3.8 Mechanics: Projectiles and Applications Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: projectile motion and combining kinematics with forces

Original content written for Revision Library.

Unless a question states otherwise, take g = 9.8 m/s^2 and model every object as a particle moving freely under gravity once released, so that gravity is the only force acting during the flight itself. For a particle projected from a point on horizontal ground with speed U at angle alpha above the horizontal, resolve the initial velocity into a horizontal component U cos(alpha), which is constant throughout the flight, and a vertical component U sin(alpha), which reduces uniformly under gravity; the horizontal and vertical motions can then be treated independently using the suvat equations, and combined using Pythagoras' theorem and trigonometry to find the resultant speed and direction at any instant. Two standard results you may quote without proof, unless a question asks you to derive them, are the time of flight on level ground, T = 2Usin(alpha)/g, and the range on level ground, R = U^2 sin(2alpha)/g; the greatest height reached is H = U^2 sin^2(alpha)/(2g). Several questions in this worksheet combine projectile motion with forces: where a particle or block is first accelerated by gravity and/or friction while in contact with a surface (for example sliding down a rough inclined plane, or being pulled by a connected particle over a pulley), use Newton's second law, F = ma, resolving forces parallel and perpendicular to the relevant surface, to find its speed at the point where it becomes a projectile, and then apply projectile motion to the remainder of its path. Vectors are given in terms of perpendicular unit vectors i and j; a position vector r(t) for a projectile can be written in terms of t by integrating, or by using the suvat equations componentwise, with constant acceleration -gj.

1 Find the horizontal range of the ball, giving your answer to 3 significant figures.

(Total for Question 1 is 3 marks)

---

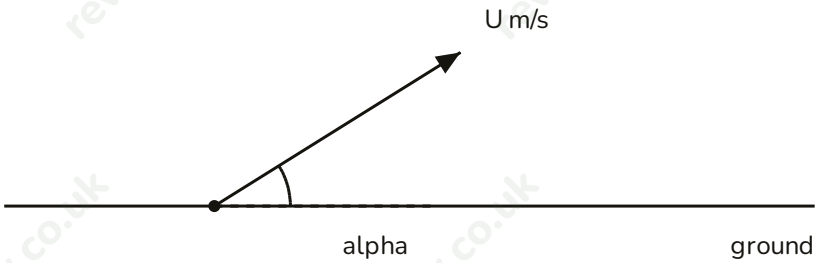
2 State the acceleration acting on a particle at every instant after it has been projected and is moving freely under gravity, giving your answer in the form ...j, using g = 9.8 m/s^2 and taking j as the vertically upward unit vector.

(Total for Question 2 is 1 mark)

© Revision Library | revisionlibrary.co.uk

3 A particle is projected from a point on horizontal ground with speed U m/s at an angle α above the horizontal, where $0 < \alpha < 90$, and lands on the same horizontal ground. You may quote the result that the horizontal range is $R = U^2 \sin(2\text{alpha})/g$ without further proof.

Diagram NOT accurately drawn



(a) Explain why, for a fixed value of U, the range R is greatest when $\alpha = 45$ degrees. **(2)**

(b) Given that U = 19.6 m/s, find the maximum possible range, giving your answer exactly. **(2)**

(c) In practice, sports scientists modelling the flight of a thrown object such as a javelin or a shot do not always find that the optimal launch angle for maximum distance is 45 degrees. Suggest two features of a real thrown object that this projectile model ignores, and for each one, briefly explain the effect it would have on the actual range achieved compared with the range predicted by the model. **(4)**

(Total for Question 3 is 8 marks)

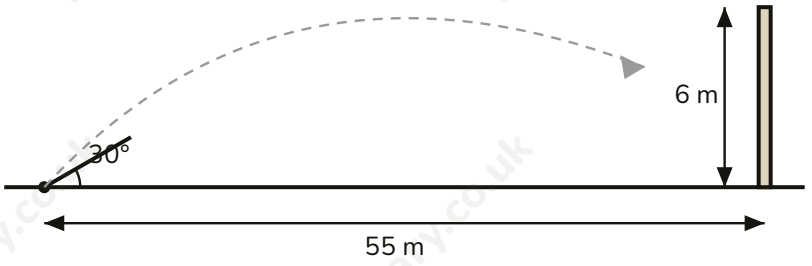
© Revision Library | revisionlibrary.co.uk

4 Discuss the validity of this projectile model for a real rugby conversion kick. Refer to at least three specific features of the real situation that the model does not take into account, and explain how each feature would affect the accuracy of the model's predicted range and/or time of flight.

(Total for Question 4 is 5 marks)

---

5 A cricketer, Amara, strikes a ball from ground level so that it leaves the bat with speed 27 m/s at an angle of 30 degrees above the horizontal, directly towards a vertical sight screen of height 6 m, which stands at a horizontal distance of 55 m from the point where the ball is struck, in the vertical plane of the ball's motion. The ball is modelled as a particle moving freely under gravity. Take $g = 9.8$ m/s^2.



(a) Find the horizontal component of the ball's initial velocity, giving your answer to 3 significant figures. **(2)**

(b) Find the time taken for the ball to travel the 55 m horizontal distance to the sight screen, giving your answer to 3 significant figures. **(2)**

(c) Find the height of the ball above the ground at this instant, giving your answer to 3 significant figures. **(3)**

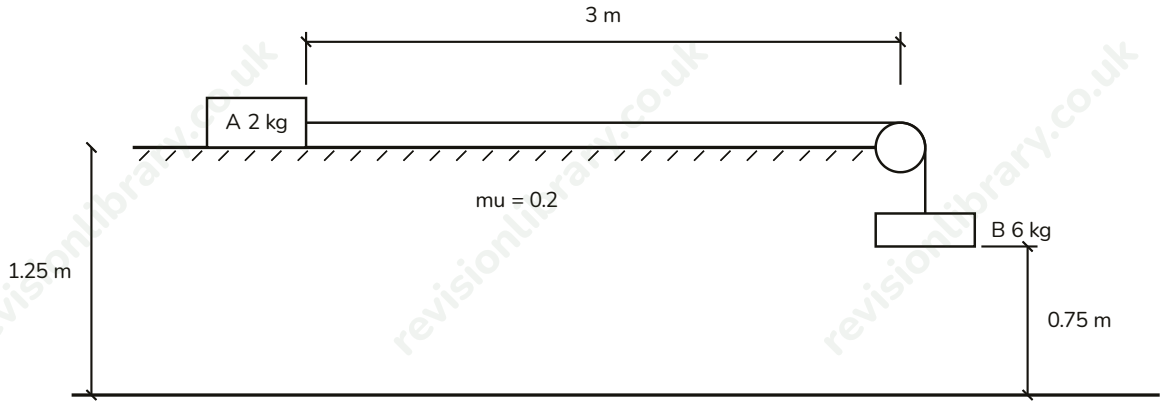
(d) Hence determine, with justification, whether the ball passes over the sight screen or strikes it. **(2)**

(Total for Question 5 is 9 marks)

© Revision Library | revisionlibrary.co.uk

6 In a warehouse, a technician called Tom sets up a demonstration. Block A, of mass 2 kg, lies on a rough horizontal table, 3 m from a smooth light pulley fixed at the edge of the table. A light inextensible string connects A, over the pulley, to block B, of mass 6 kg, which hangs freely 0.75 m above the floor. The coefficient of friction between A and the table is 0.2. The table edge is 1.25 m above the floor. The system is released from rest with the string taut, and B descends, pulling A towards the pulley. Take $g = 9.8\ m/s^2$.

Diagram NOT accurately drawn



(a) Show that, while both blocks are moving and the string remains taut, the acceleration of the system is $6.86\ m\ s^{-2}$. **(3)**

(b) Find the tension in the string while both blocks are moving. **(2)**

(c) Given that B strikes the floor after falling 0.75 m, find the speed of the blocks immediately before B strikes the floor, giving your answer to 3 significant figures. **(3)**

(d) Once B lands, it remains on the floor and the string becomes slack. Find the speed of A as it reaches the edge of the table, given that A has a further 2.25 m to travel and now decelerates under friction alone, giving your answer to 3 significant figures. **(3)**

(e) A leaves the table edge horizontally with this speed and falls to the floor, which is 1.25 m below the table edge. Find the horizontal distance travelled by A from the table edge to the point where it lands, giving your answer to 3 significant figures. **(3)**

(Total for Question 6 is 14 marks)

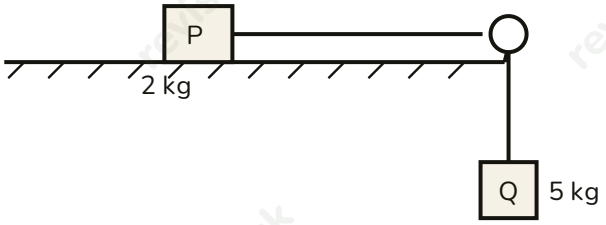
© Revision Library | revisionlibrary.co.uk

7 A ball is thrown horizontally with speed 14 m/s from the top of a cliff 44.1 m above the sea, which may be modelled as a horizontal plane. The ball moves freely under gravity. Take g = 9.8 m/s^2. Find the speed of the ball as it hits the sea, giving your answer to 3 significant figures.

(Total for Question 7 is 3 marks)

---

8 Block P, of mass 2 kg, lies on a rough horizontal table. A light inextensible string connects P, over a smooth light pulley fixed at the edge of the table, to block Q, of mass 5 kg, which hangs freely. The coefficient of friction between P and the table is 0.25. The system is released from rest with the string taut, and Q descends, pulling P towards the pulley. Take g = 9.8 m/s^2. Find the acceleration of the system, and the tension in the string, while both blocks are moving.



(Total for Question 8 is 4 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 3.8 Mechanics: Projectiles and Applications Depth: Test Pack

Question 1

substitutes U = 18, α = 55 into R = U^2 sin(2alpha)/g (oe) **M1**

correct unsimplified expression, e.g. R = 324 sin(110)/9.8 **A1**

R = awrt 31.1 (m), cao **A1**

Answer: R = awrt 31.1 m

Question 2

-9.8j cao **B1**

Answer: -9.8j m/s^2

Question 3

(a) sin(2alpha) has its greatest possible value, 1, when 2alpha = 90 degrees, since the sine function has maximum value 1 **B1**

(a) hence α = 45 degrees gives the maximum range, for a fixed value of U **B1**

(a) Answer: R is greatest when α = 45 degrees, since this makes sin(2alpha) = sin(90) = 1, its maximum possible value

(b) substitutes α = 45 into R = U^2 sin(90)/g = U^2/g **M1**

(b) R = 39.2 (m), exact, cao **A1**

(b) Answer: R = 39.2 m (exact)

(c) feature 1, e.g. air resistance (drag) on the object is ignored by the model **B1**

(c) effect of feature 1, e.g. including drag would reduce the object's horizontal velocity throughout the flight, so the actual range would be smaller than the model predicts, and the optimal angle would typically be less than 45 degrees **B1**

(c) feature 2, e.g. the object is released above ground level (from the thrower's hand or the kicker's boot), not from ground level as the model assumes; or the object's shape or spin can generate aerodynamic lift **B1**

(c) effect of feature 2, e.g. a release height above ground level means the object stays airborne longer than the model predicts, increasing the actual range for the same speed and angle; or aerodynamic lift would increase the range and change the optimal angle **B1**

(c) Answer: E.g. air resistance (reduces range, lowers optimal angle) and release height above ground (increases range for the same speed and angle)

Question 4

Level 1 Identifies one or two relevant features of the real kick that the model ignores, with little or no discussion of their effect on the modelled flight. **1-2**

Level 2 Identifies at least three relevant features and gives a generally correct explanation of the effect of at least one on the accuracy of the model, with some imprecision. **3-4**

Level 3 Identifies at least three relevant features and explains clearly and correctly how each affects the accuracy of the model's predicted range and/or time of flight, using appropriate mechanics reasoning throughout. **5**

Indicative content:

- the ball is modelled as a particle, so its size, shape and rotation (e.g. spin imparted by the kicker) are ignored
- air resistance (drag) is assumed negligible, but a real rugby ball experiences significant air resistance, which would reduce its range and time of flight compared with the model's prediction
- the ball is released above ground level, from the kicking tee or the kicker's boot, rather than from ground level as the model assumes, so the true range would be greater than predicted for the same speed and angle
- a spinning or asymmetric ball can experience aerodynamic forces (such as swerve or additional lift) that are not included in a simple projectile model
- wind is not accounted for in the model, but a real kick can be significantly affected by a headwind, tailwind or crosswind, changing both the range and the direction of the ball's flight
- g is assumed constant throughout the flight, which is a reasonable assumption given the heights and distances involved, so this assumption has negligible effect on the comparison

Question 5

(a) u_x = 27cos(30) (oe) **M1**

(a) awrt 23.4 (m/s) **A1**

(a) Answer: u_x = awrt 23.4 m/s

(b) t = 55/u_x, ft (a) **M1**

(b) awrt 2.35 (s) **A1**

(b) Answer: t = awrt 2.35 s

(c) identifies the initial vertical component u_y = 27sin(30) = 13.5 **M1**

(c) substitutes into y = u_y t - 4.9t^2, using their t from (b), dependent on the previous mark **dM1**

(c) awrt 4.64 (m) **A1**

(c) Answer: awrt 4.64 m

(d) compares their height from (c) with the screen height of 6 m **M1**

(d) correct conclusion with reason: since 4.64 m < 6 m, the ball is below the top of the screen at this horizontal distance, so it strikes the sight screen rather than passing over it, ft **A1**

(d) Answer: The ball strikes the sight screen (its height there, awrt 4.64 m, is less than the screen's height of 6 m)

© Revision Library | revisionlibrary.co.uk

Question 6

(a) equation of motion for B (down positive): $6g - T = 6a$ **M1**

(a) equation of motion for A: $T - 0.2(2)(9.8) = 2a$, using $R = 2g$ for the normal reaction on the horizontal table **M1**

(a) adds the two equations to eliminate T; mass simplifies to give $a = 6.86$, cso **A1**

(a) Answer: a = 6.86 m s^{-2} (shown)

(b) substitutes $a = 6.86$ into either equation of motion **M1**

(b) $T = 17.64$ (N), cao **A1**

(b) Answer: T = 17.64 N

(c) uses $v1^2 = u^2 + 2as$ with $u = 0$, $s = 0.75$, and $a = 6.86$ **M1**

(c) $v1^2 = 10.29$, unsimplified **A1**

(c) v1 = awrt 3.21 (m/s) **A1**

(c) Answer: v1 = awrt 3.21 m/s

(d) new deceleration $a2 = 0.2(9.8) = 1.96$ (m/s^2), since the normal reaction and hence friction on A are unchanged once the string is slack **B1**

(d) uses $v2^2 = v1^2 - 2(1.96)(2.25)$, ft their v1 from (c), dependent on the previous mark **dM1**

(d) v2 = awrt 1.21 (m/s) **A1**

(d) Answer: v2 = awrt 1.21 m/s

(e) uses $1.25 = 4.9t^2$ (or equivalent) to find the time of fall **M1**

(e) horizontal distance = v2 x t, ft their v2 from (d) **M1**

(e) awrt 0.612 (m) **A1**

(e) Answer: awrt 0.612 m

Question 7

uses $s = 4.9t^2 = 44.1$ to find the time of fall, $t = 3$ (s) **M1**

finds the vertical speed at impact $v_y = gt = 9.8 \times 3 = 29.4$, and combines with the horizontal speed using Pythagoras, speed = $\sqrt{14^2 + 29.4^2}$ **M1**

awrt 32.6 (m/s), cao **A1**

Answer: 32.6 m/s (3sf)

Question 8

equation of motion for Q (down positive): $5g - T = 5a$ **M1**

equation of motion for P: $T - 0.25(2)(9.8) = 2a$, i.e. $T - 4.9 = 2a$, using $R = 2g$ for the normal reaction on the table **M1**

adds the two equations to eliminate T and solves: $a = 6.3$ (m/s^2), cao **A1**

substitutes back to find $T = 17.5$ (N), cao (ft) **A1**

Answer: a = 6.3 m/s^2; T = 17.5 N

© Revision Library | revisionlibrary.co.uk