3.5 Mechanics: Kinematics Depth: 2-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A particle P moves along the x-axis with displacement $x = t^2 - 5t$ metres at time t seconds. Find the velocity of P when $t = 0$.

(Total for Question 1 is 1 mark)

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2 A particle P moves along the x-axis. Its displacement from the origin at time t seconds ($t \geq 0$) is $x = t^3 - 12t$ metres. Find the value of t at which P is instantaneously at rest.

(Total for Question 2 is 2 marks)

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3 A particle has velocity $v = 3\sin(t)$ m/s at time t seconds. Find an expression for the acceleration of the particle at time t.

(Total for Question 3 is 2 marks)

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4 A particle P moves along the x-axis. At time t seconds ($t \geq 0$), the velocity of P is $v = t^2 - 6t + 8$ m/s. Given that P is at the origin when $t = 0$, find the values of t at which P is instantaneously at rest, and hence find the total distance travelled by P in the interval $0 \leq t \leq 5$.

(Total for Question 4 is 4 marks)

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5 Two cars, A and B, pass the same road sign at the same instant, travelling in the same direction along a straight road. Car A passes the sign with speed 22 m/s and then travels at this constant speed. Car B passes the sign from rest at this same instant, with constant acceleration 2 m/s^2. Let t seconds be the time after the cars pass the sign.

(a) Write down expressions, in terms of t, for the displacement from the sign of car A and of car B. **(2)**

(b) Find the value of t at which car B catches up with car A, and find the distance from the sign at which this happens. **(3)**

(c) Find the speed of car B at the instant it catches up with car A. **(2)**

(Total for Question 5 is 7 marks)

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6 A particle P moves along the x-axis. At time t seconds ($t \geq 0$), the displacement of P from a fixed origin O is x metres, where $x = t^3 - 6t^2 + 9t + 2$.

(a) Find an expression for the velocity, v m/s, of P at time t seconds. **(2)**

(b) Find an expression for the acceleration, a m/s^2, of P at time t seconds. **(2)**

(c) Find the values of t for which P is instantaneously at rest. **(2)**

(d) Find the acceleration of P at the instant when $t = 3$. **(2)**

(Total for Question 6 is 8 marks)

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7 A particle is projected vertically upwards from the top of a tower 25 m high, with initial speed 18 m/s. Take the ground at the foot of the tower as the origin, with distances measured vertically upwards, and use $g = 9.8$ m/s^2.

(a) Find the greatest height above the ground reached by the particle. **(3)**

(b) Find the time taken for the particle to reach its greatest height, giving your answer to 3 significant figures. **(2)**

(c) Show that the times at which the particle is at a height of 40 m above the ground satisfy $4.9t^2 - 18t + 15 = 0$, and hence find both times, giving your answers to 3 significant figures. **(4)**

(d) Find the time taken for the particle to hit the ground, giving your answer to 3 significant figures. **(3)**

(Total for Question 7 is 12 marks)

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8 A train moves in a straight line with constant acceleration. It passes a point A with speed 11 m/s and reaches a point B with speed 35 m/s, after travelling a distance of 184 m from A.

(a) Find the acceleration. **(3)**

(b) Find the time taken to travel from A to B. **(2)**

(c) Find the distance travelled in the 4th second after passing A. **(3)**

(Total for Question 8 is 8 marks)

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9 At time $t = 0$ a particle P has position vector $(-5\mathbf{i} - 3\mathbf{j})$ m relative to a fixed origin O, and velocity $(5\mathbf{i} - 3\mathbf{j})$ m/s. P moves with constant acceleration $(\mathbf{i} - \mathbf{j})$ m/s^2.

(a) Find the velocity of P when $t = 2$, and its speed, to 3 significant figures. **(3)**

(b) Find the position vector of P when $t = 2$. **(2)**

(Total for Question 9 is 5 marks)

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10 A particle P moves in a horizontal plane. Its position vector relative to a fixed origin O at time t seconds is $\mathbf{r} = (t^2)\mathbf{i} + (-3t^3 + 4t)\mathbf{j}$ metres.

(a) Find the velocity and the acceleration of P at time t. **(3)**

(b) Find the speed of P when t = 2, to 3 significant figures. **(2)**

(c) Find the magnitude of the acceleration of P when t = 2, to 3 significant figures. **(2)**

(Total for Question 10 is 7 marks)

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11 A particle P moves along a straight line. At time t seconds, $t \geq 0$, its velocity is $v = t^2 - 6t + 5$ m/s.

(a) Find the times at which P is instantaneously at rest. **(2)**

(b) Find the acceleration of P when t = 5. **(3)**

(c) Find the distance travelled by P in the interval $0 \leq t \leq 5$. **(4)**

(Total for Question 11 is 9 marks)

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12 A particle P moves in a horizontal plane. Its position vector relative to a fixed origin O at time t seconds is $\mathbf{r} = (t^2 + 5t)\mathbf{i} + (3t^3 - 3t)\mathbf{j}$ metres.

(a) Find the velocity and the acceleration of P at time t. **(3)**

(b) Find the speed of P when t = 3, to 3 significant figures. **(2)**

(c) Find the magnitude of the acceleration of P when t = 3, to 3 significant figures. **(2)**

(Total for Question 12 is 7 marks)

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13 A train moves in a straight line with constant acceleration. It passes a point A with speed 2 m/s and reaches a point B with speed 6 m/s, after travelling a distance of 16 m from A.

(a) Find the acceleration. **(3)**

(b) Find the time taken to travel from A to B. **(2)**

(c) Find the distance travelled in the 3rd second after passing A. **(3)**

(Total for Question 13 is 8 marks)

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14 At time t = 0 a particle P has position vector $(\mathbf{i} - 3\mathbf{j})$ m relative to a fixed origin O, and velocity $(\mathbf{i} + 4\mathbf{j})$ m/s. P moves with constant acceleration $(\mathbf{i} + \mathbf{j})$ m/s^2.

(a) Find the velocity of P when t = 2, and its speed, to 3 significant figures. **(3)**

(b) Find the position vector of P when t = 2. **(2)**

(c) Find the value of t for which the velocity of P is parallel to $(2\mathbf{i} + 3\mathbf{j})$. **(3)**

(Total for Question 14 is 8 marks)

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15 A particle R moves in a straight line so that its velocity, v m/s, at time t seconds ($0 \leq t \leq 6$) is given by $v = t^2 - 6t + 8$.

(a) Find the values of t at which R is instantaneously at rest. **(2)**

(b) Find the acceleration of R at time $t = 5$ seconds, and state, with a reason, whether R is speeding up or slowing down at this instant. **(3)**

(c) Find the total distance travelled by R in the interval $0 \leq t \leq 6$. **(4)**

(Total for Question 15 is 9 marks)

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16 A particle is projected vertically upwards from the top of a tower 20 m high, with initial speed 15 m/s. Taking the ground at the foot of the tower as the origin, with distances measured vertically upwards, and using $g = 9.8$ m/s^2, show that the times at which the particle is at a height of 22 m above the ground satisfy $4.9t^2 - 15t + 2 = 0$, and hence find both times, giving your answers to 3 significant figures.

(Total for Question 16 is 4 marks)

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Question 1

-5 cao **B1**

Answer: -5 m/s

Question 2

differentiate and set $v = 3t^2 - 12 = 0$ **M1**

$t = 2$ cao (rejecting the negative root as $t \geq 0$) **A1**

Answer: t = 2 s

Question 3

differentiate v with respect to t **M1**

$a = 3\cos(t)$ cao **A1**

Answer: a = 3cos(t)

Question 4

set v=0 and solve: (t-2)(t-4)=0 **M1**

t = 2 and t = 4 cao **A1**

integrate v to find $x = t^3/3 - 3t^2 + 8t$ (using x(0)=0), and evaluate at t=0,2,4,5 **M1**

total distance = |20/3 - 0| + |16/3 - 20/3| + |20/3 - 16/3| = awrt 9.33 m **A1**

Answer: t = 2 s and t = 4 s; total distance = 9.33 m (3 s.f.)

Question 5

(a) displacement of A: $s_A = 22t$ **B1**

(a) displacement of B: $s_B = t^2$ (from $s = 0 + 0.5(2)t^2$) **B1**

(a) Answer: $s_A = 22t$; $s_B = t^2$

(b) sets $s_A = s_B$: $22t = t^2$ **M1**

(b) t = 22 (s), rejecting t = 0 **A1**

(b) distance from sign = 484 (m) cao **A1**

(b) Answer: t = 22 s; distance = 484 m

(c) uses v = u + at with u = 0, a = 2, their t = 22 **M1**

(c) v = 44 (m/s) cao **A1**

(c) Answer: 44 m/s

Question 6

(a) differentiates x with respect to t **M1**

(a) $v = 3t^2 - 12t + 9$ cao **A1**

(a) Answer: $v = 3t^2 - 12t + 9$

(b) differentiates their v with respect to t **M1**

(b) $a = 6t - 12$ cao **A1**

(b) Answer: a = 6t - 12

(c) sets v = 0 and divides by 3: $t^2 - 4t + 3 = 0$, then factorises **M1**

(c) t = 1 and t = 3 cao **A1**

(c) Answer: t = 1 s and t = 3 s

(d) substitutes t = 3 into their expression for a **M1**

(d) $a = 6$ (m/s^2) cao **A1**

(d) Answer: 6 m/s^2

Question 7

(a) uses $v^2 = u^2 - 2gh$ with v = 0, u = 18, to find height above the tower **M1**

(a) height above tower = awrt 16.5 (m) **A1**

(a) adds 25 m: greatest height above ground = awrt 41.5 (m) cao **A1**

(a) Answer: 41.5 m (3sf)

(b) uses v = u - gt with v = 0, u = 18 **M1**

(b) t = awrt 1.84 (s) cao **A1**

(b) Answer: 1.84 s (3sf)

(c) sets height $25 + 18t - 4.9t^2 = 40$ and rearranges **M1**

(c) $4.9t^2 - 18t + 15 = 0$ cso **A1**

(c) uses the quadratic formula, with discriminant $18^2 - 4(4.9)(15) = 30$ **M1**

(c) t = awrt 1.28 (s) and t = awrt 2.40 (s), both values given **A1**

(c) Answer: $4.9t^2 - 18t + 15 = 0$ (shown); t = 1.28 s and t = 2.40 s (3sf)

(d) sets $25 + 18t - 4.9t^2 = 0$, i.e. $4.9t^2 - 18t - 25 = 0$ **M1**

(d) uses the quadratic formula with discriminant $18^2 + 4(4.9)(25) = 814$ **A1**

(d) t = awrt 4.75 (s), rejecting the negative root **A1**

(d) Answer: 4.75 s (3sf)

Question 8

(a) uses $v^2 = u^2 + 2as$ **M1**

(a) $35^2 = 11^2 + 2a(184)$ **A1**

(a) a = 3 m/s^2 **A1**

(a) Answer: 3 m/s^2

(b) uses v = u + at **M1**

(b) t = (35 - 11)/3 = 8 s **A1**

(b) Answer: 8 s

(c) distance in first 4 s minus distance in first 3 s **M1**

(c) 68 - 46.5 **A1**

(c) 21.5 m **A1**

(c) Answer: 21.5 m

Question 9

(a) uses v = u + at **M1**

(a) 7i - 5j **A1**

(a) speed = 8.60 m/s **A1**

(a) Answer: (7i - 5j) m/s, speed 8.60 m/s

(b) uses $r = r0 + ut + (1/2)at^2$ **M1**

(b) (7i - 11j) m **A1**

(b) Answer: (7i - 11j) m

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Question 10

(a) differentiates the position vector **M1**

(a) $v = (2t)i + (-9t^2 + 4)j$ **A1**

(a) $a = 2i - 18t\ j$ **A1**

(a) Answer: $v = (2t)i + (-9t^2 + 4)j$; $a = 2i - 18t\ j$

(b) substitutes t = 2: $v = 4i - 32j$ **M1**

(b) speed = 32.2 m/s **A1**

(b) Answer: 32.2 m/s

(c) $a = 2i - 18t\ j$ with t = 2 **M1**

(c) $|a| = 36.1\ m/s^2$ **A1**

(c) Answer: 36.1 m/s^2

Question 11

(a) sets v = 0 and factorises **M1**

(a) t = 1 and t = 5 **A1**

(a) Answer: t = 1 s and t = 5 s

(b) differentiates v with respect to t **M1**

(b) $a = 2t - 6$ **A1**

(b) $a = 4\ m/s^2$ **A1**

(b) Answer: 4 m/s^2

(c) integrates v to obtain the displacement function **M1**

(c) evaluates the displacements over 0 to 1 and 1 to 5 separately, since v changes sign **M1**

(c) displacements 7/3 and -32/3 **A1**

(c) total distance = 13 m **A1**

(c) Answer: 13 m

Question 12

(a) differentiates the position vector **M1**

(a) $v = (2t + 5)i + (9t^2 - 3)j$ **A1**

(a) $a = 2i + 18t\ j$ **A1**

(a) Answer: $v = (2t + 5)i + (9t^2 - 3)j$; $a = 2i + 18t\ j$

(b) substitutes t = 3: $v = 11i + 78j$ **M1**

(b) speed = 78.8 m/s **A1**

(b) Answer: 78.8 m/s

(c) $a = 2i + 18t\ j$ with t = 3 **M1**

(c) $|a| = 54.0\ m/s^2$ **A1**

(c) Answer: 54.0 m/s^2

Question 13

(a) uses $v^2 = u^2 + 2as$ **M1**

(a) $6^2 = 2^2 + 2a(16)$ **A1**

(a) $a = 1\ m/s^2$ **A1**

(a) Answer: 1 m/s^2

(b) uses v = u + at **M1**

(b) t = (6 - 2) = 4 s **A1**

(b) Answer: 4 s

(c) distance in first 3 s minus distance in first 2 s **M1**

(c) 10.5 - 6 **A1**

(c) 4.5 m **A1**

(c) Answer: 4.5 m

Question 14

(a) uses v = u + at **M1**

(a) 3i + 6j **A1**

(a) speed = 6.71 m/s **A1**

(a) Answer: (3i + 6j) m/s, speed 6.71 m/s

(b) uses $r = r0 + ut + (1/2)at^2$ **M1**

(b) (5i + 7j) m **A1**

(b) Answer: (5i + 7j) m

(c) equates the ratio of the velocity components to the ratio in the given vector **M1**

(c) forms and solves a linear equation in t **M1**

(c) t = 5 **A1**

(c) Answer: t = 5

Question 15

(a) sets v = 0 and factorises: (t-2)(t-4) = 0 **M1**

(a) t = 2 and t = 4 cao **A1**

(a) Answer: t = 2 s and t = 4 s

(b) differentiates v to find a = 2t - 6, and substitutes t = 5 **M1**

(b) a = 4 (m/s^2) cao **A1**

(b) speeding up, since v(5) = 3 (> 0) and a = 4 (> 0) act in the same direction (dependent on correct a and correct v(5)) **B1**

(b) Answer: a = 4 m/s^2; R is speeding up

(c) integrates v to find $s(t) = t^3/3 - 3t^2 + 8t$, with s(0) = 0 **M1**

(c) evaluates s at t = 2, t = 4 and t = 6, recognising R changes direction at t = 2 and t = 4 **dM1**

(c) distances 20/3 m, 4/3 m and 20/3 m for the three stages (oe, e.g. decimals) **A1**

(c) total distance = 44/3 (m) (oe, awrt 14.7 m) cao **A1**

(c) Answer: 44/3 m (awrt 14.7 m)

Question 16

height above ground = $20 + 15t - 4.9t^2$, set equal to 22 **M1**

cso: rearrange $15t - 4.9t^2 = 2$ to give $4.9t^2 - 15t + 2 = 0$ **A1**

apply the quadratic formula to solve for t **M1**

awrt 0.140 s and 2.92 s **A1**

Answer: t = 0.140 s and t = 2.92 s (3 s.f.)

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