3.7 Mechanics: Moments and Statics Depth

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A uniform rod AB has weight 90 N and rests horizontally in equilibrium on a single smooth pivot at its midpoint. State the reaction at the pivot.

(Total for Question 1 is 1 mark)

2 A moment of 96 N m is required to turn a lever about its pivot. A force of 160 N is applied perpendicular to the lever. Find the minimum distance from the pivot at which the force must act.

(Total for Question 2 is 1 mark)

3 Two weights of 20 N and 30 N hang from a light rod at distances 0.6 m and 0.4 m from a pivot O, both on the same side of the pivot. Find the total moment of these two weights about O.

(Total for Question 3 is 1 mark)

4 A light rod AB rests horizontally in equilibrium on a pivot at C, under three weights of 15 N, 25 N and 42.5 N hanging from it (and the pivot reaction). Find the reaction at the pivot C.

(Total for Question 4 is 2 marks)

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5 A non-uniform rod AB of length 3 m is held horizontal by two vertical strings, one at A and one at B. The tension in the string at A is twice the tension in the string at B, and the weight of the rod is 90 N. Find the tension in the string at B.

(Total for Question 5 is 2 marks)

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6 A non-uniform rod is held horizontal by two vertical strings, with tensions T_A and T_B such that $T_A = 2T_B$ and $T_A + T_B = 90$ N. Find T_A.

(Total for Question 6 is 2 marks)

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7 A uniform rectangular lamina ABCD has AB = 4 cm and BC = 10 cm. Point masses of 2 kg are attached at each of the four corners, and the lamina itself has negligible mass. Taking A as the origin, with AB along the x-axis and AD along the y-axis, find the coordinates of the centre of mass of the system of four point masses.

(Total for Question 7 is 2 marks)

8 Two point masses of 3 kg and 5 kg are placed on a light rod at x = 0 and x = 8 m respectively. Find the x-coordinate of the centre of mass of the system.

(Total for Question 8 is 1 mark)

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9 A light rod AB rests horizontally on a pivot at C. A weight of 25 N hangs at a distance 0.8 m from C on one side. Find the distance from C at which a weight of 40 N must hang on the other side, for the rod to be in equilibrium.

(Total for Question 9 is 2 marks)

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10 A light rod AB rests horizontally in equilibrium on a pivot at C. Weights of 15 N and 25 N hang from the rod at distances 0.3 m and 0.5 m from C, on the same side of the pivot. A weight W hangs at a distance 0.4 m from C on the other side. By taking moments about C, find W.

(Total for Question 10 is 2 marks)

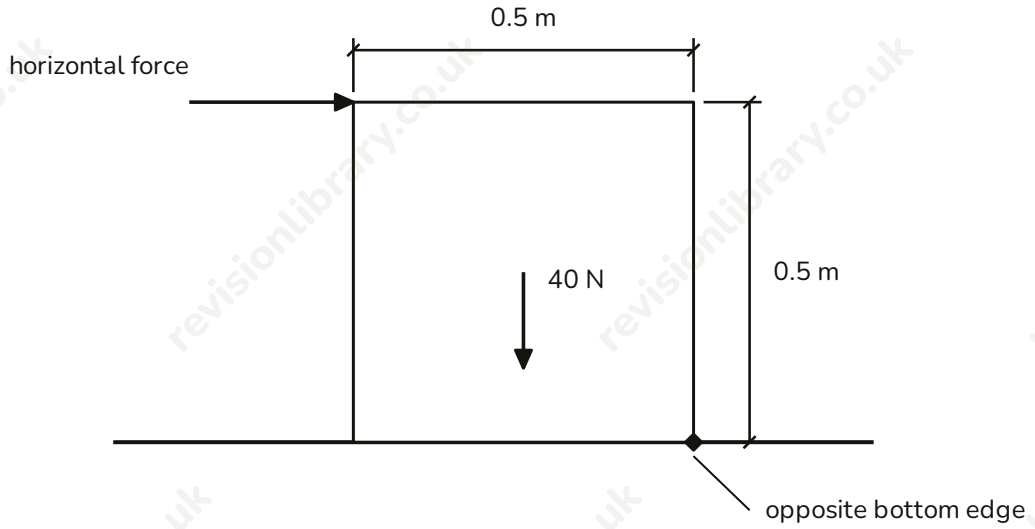
11 A bent rod OAB has two straight arms, OA and AB, rigidly joined at right angles at A. When the rod is laid flat with OA horizontal and AB vertical, its centre of mass is a horizontal distance 0.3 m and a vertical distance 0.4 m from O. The rod is suspended freely from O and hangs in equilibrium. Find the angle OA makes with the vertical, giving your answer to 3 significant figures.

(Total for Question 11 is 2 marks)

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12 A uniform cube of side 0.5 m and weight 40 N rests on rough horizontal ground. A horizontal force is applied to the top edge, increasing until the cube is on the point of toppling about the opposite bottom edge. Find the moment of the weight about this edge at the point of toppling.

Diagram NOT accurately drawn



(Total for Question 12 is 2 marks)

13 A light rigid rod AB has length 1.2 m and rests horizontally in equilibrium on a smooth pivot at point C on the rod. Weights of 30 N and 50 N hang from the rod at distances 0.2 m and 0.5 m from C, on one side of the pivot. A weight of magnitude W hangs at a distance 0.8 m from C, on the other side. By taking moments about C, find W, and hence find the reaction at the pivot.

(Total for Question 13 is 3 marks)

14 A uniform beam AB has length 6 m and weight 240 N. The beam rests horizontally in equilibrium on two smooth supports, one at C, 1 m from A, and one at D, 1 m from B. By taking moments about C, find the reaction at D, and hence find the reaction at C.

(Total for Question 14 is 3 marks)

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15 A light rigid rod AB has length 1.1 m. It rests horizontally in equilibrium on a smooth pivot at the point C on the rod. Weights of 40 N and 70 N hang from hooks on the rod at distances 0.25 m and 0.5 m from C, on one side of the pivot. A weight of magnitude W newtons hangs from a hook at a distance 0.6 m from C, on the other side of the pivot.

(a) By taking moments about C, show that W = 75. **(3)**

(b) Find the reaction at the pivot C. **(2)**

(Total for Question 15 is 5 marks)

16 A uniform beam AB has length 6 m and weight 210 N. The beam rests horizontally in equilibrium on two smooth supports, one at C, 1 m from A, and one at D, 4 m from A.

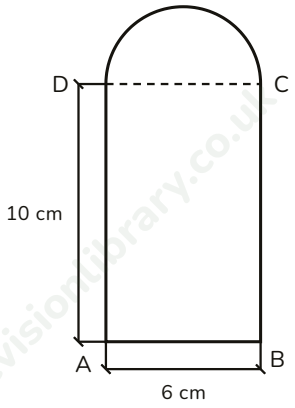
(a) By taking moments about C, show that the reaction at D, R_D, is 140 N. **(3)**

(b) Find the reaction at C, R_C. **(3)**

(Total for Question 16 is 6 marks)

17 A uniform lamina is formed from a rectangle ABCD, where AB = 6 cm and BC = 10 cm, together with a uniform semicircular lamina of the same material and thickness, of diameter 6 cm, attached along the edge CD, so that the semicircle bulges outward from the rectangle. The centre of mass of a uniform semicircular lamina of radius r lies a distance $4r/(3*\pi)$ from the centre of its diameter, along the axis of symmetry.

Diagram NOT accurately drawn



(a) Show that the area of the semicircular portion is $4.5*\pi$ cm^2. **(2)**

(b) Find the distance of the centre of mass of the whole lamina from the edge AB (the edge of the rectangle furthest from the semicircle). **(4)**

(Total for Question 17 is 6 marks)

18 A non-uniform rod AB has length 5 m and weight 150 N. The rod is held in equilibrium in a horizontal position by two vertical strings, one at A and one at B. The tension in the string at A is twice the tension in the string at B. Find the tension in each string, and hence, by taking moments about A, find the distance of the centre of mass of the rod from A, giving your answer to 3 significant figures.

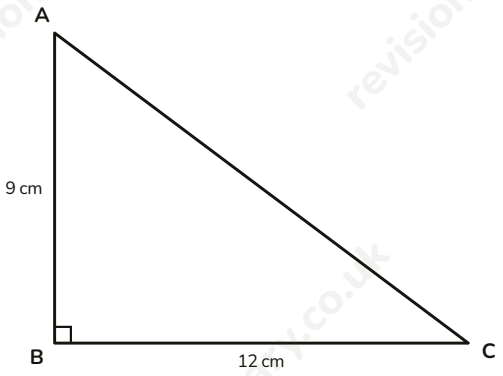
(Total for Question 18 is 4 marks)

19 A particle of weight 60 N hangs in equilibrium, attached to two light inextensible strings whose other ends are fixed to a horizontal ceiling, on opposite sides of the vertical through the particle. String 1 makes an angle of 55 degrees with the ceiling, and string 2 makes an angle of 40 degrees with the ceiling. Find the tension in each string, giving your answers to 3 significant figures.

(Total for Question 19 is 4 marks)

20 A uniform triangular lamina ABC has a right angle at B, with AB = 9 cm and BC = 12 cm.

Diagram NOT accurately drawn



(a) Show that AC = 15 cm. **(1)**

(b) Taking B as the origin, with BC along the x-axis and BA along the y-axis, find the coordinates of the centre of mass of the lamina. **(3)**

(c) The lamina is freely suspended from the vertex A and hangs in equilibrium. Find the angle that AB makes with the vertical, giving your answer to 3 significant figures. **(5)**

(Total for Question 20 is 9 marks)

21 A uniform beam AB has length 7 m and weight 230 N. It rests horizontally on two supports at C and D, where AC = 3 m and AD = 4 m. A particle of weight 45 N is placed on the beam at the point E, where AE = 4 m.

(a) Find the magnitudes of the reactions at C and D. **(5)**

(b) The particle is now moved along the beam towards B. Find the greatest distance from A at which it can be placed so that the beam remains in equilibrium, resting on both supports. **(3)**

(Total for Question 21 is 8 marks)

22 A uniform plank AB has length 3 m and weight 40 N. It rests horizontally on a single support at the point C, where AC = 3/2 m. A load of weight w N is hung from A so that the plank is in equilibrium and about to tilt about C.

(a) Find the value of w. **(3)**

(b) Find the magnitude of the reaction of the support on the plank. **(2)**

(Total for Question 22 is 5 marks)

23 A uniform ladder of length 6 m and weight 100 N rests with its upper end against a smooth vertical wall and its lower end on rough horizontal ground. The ladder makes an angle α with the ground, where $\tan \alpha = 5/12$. A man of weight 150 N stands on the ladder at a distance 3 m from its foot.

(a) Find the magnitude of the normal reaction of the wall on the ladder. **(3)**

(b) Find the magnitude of the friction force at the ground and the normal reaction at the ground. **(4)**

(c) Given that the ladder is in limiting equilibrium, find the coefficient of friction between the ladder and the ground. **(2)**

(Total for Question 23 is 9 marks)

24 A uniform beam AB has length 5 m and weight 210 N. It rests horizontally on two supports at C and D, where AC = 2 m and AD = 3 m. A particle of weight 95 N is placed on the beam at the point E, where AE = 2 m.

(a) Find the magnitudes of the reactions at C and D. **(5)**

(b) The particle is now moved along the beam towards B. Find the greatest distance from A at which it can be placed so that the beam remains in equilibrium, resting on both supports. **(3)**

(Total for Question 24 is 8 marks)

25 A uniform plank AB has length 3 m and weight 60 N. It rests horizontally on a single support at the point C, where AC = 1 m. A load of weight w N is hung from A so that the plank is in equilibrium and about to tilt about C.

(a) Find the value of w. **(3)**

(b) Find the magnitude of the reaction of the support on the plank. **(2)**

(Total for Question 25 is 5 marks)

26 A uniform beam AB has length 12 m and weight 110 N. It rests horizontally on two supports at C and D, where AC = 4 m and AD = 8 m. A particle of weight 60 N is placed on the beam at the point E, where AE = 2 m.

(a) Find the magnitudes of the reactions at C and D. **(5)**

(b) The particle is now moved along the beam towards B. Find the greatest distance from A at which it can be placed so that the beam remains in equilibrium, resting on both supports. **(3)**

(Total for Question 26 is 8 marks)

27 A uniform ladder of length 4 m and weight 200 N rests with its upper end against a smooth vertical wall and its lower end on rough horizontal ground. The ladder makes an angle α with the ground, where $\tan \alpha = 5/12$. A man of weight 100 N stands on the ladder at a distance 4/3 m from its foot.

(a) Find the magnitude of the normal reaction of the wall on the ladder. **(3)**

(b) Find the magnitude of the friction force at the ground and the normal reaction at the ground. **(4)**

(c) Given that the ladder is in limiting equilibrium, find the coefficient of friction between the ladder and the ground. **(2)**

(Total for Question 27 is 9 marks)

28 A uniform beam AB has length 7 m and weight 100 N. It rests horizontally on two supports at C and D, where AC = 3 m and AD = 4 m. A particle of weight 30 N is placed on the beam at the point E, where AE = 5 m.

(a) Find the magnitudes of the reactions at C and D. **(5)**

(b) The particle is now moved along the beam towards B. Find the greatest distance from A at which it can be placed so that the beam remains in equilibrium, resting on both supports. **(3)**

(Total for Question 28 is 8 marks)

29 A uniform plank AB has length 6 m and weight 120 N. It rests horizontally on a single support at the point C, where AC = 2 m. A load of weight w N is hung from A so that the plank is in equilibrium and about to tilt about C.

(a) Find the value of w. **(3)**

(b) Find the magnitude of the reaction of the support on the plank. **(2)**

(Total for Question 29 is 5 marks)

30 A uniform beam AB has length 7 m and weight 220 N. It rests horizontally on two supports at C and D, where AC = 1 m and AD = 4 m. A particle of weight 70 N is placed on the beam at the point E, where AE = 2 m.

(a) Find the magnitudes of the reactions at C and D. **(5)**

(b) The particle is now moved along the beam towards B. Find the greatest distance from A at which it can be placed so that the beam remains in equilibrium, resting on both supports. **(3)**

(Total for Question 30 is 8 marks)

31 A shop sign of weight 80 N hangs in equilibrium from a single point on the sign. Two light inextensible strings are attached to this point, with their other ends fixed to a horizontal ceiling. String 1 makes an angle of 50 degrees with the ceiling, and string 2 makes an angle of 65 degrees with the ceiling, on opposite sides of the vertical through the sign. Model the sign as a particle.

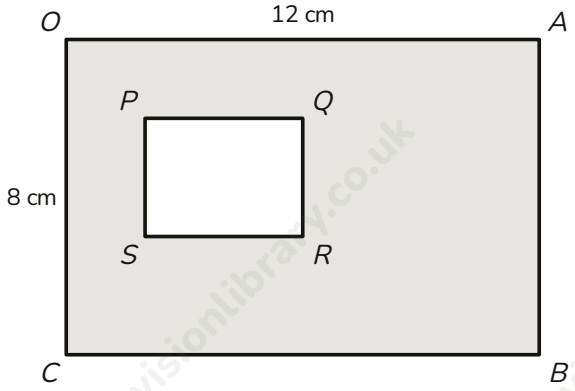
(a) Find the tension in each string, giving your answers to 3 significant figures. **(7)**

(b) The sign is now instead supported by string 1 alone (still at 50 degrees to the ceiling) together with a smooth horizontal strut, fixed to a wall, which pushes horizontally on the sign. Find the tension in string 1 and the force in the strut. **(4)**

(Total for Question 31 is 11 marks)

32 A uniform lamina occupies the region of a rectangle OABC, where O is the origin, OA lies along the x-axis with OA = 12 cm, and OC lies along the y-axis with OC = 8 cm. A rectangular hole PQRS is removed from the lamina, where P = (2, 2), Q = (6, 2), R = (6, 5) and S = (2, 5), all coordinates in cm.

Diagram NOT accurately drawn



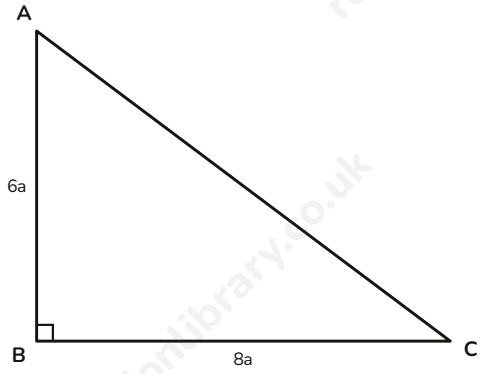
(a) Find the coordinates of the centre of mass of the remaining lamina. **(6)**

(b) The lamina is freely suspended from the point A and hangs in equilibrium. Find the angle that AB makes with the vertical, giving your answer to 3 significant figures. **(3)**

(Total for Question 32 is 9 marks)

33 A framework is formed from three uniform rods AB, BC and CA, rigidly joined at their ends to form a triangle with a right angle at B. The rods are made of the same material, with the same mass per unit length, and AB = 6a, BC = 8a.

Diagram NOT accurately drawn



(a) Show that CA = 10a. **(1)**

(b) Taking B as the origin, with BC along the x-axis and BA along the y-axis, find the coordinates of the centre of mass of the framework, giving your answer in terms of a. **(7)**

(c) The framework is freely suspended from A and hangs in equilibrium. Find, to 3 significant figures, the angle that AB makes with the vertical. **(4)**

(Total for Question 33 is 12 marks)

34 A uniform rod AB has length 2.4 m and weight 100 N. The rod rests with end A on rough horizontal ground and passes over a small smooth peg fixed at the point P on the rod, where AP = 1.6 m. The peg supports the rod from below, exerting a force on the rod perpendicular to the rod. The rod is inclined to the horizontal at an angle θ, where $\tan(\theta) = 3/4$, and rests in equilibrium.

(a) By taking moments about A, show that the normal reaction at the peg, N, has magnitude 60 N. **(5)**

(b) Find the normal reaction at A, N_A. **(3)**

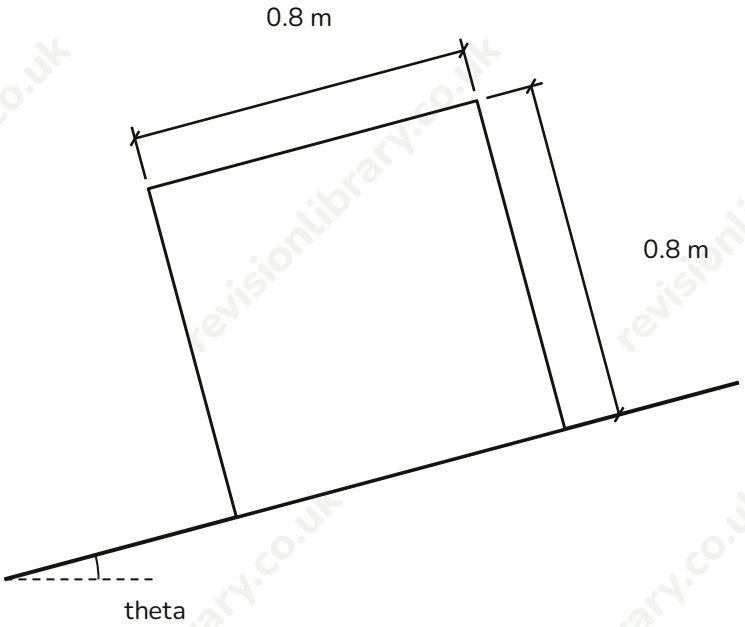
(c) Find the friction force at A. **(2)**

(d) Given that the rod is on the point of slipping at A, find the coefficient of friction between the rod and the ground. **(2)**

(Total for Question 34 is 12 marks)

35 A uniform cube of side 0.8 m rests on a rough plane inclined at angle θ to the horizontal, which is slowly increased from 0. The coefficient of friction between the cube and the plane is 0.5. Find the angle θ at which the cube would be on the point of toppling about its lower edge (assuming it does not slide first), and find the angle at which it would be on the point of sliding (assuming it does not topple first). Hence determine, with justification, whether the cube slides or topples first as θ increases from 0.

Diagram NOT accurately drawn



mu = 0.5

(Total for Question 35 is 4 marks)

Mark scheme · 3.7 Mechanics: Moments and Statics Depth

Question 1

90 cao **B1**

Answer: 90 N

Question 2

0.6 cao **B1**

Answer: 0.6 m

Question 3

24 cao **B1**

Answer: 24 N m

Question 4

resolve vertically: reaction = sum of the three weights (the rod is light) **M1**

82.5 N cao **A1**

Answer: 82.5 N

Question 5

resolve vertically: $T_A + T_B = 90$, with $T_A = 2T_B$, so $3T_B = 90$ **M1**

30 N cao **A1**

Answer: 30 N

Question 6

substitute $T_A = 2T_B$ into $T_A + T_B = 90$ to find $T_B = 30$, then $T_A = 2(30)$ **M1**

60 N cao **A1**

Answer: 60 N

Question 7

recognise that four equal masses at the corners of a rectangle have their centre of mass at the geometric centre **M1**

(2, 5) cao **A1**

Answer: (2, 5)

Question 8

5 cao **B1**

Answer: x = 5 m

Question 9

take moments about C: 25 x 0.8 = 40 x d **M1**

0.5 m cao **A1**

Answer: 0.5 m

Question 10

take moments about C: 15(0.3) + 25(0.5) = 0.4W **M1**

42.5 N cao **A1**

Answer: 42.5 N

Question 11

the rod hangs with its centre of mass directly below O, so the angle OA makes with the vertical satisfies tan(angle) = horizontal distance / vertical distance = 0.3/0.4 **M1**

awrt 36.9 degrees **A1**

Answer: 36.9 degrees (3 s.f.)

Question 12

at the point of toppling, the weight still acts through the centre of the cube, a horizontal distance of half the side (0.25 m) from the toppling edge **M1**

10 N m cao **A1**

Answer: 10 N m

Question 13

take moments about C: 30(0.2) + 50(0.5) = 0.8W **M1**

W = 38.75 N cao **A1**

reaction = 30 + 50 + 38.75 = 118.75 N cao (ft) **A1**

Answer: W = 38.75 N; reaction at pivot = 118.75 N

Question 14

the weight acts at the midpoint (3 m from A, so 2 m from C); D is 5 m from A, so 4 m from C; take moments about C: $R_D(4) = 240(2)$ **M1**

$R_D = 120$ N cao **A1**

resolve vertically: $R_C = 240 - 120 = 120$ N cao (ft) **A1**

Answer: $R_D = 120$ N, $R_C = 120$ N

Question 15

(a) sets up the moments equation about C for the two sides of the pivot **M1**

(a) 40 * 0.25 + 70 * 0.5 = W * 0.6 (i.e. 10 + 35 = 0.6W) **A1**

(a) W = 75 cso **A1**

(a) Answer: W = 75 N (shown)

(b) resolves vertically: R = 40 + 70 + 75 **M1**

(b) R = 185 N cao **A1**

(b) Answer: 185 N

Question 16

(a) takes moments about C, using CD = 3 m and the beam's centre of mass 2 m from C **M1**

(a) $R_D * 3 = 210 * 2$ (correct equation) **A1**

(a) $R_D = 140$ (N) cso **A1**

(a) Answer: $R_D = 140$ N (shown)

(b) resolves vertically: $R_C + R_D = 210$ **M1**

(b) $R_C = 210 - 140$ **A1**

(b) $R_C = 70$ (N) cao **A1**

(b) Answer: 70 N

Question 17

(a) radius = 3 cm (half of the 6 cm diameter); uses area = $0.5*\pi*r^2$ **M1**

(a) = $0.5*\pi*9 = 4.5*\pi$ cm^2 cso **A1**

(a) Answer: $4.5*\pi$ cm^2 (shown)

(b) identifies the rectangle's area (60 cm^2) at distance 5 cm from AB, and the semicircle's area ($4.5*\pi$) at distance $10 + 4*3/(3*\pi)$ from AB **M1**

(b) correct areas and positions listed **A1**

(b) combines: xbar = $(60*5 + 4.5*\pi*(10 + 4/\pi)) / (60 + 4.5*\pi)$ **M1**

(b) xbar = awrt 6.20 (cm) cao **A1**

(b) Answer: awrt 6.20 cm from AB

Question 18

$T_A + T_B = 150$ with $T_A = 2T_B$, so $3T_B = 150$ **M1**

$T_B = 50$ N, $T_A = 100$ N **A1**

take moments about A: $T_B(5) = 150(d)$, where d is the distance of the centre of mass from A **M1**

awrt 1.67 m **A1**

Answer: $T_A = 100$ N, $T_B = 50$ N; centre of mass is 1.67 m from A (3 s.f.)

Question 19

resolve horizontally: T1 cos55 = T2 cos40 **M1**

resolve vertically: T1 sin55 + T2 sin40 = 60, and eliminate T1 (or T2) using the horizontal equation **M1**

awrt T1 = 46.1 N **A1**

awrt T2 = 34.5 N **A1**

Answer: T1 = 46.1 N, T2 = 34.5 N (both 3 s.f.)

Question 20

(a) AC = $\sqrt{9^2+12^2} = \sqrt{225} = 15$, using Pythagoras' theorem, cso **B1**

(a) Answer: AC = 15 cm (shown)

(b) uses the standard result that the centre of mass of a uniform triangular lamina is the average of the coordinates of its vertices **M1**

(b) xbar = (0+0+12)/3 = 4 **A1**

(b) ybar = (9+0+0)/3 = 3 **A1**

(b) Answer: (4, 3) cm

(c) recognises that AG becomes vertical, and that AB is initially vertical in the lamina's own frame, so the required angle is between AG and AB **B1**

(c) finds the horizontal and vertical displacement of G from A **M1**

(c) 4 cm horizontally and 6 cm vertically (or magnitude $2*\sqrt{13}$ cm) **A1**

(c) tan(angle) = 4/6 (oe) **M1**

(c) angle = awrt 33.7 (degrees) cao **A1**

(c) Answer: awrt 33.7 degrees

Question 21

(a) takes moments about C **M1**

(a) $R_D(1) = 230(1/2) + 45(1)$ **A1**

(a) $R_D = 160$ N **A1**

(a) resolves vertically: $R_C + R_D = W + w$ **M1**

(a) $R_C = 115$ N **A1**

(a) Answer: $R_C = 115$ N, $R_D = 160$ N

(b) when the beam is about to tilt about D, $R_C = 0$, so takes moments about D **M1**

(b) $230(4 - 7/2) = 45(y - 4)$ where y is the distance of the particle from A **M1**

(b) y = 59/9 m **A1**

(b) Answer: 59/9 m from A

Question 22

(a) takes moments about C **M1**

(a) $w(3/2) = 40(0)$ **A1**

(a) w = 0 **A1**

(a) Answer: w = 0

(b) resolves vertically **M1**

(b) R = 40 + 0 = 40 N **A1**

(b) Answer: 40 N

Question 23

(a) takes moments about the foot of the ladder **M1**

(a) $N(6) \sin\alpha = 100(3) \cos\alpha + 150(3) \cos\alpha$ **A1**

(a) N = 300 N **A1**

(a) Answer: 300 N

(b) resolves horizontally: F = N **M1**

(b) F = 300 N **A1**

(b) resolves vertically: R = weights **M1**

(b) R = 250 N **A1**

(b) Answer: F = 300 N, R = 250 N

(c) uses $F = \mu R$ in limiting equilibrium **M1**

(c) $\mu = 300/250 = 6/5$ **A1**

(c) Answer: 6/5

Question 24

(a) takes moments about C **M1**

(a) $R_D(1) = 210(1/2) + 95(0)$ **A1**

(a) $R_D = 105$ N **A1**

(a) resolves vertically: $R_C + R_D = W + w$ **M1**

(a) $R_C = 200$ N **A1**

(a) Answer: $R_C = 200$ N, $R_D = 105$ N

(b) when the beam is about to tilt about D, $R_C = 0$, so takes moments about D **M1**

(b) $210(3 - 5/2) = 95(y - 3)$ where y is the distance of the particle from A **M1**

(b) y = 78/19 m **A1**

(b) Answer: 78/19 m from A

Question 25

(a) takes moments about C **M1**

(a) $w(1) = 60(1/2)$ **A1**

(a) w = 30 **A1**

(a) Answer: w = 30

(b) resolves vertically **M1**

(b) R = 60 + 30 = 90 N **A1**

(b) Answer: 90 N

Question 26

(a) takes moments about C **M1**

(a) $R_D(4) = 110(2) + 60(-2)$ **A1**

(a) $R_D = 25$ N **A1**

(a) resolves vertically: $R_C + R_D = W + w$ **M1**

(a) $R_C = 145$ N **A1**

(a) Answer: $R_C = 145$ N, $R_D = 25$ N

(b) when the beam is about to tilt about D, $R_C = 0$, so takes moments about D **M1**

(b) $110(8 - 6) = 60(y - 8)$ where y is the distance of the particle from A **M1**

(b) $y = 35/3$ m **A1**

(b) Answer: 35/3 m from A

Question 27

(a) takes moments about the foot of the ladder **M1**

(a) $N(4) \sin \alpha = 200(2) \cos \alpha + 100(4/3) \cos \alpha$ **A1**

(a) $N = 320$ N **A1**

(a) Answer: 320 N

(b) resolves horizontally: $F = N$ **M1**

(b) $F = 320$ N **A1**

(b) resolves vertically: R = weights **M1**

(b) $R = 300$ N **A1**

(b) Answer: F = 320 N, R = 300 N

(c) uses $F = \mu R$ in limiting equilibrium **M1**

(c) $\mu = 320/300 = 16/15$ **A1**

(c) Answer: 16/15

Question 28

(a) takes moments about C **M1**

(a) $R_D(1) = 100(1/2) + 30(2)$ **A1**

(a) $R_D = 110$ N **A1**

(a) resolves vertically: $R_C + R_D = W + w$ **M1**

(a) $R_C = 20$ N **A1**

(a) Answer: $R_C = 20$ N, $R_D = 110$ N

(b) when the beam is about to tilt about D, $R_C = 0$, so takes moments about D **M1**

(b) $100(4 - 7/2) = 30(y - 4)$ where y is the distance of the particle from A **M1**

(b) $y = 17/3$ m **A1**

(b) Answer: 17/3 m from A

Question 29

(a) takes moments about C **M1**

(a) $w(2) = 120(1)$ **A1**

(a) $w = 60$ **A1**

(a) Answer: w = 60

(b) resolves vertically **M1**

(b) $R = 120 + 60 = 180$ N **A1**

(b) Answer: 180 N

Question 30

(a) takes moments about C **M1**

(a) $R_D(3) = 220(5/2) + 70(1)$ **A1**

(a) $R_D = 620/3$ N **A1**

(a) resolves vertically: $R_C + R_D = W + w$ **M1**

(a) $R_C = 250/3$ N **A1**

(a) Answer: $R_C = 250/3$ N, $R_D = 620/3$ N

(b) when the beam is about to tilt about D, $R_C = 0$, so takes moments about D **M1**

(b) $220(4 - 7/2) = 70(y - 4)$ where y is the distance of the particle from A **M1**

(b) $y = 39/7$ m **A1**

(b) Answer: 39/7 m from A

Question 31

(a) resolves horizontally: T1*cos(50) = T2*cos(65) **M1**

(a) correct equation **A1**

(a) resolves vertically: T1*sin(50) + T2*sin(65) = 80 **M1**

(a) correct equation **A1**

(a) eliminates one tension using the horizontal equation and solves the resulting equation in one unknown **M1**

(a) T2 = awrt 56.7 (N) **A1**

(a) T1 = awrt 37.3 (N) **A1**

(a) Answer: T1 = 37.3 N (string at 50 degrees); T2 = 56.7 N (string at 65 degrees), both 3sf

(b) resolves vertically: T1'*sin(50) = 80 **M1**

(b) T1' = 80/sin(50) = awrt 104 (N) **A1**

(b) resolves horizontally: strut force = T1'*cos(50) **M1**

(b) strut force = awrt 67.1 (N) **A1**

(b) Answer: T1' = 104 N; strut force = 67.1 N (both 3sf)

Question 32

(a) area of the hole = 12 cm^2, with its centroid at (4, 3.5) **B1**

(a) remaining area = 96 - 12 = 84 cm^2 **B1**

(a) xbar = (96*6 - 12*4)/84 **M1**

(a) xbar = 44/7 (awrt 6.29) **A1**

(a) ybar = (96*4 - 12*3.5)/84 **M1**

(a) ybar = 57/14 (awrt 4.07) **A1**

(a) Answer: (44/7, 57/14) cm, i.e. awrt (6.29, 4.07) cm

(b) finds the horizontal and vertical displacement of the centre of mass from A: 40/7 cm and 57/14 cm (or equivalent decimals) **B1**

(b) tan(angle) = (40/7) / (57/14), using AB as the local vertical direction **M1**

(b) angle = awrt 54.5 (degrees) cao **A1**

(b) Answer: awrt 54.5 degrees

Question 33

(a) CA = $\sqrt{(6a)^2+(8a)^2}$ = $\sqrt{100a^2}$ = 10a, using Pythagoras' theorem, cso **B1**

(a) Answer: CA = 10a (shown)

(b) masses (lengths) of the three rods are proportional to 6a, 8a, 10a, with total 24a **B1**

(b) identifies the midpoint of each rod: AB -> (0,3a), BC -> (4a,0), CA -> (4a,3a) **M1**

(b) all three midpoints correct **A1**

(b) xbar = (6a*0 + 8a*4a + 10a*4a)/24a **M1**

(b) xbar = 3a **A1**

(b) ybar = (6a*3a + 8a*0 + 10a*3a)/24a **M1**

(b) ybar = 2a **A1**

(b) Answer: (3a, 2a)

(c) identifies the horizontal and vertical displacement of G from A: 3a horizontally and 6a-2a=4a vertically **B1**

(c) sets up tan(angle) = horizontal/vertical = 3a/4a **M1**

(c) = 3/4 (oe) **A1**

(c) angle = awrt 36.9 (degrees) cao **A1**

(c) Answer: awrt 36.9 degrees

Question 34

(a) recognises that the moment of N about A is N * AP, since N acts perpendicular to the rod at the point P on the rod **M1**

(a) recognises that the moment of the weight about A is W * (AB/2) * cos(θ), since the weight acts vertically at the midpoint of the rod **A1**

(a) uses the 3-4-5 triangle from tan(θ) = 3/4 to state cos(θ) = 4/5 **M1**

(a) forms N * 1.6 = 100 * 1.2 * 4/5 (= 96) **A1**

(a) N = 96/1.6 = 60 cso **A1**

(a) Answer: N = 60 N (shown)

(b) resolves vertically: N_A + N*cos(θ) = 100 **M1**

(b) N_A = 100 - 60*(4/5) = 100 - 48 **A1**

(b) N_A = 52 (N) cao **A1**

(b) Answer: 52 N

(c) resolves horizontally: F_A = N*sin(θ) **M1**

(c) F_A = 60*(3/5) = 36 (N) cao **A1**

(c) Answer: 36 N

(d) μ = F_A / N_A **M1**

(d) μ = 36/52 = 9/13 (awrt 0.692) cao **A1**

(d) Answer: μ = 9/13 (awrt 0.692)

Question 35

for a cube, toppling occurs when the line of action of the weight passes through the lower edge, i.e. when tan(θ) = (half the side)/(half the side) = 1 **M1**

toppling angle = 45 degrees **A1**

sliding occurs when tan(θ) = μ = 0.5 **M1**

sliding angle = awrt 26.6 degrees; since 26.6 < 45, the cube slides first **A1**

Answer: Toppling angle = 45 degrees; sliding angle = 26.6 degrees (3 s.f.); the cube slides before it would topple