA = x / cos(20) (oe, or use Pythagoras with $y = x \tan(20)$) **M1**

(c) correct method with ft value of x from (b) **A1**

(c) OA = awrt 119 m **A1**

(c) Answer: awrt 119 m

(d) use $x = u \cos(50) \times t$ **M1**

(d) rearrange for t, ft value of x from (b) **M1**

(d) t = awrt 4.34 s **A1**

(d) Answer: awrt 4.34 s

Question 37

(a) uses $s = ut + (1/2)at^2$ vertically: $19.6 = (1/2)(9.8)t^2$ **M1**

(a) t = 2.00 s **A1**

(a) Answer: 2.00 s

(b) horizontal speed is constant: distance = u t **M1**

(b) 30.0 m **A1**

(b) Answer: 30.0 m

(c) vertical speed = gt = 9.8 x 2.000 = 19.600 **M1**

(c) combines the components: speed = $\sqrt{15^2 + 19.600^2}$ **M1**

(c) speed = 24.7 m/s **A1**

(c) angle 53 degrees below the horizontal from tan(angle) = 19.600/15 **A1**

(c) Answer: 24.7 m/s at 53 degrees below the horizontal

Question 38

(a) use $v = u + at$ with correct substitution **M1**

(a) v = (i + 7j) m/s (cao) **A1**

(a) Answer: (i + 7j) m/s

(b) use $r = r0 + ut + 0.5at^2$ **M1**

(b) correct substitution of all vectors and t = 3 **M1**

(b) r = (9.5i + 9j) m (cao) **A1**

(b) Answer: (9.5i + 9j) m

(c) v(t) = (4-t)i + (1+2t)j **M1**

(c) equate components: 4 - t = 1 + 2t (oe) **M1**

(c) t = 1 (cao) **A1**

(c) Answer: t = 1 s

(d) v = 3i + 3j, speed = $\sqrt{3^2 + 3^2}$ **M1**

(d) $3\sqrt{2}$ m/s, awrt 4.24 m/s **A1**

(d) Answer: $3\sqrt{2}$ m/s = awrt 4.24 m/s

Question 39

(a) initial vertical speed = 28 sin(30) = 14 (oe) **M1**

(a) T = 2 x 14 / 9.8 (oe) **M1**

(a) T = 20/7 s = awrt 2.86 s **A1**

(a) Answer: T = 20/7 s = awrt 2.86 s

(b) use $v^2 = u^2 - 2gs$ with v = 0, u = 14 **M1**

(b) rearrange for s **M1**

(b) H = 10 m (cao) **A1**

(b) Answer: 10 m

(c) horizontal speed = 28 cos(30) (oe) **M1**

(c) R = horizontal speed x T **M1**

(c) $R = 40\sqrt{3}$ m, awrt 69.3 m **A1**

(c) Answer: $40\sqrt{3}$ m = awrt 69.3 m

(d) time of flight $T = 2u \sin(\theta)/g$ stated or derived **M1**

(d) $R = (u \cos(\theta)) \times T$ **M1**

(d) substitute and use $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$ **M1**

(d) $R = u^2 \sin(2\theta)/g$ (cso, no errors seen) **A1**

(d) Answer: $R = u^2 \sin(2\theta)/g$ (shown)

Question 40

(a) differentiate v to obtain a = 6ti + 4j **M1**

(a) correct expression for a(t) **A1**

(a) a(2) = 12i + 4j m/s^2 (cao) **A1**

(a) Answer: a = 6ti + 4j; at t = 2, a = 12i + 4j m/s^2

(b) integrate v(t) componentwise, including constants of integration **M1**

(b) i-component t^3 - 6t + C1 **A1**

(b) j-component $2t^2$ - 2t + C2 **A1**

(b) use r(0) = 5i + 2j to find constants, giving r(2) = (i + 6j) m (cao) **A1**

(b) Answer: (i + 6j) m

(c) set j-component of v to zero: 4t - 2 = 0 **M1**

(c) t = 0.5 (cso) **A1**

(c) Answer: t = 0.5 s

(d) v(0.5) = $(3(0.5)^2$ - 6)i + 0j = -5.25i **M1**

(d) speed = 5.25 m/s (cao) **A1**

(d) Answer: 5.25 m/s

Question 41

(a) a = dv/dt = 12 - 6t **M1**

(a) a(1) = 6 m/s^2 (cao) **A1**

(a) Answer: 6 m/s^2

(b) set v = 0: 12t - $3t^2$ = 0, factorise (oe) **M1**

(b) t = 4 (cao) **A1**

(b) Answer: t = 4 s

(c) recognise v ≥ 0 throughout so distance = displacement **M1**

(c) integrate: integral of (12t - $3t^2$) dt = $6t^2$ - t^3 **M1**

(c) substitute limits t = 0 and t = 4 **M1**

(c) distance = 32 m (cao) **A1**

(c) Answer: 32 m

(d) set a = 0: 12 - 6t = 0, t = 2 **M1**

(d) justify maximum (a changes from positive to negative, oe) **M1**

(d) v(2) = 12 m/s (cao) **A1**

(d) Answer: 12 m/s

Question 42

(a) a = F/m **M1**

(a) a = (12t - 6)i + 8j m/s^2 (cao) **A1**

(a) Answer: a = (12t - 6)i + 8j m/s^2

(b) integrate a(t) to find v(t) with a constant vector **M1**

(b) correct unsimplified expression, e.g. ($6t^2$-6t+C1)i + (8t+C2)j **A1**

(b) use v(0) = 2i - j to find constants and complete (cso) **A1**

(b) Answer: v = ($6t^2$ - 6t + 2)i + (8t - 1)j (shown)

(c) set j-component of v to zero: 8t - 1 = 0 **M1**

(c) solve for t **M1**

(c) t = 0.125 s (= 1/8 s, cao) **A1**

(c) Answer: t = 0.125 s

(d) a(0.125) = (12x0.125-6)i + 8j = -4.5i + 8j **M1**

(d) F = ma, magnitude = 0.5 x $\sqrt{4.5^2 + 8^2}$ **M1**

(d) F = awrt 4.59 N **A1**

(d) Answer: awrt 4.59 N