3.8 Mechanics: Projectiles and Applications Depth: 1-hour Pack

Total Marks

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: projectile motion and combining kinematics with forces

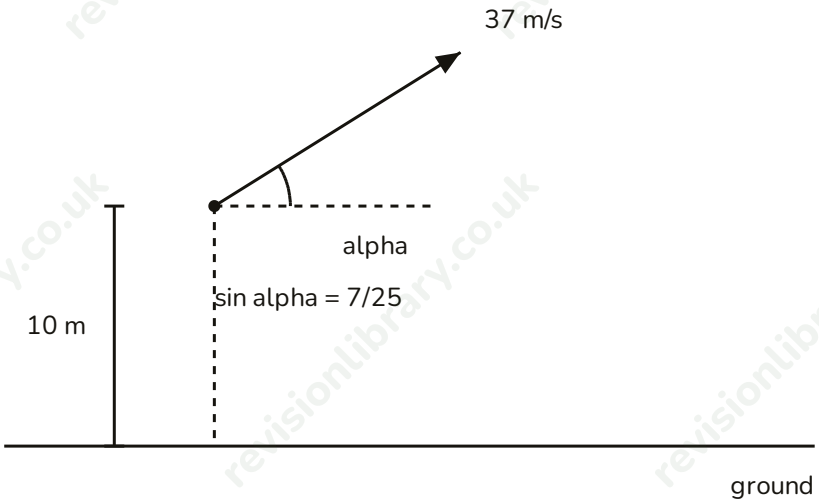
Original content written for Revision Library.

Unless a question states otherwise, take g = 9.8 m/s^2 and model every object as a particle moving freely under gravity once released, so that gravity is the only force acting during the flight itself. For a particle projected from a point on horizontal ground with speed U at angle alpha above the horizontal, resolve the initial velocity into a horizontal component U cos(alpha), which is constant throughout the flight, and a vertical component U sin(alpha), which reduces uniformly under gravity; the horizontal and vertical motions can then be treated independently using the suvat equations, and combined using Pythagoras' theorem and trigonometry to find the resultant speed and direction at any instant. Two standard results you may quote without proof, unless a question asks you to derive them, are the time of flight on level ground, T = 2Usin(alpha)/g, and the range on level ground, R = U^2 sin(2alpha)/g; the greatest height reached is H = U^2 sin^2(alpha)/(2g). Several questions in this worksheet combine projectile motion with forces: where a particle or block is first accelerated by gravity and/or friction while in contact with a surface (for example sliding down a rough inclined plane, or being pulled by a connected particle over a pulley), use Newton's second law, F = ma, resolving forces parallel and perpendicular to the relevant surface, to find its speed at the point where it becomes a projectile, and then apply projectile motion to the remainder of its path. Vectors are given in terms of perpendicular unit vectors i and j; a position vector r(t) for a projectile can be written in terms of t by integrating, or by using the suvat equations componentwise, with constant acceleration -gj.

© Revision Library | revisionlibrary.co.uk

1 A ball is projected from a point 10 m above horizontal ground with speed 37 m/s at an angle α above the horizontal, where $\sin \alpha = 7/25$. The ball moves freely under gravity and is modelled as a particle. Take $g = 9.8 \text{ m/s}^2$.

Diagram NOT accurately drawn



(a) Find the horizontal and vertical components of the initial velocity. **(2)**

(b) Find the greatest height of the ball above the ground, to 3 significant figures. **(3)**

(c) Find the time of flight and the horizontal distance travelled before the ball hits the ground, each to 3 significant figures. **(4)**

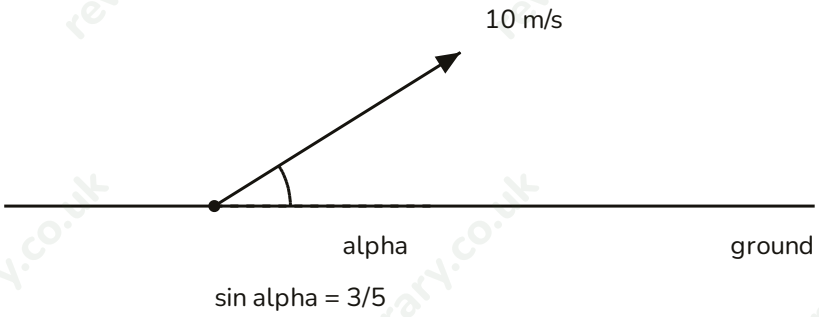
(d) Find the speed of the ball 2 s after projection, and the angle its direction of motion makes with the horizontal, to 3 significant figures and the nearest degree. **(3)**

(Total for Question 1 is 12 marks)

© Revision Library | revisionlibrary.co.uk

2 A ball is projected from ground level with speed 10 m/s at an angle α above the horizontal, where sin α = 3/5. The ball moves freely under gravity and is modelled as a particle. Take g = 9.8 m/s^2.

Diagram NOT accurately drawn



(a) Find the horizontal and vertical components of the initial velocity. **(2)**

(b) Find the greatest height of the ball above the ground, to 3 significant figures. **(3)**

(c) Find the time of flight and the horizontal distance travelled before the ball hits the ground, each to 3 significant figures. **(4)**

(d) Find the speed of the ball 1 s after projection, and the angle its direction of motion makes with the horizontal, to 3 significant figures and the nearest degree. **(3)**

(Total for Question 2 is 12 marks)

---

3 A stone is thrown horizontally with speed 8 m/s from a bridge 19.6 m above a river, and moves freely under gravity until it lands in the river. Take g = 9.8 m/s^2. Find the horizontal distance from the bridge at which the stone lands, giving your answer exactly.

(Total for Question 3 is 2 marks)

© Revision Library | revisionlibrary.co.uk

4 A particle is projected from a point on horizontal ground with speed U at an angle α above the horizontal, and moves freely under gravity. Take g = 9.8 m/s^2.

(a) By considering the vertical component of the initial velocity, show that the greatest height reached by the particle is H = U^2 sin^2(α) / (2g). **(3)**

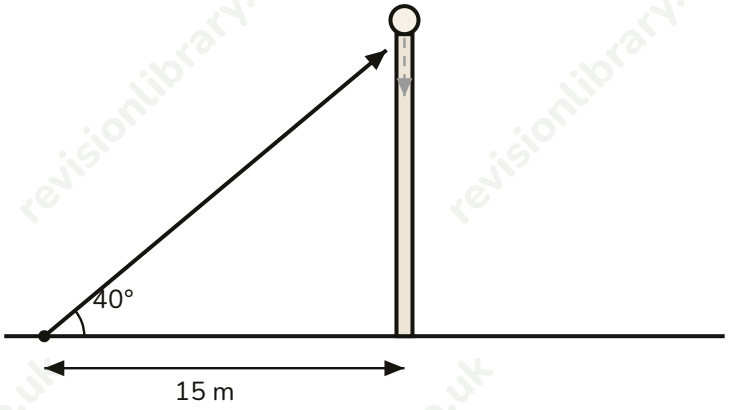
(b) Given that U = 26 m/s and α = 38 degrees, find the greatest height reached, giving your answer to 3 significant figures. **(2)**

(c) By first finding the time taken to reach the greatest height, use a different suvat equation to verify your value of H from part (b). **(3)**

(Total for Question 4 is 8 marks)

---

5 Two students, Ryan and Sofia, are experimenting in a field. At the instant Ryan throws a beanbag from ground level with speed 25 m/s at an angle of 40 degrees above the horizontal, aimed directly at a second beanbag balanced on top of a vertical post standing at a horizontal distance of 15 m from Ryan, in the vertical plane of the throw, Sofia releases that second beanbag from rest, so that it falls freely under gravity. Both beanbags are modelled as particles moving freely under gravity, and they collide in mid-air. Take g = 9.8 m/s^2.



(a) By setting up expressions for the horizontal and vertical displacements of each beanbag, and using the fact that Ryan's beanbag is aimed directly at the second beanbag's initial position, find the height of the post, giving your answer to 3 significant figures. **(5)**

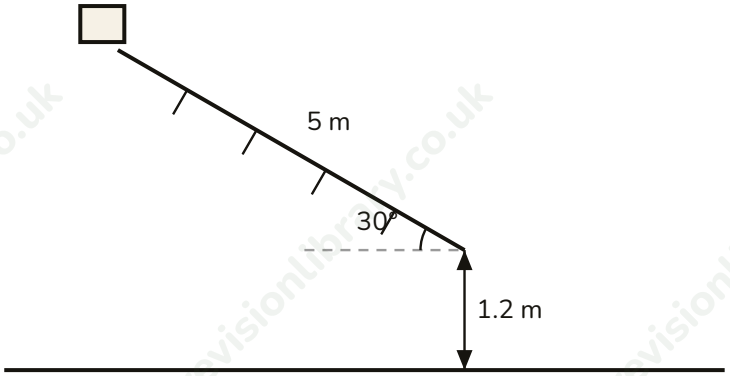
(b) Find the time after the beanbags are released at which they collide, giving your answer to 3 significant figures. **(2)**

(c) State two modelling assumptions, beyond treating each beanbag as a particle moving freely under gravity, that are needed for the collision to occur exactly as described. **(2)**

(Total for Question 5 is 9 marks)

© Revision Library | revisionlibrary.co.uk

6 A small block of mass 2 kg is released from rest at the top of a rough plane inclined at 30 degrees to the horizontal. The coefficient of friction between the block and the plane is 0.2. The plane has length 5 m, measured along a line of greatest slope, and the bottom edge of the plane is 1.2 m above horizontal ground. Beyond this edge, the block leaves the plane and moves freely under gravity as a projectile until it lands on the ground. Take $g = 9.8\ m/s^2$.



(a) By resolving forces perpendicular and parallel to the plane, show that the acceleration of the block down the plane is given by $a = g(\sin(30) - 0.2\cos(30))$. **(3)**

(b) Find the value of this acceleration, giving your answer to 3 significant figures. **(2)**

(c) Find the speed of the block at the instant it leaves the plane, giving your answer to 3 significant figures. **(2)**

(d) At the instant the block leaves the plane, its velocity is directed at 30 degrees below the horizontal, with the magnitude found in part (c). Find the horizontal distance from the point where the block leaves the plane to the point where it lands on the ground, giving your answer to 3 significant figures. **(6)**

(Total for Question 6 is 13 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 3.8 Mechanics: Projectiles and Applications Depth: 1-hour Pack

Question 1

(a) resolves: U cos α and U sin α	**M1**
(a) horizontal 35.5 m/s, vertical 10.4 m/s	**A1**
(a) Answer: 35.5 m/s and 10.4 m/s	
(b) uses $v^2 = u^2$ - 2gs for the vertical motion with v = 0	**M1**
(b) s = 5.476 m above the point of projection	**A1**
(b) greatest height = 15.5 m	**A1**
(b) Answer: 15.5 m	
(c) vertical displacement -10 = 10.360t - $4.9t^2$	**M1**
(c) T = 2.83 s	**A1**
(c) horizontal distance = ux x T	**M1**
(c) range = 101 m	**A1**
(c) Answer: T = 2.83 s, range = 101 m	
(d) vertical velocity = 10.360 - 9.8 x 2 = -9.240	**M1**
(d) speed = 36.7 m/s	**A1**
(d) angle = 15 degrees below the horizontal	**A1**
(d) Answer: 36.7 m/s, 15 degrees below the horizontal	

Question 2

(a) resolves: U cos α and U sin α	**M1**
(a) horizontal 8.00 m/s, vertical 6.00 m/s	**A1**
(a) Answer: 8.00 m/s and 6.00 m/s	
(b) uses $v^2 = u^2$ - 2gs for the vertical motion with v = 0	**M1**
(b) s = 1.837 m above the point of projection	**A1**
(b) greatest height = 1.84 m	**A1**
(b) Answer: 1.84 m	
(c) s = 0: 6.000t - $4.9t^2$ = 0	**M1**
(c) T = 1.22 s	**A1**
(c) horizontal distance = ux x T	**M1**
(c) range = 9.80 m	**A1**
(c) Answer: T = 1.22 s, range = 9.80 m	
(d) vertical velocity = 6.000 - 9.8 x 1 = -3.800	**M1**
(d) speed = 8.86 m/s	**A1**
(d) angle = 25 degrees below the horizontal	**A1**
(d) Answer: 8.86 m/s, 25 degrees below the horizontal	

Question 3

uses s = $4.9t^2$ = 19.6 to find the time of fall, t = 2 (s)	**M1**
16 (m), cao	**A1**
Answer: 16 m (exact)	

Question 4

(a) identifies the initial vertical component of velocity, u_y = Usin(α)	**M1**
(a) uses $v^2 = u^2$ - 2gs with v = 0 (at the greatest height) and their u_y	**M1**
(a) rearranges to H = $U^2 \sin^2(\alpha)/(2g)$, cso	**A1**
(a) Answer: H = $U^2 \sin^2(\alpha) / (2g)$ (shown)	
(b) substitutes U = 26, α = 38 into H = $U^2 \sin^2(\alpha)/(2g)$	**M1**
(b) H = awrt 13.1 (m), cao	**A1**
(b) Answer: H = awrt 13.1 m	
(c) finds the time to greatest height, t = u_y/g = 26sin(38)/9.8, ft	**M1**
(c) uses a different suvat equation, e.g. s = ((u+v)/2)t with u = u_y, v = 0, and their t	**M1**
(c) H = awrt 13.1 (m), matching (b), ft	**A1**
(c) Answer: H = awrt 13.1 m (verified)	

Question 5

(a) horizontal displacement of the thrown beanbag: x = 25cos(40) t	**M1**
(a) vertical displacements: y1 = 25sin(40) t - $4.9t^2$ (thrown beanbag) and y2 = h - $4.9t^2$ (falling beanbag, released from height h)	**M1**
(a) uses the fact that Ryan aims directly at the post-top, i.e. tan(40) = h/15, or equivalently finds t = 15/(25cos(40)) from x = 15	**M1**
(a) sets y1 = y2 at the collision instant; the -$4.9t^2$ terms cancel, giving h = 25sin(40) x t = 15tan(40)	**M1**
(a) h = awrt 12.6 (m)	**A1**
(a) Answer: h = awrt 12.6 m	
(b) t = 15/(25cos(40)), ft	**M1**
(b) awrt 0.783 (s)	**A1**
(b) Answer: t = awrt 0.783 s	
(c) the second beanbag is released from rest with no horizontal velocity, so it falls vertically downward only	**B1**
(c) nothing obstructs the path of either beanbag before the collision point, and the beanbags are released and thrown at precisely the same instant	**B1**
(c) Answer: E.g. the released beanbag falls with no horizontal velocity; there is no obstruction between the two beanbags before they collide	

© Revision Library | revisionlibrary.co.uk

Question 6

(a) resolves perpendicular to the plane: R = 2g cos(30) **M1**

(a) resolves along the plane (down positive), using F = 0.2R: 2g sin(30) - 0.2(2g cos(30)) = 2a **M1**

(a) mass cancels throughout, giving a = g(sin(30) - 0.2cos(30)), cso **A1**

(a) Answer: a = g(sin(30) - 0.2cos(30)) (shown)

(b) substitutes g = 9.8 and evaluates sin(30), cos(30) **M1**

(b) a = awrt 3.20 (m/s^2), cao **A1**

(b) Answer: a = awrt 3.20 m/s^2

(c) uses $v^2 = u^2 + 2as$ with u = 0, s = 5, and their a from (a)/(b), ft **M1**

(c) v = awrt 5.66 (m/s), cao **A1**

(c) Answer: v = awrt 5.66 m/s

(d) resolves the velocity into components: $u_x = v \cos(30)$, $u_y = v \sin(30)$, ft their v from (c) **M1**

(d) u_x = awrt 4.90 (m/s) and u_y = awrt 2.83 (m/s) **A1**

(d) sets up the vertical equation of motion for the 1.2 m fall: $1.2 = u_y t + 4.9t^2$, oe **M1**

(d) t = awrt 0.284 (s) **A1**

(d) horizontal distance = $u_x \times t$, ft **M1**

(d) awrt 1.39 (m) **A1**

(d) Answer: awrt 1.39 m

© Revision Library | revisionlibrary.co.uk