3.8 Mechanics: Projectiles and Applications Depth

EDEXCEL 9MA0 · Calculator allowed · about 230 minutes · Write-in edition

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Key results: projectile motion and combining kinematics with forces

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Unless a question states otherwise, take g = 9.8 m/s^2 and model every object as a particle moving freely under gravity once released, so that gravity is the only force acting during the flight itself. For a particle projected from a point on horizontal ground with speed U at angle alpha above the horizontal, resolve the initial velocity into a horizontal component U cos(alpha), which is constant throughout the flight, and a vertical component U sin(alpha), which reduces uniformly under gravity; the horizontal and vertical motions can then be treated independently using the suvat equations, and combined using Pythagoras' theorem and trigonometry to find the resultant speed and direction at any instant. Two standard results you may quote without proof, unless a question asks you to derive them, are the time of flight on level ground, T = 2Usin(alpha)/g, and the range on level ground, R = U^2 sin(2alpha)/g; the greatest height reached is H = U^2 sin^2(alpha)/(2g). Several questions in this worksheet combine projectile motion with forces: where a particle or block is first accelerated by gravity and/or friction while in contact with a surface (for example sliding down a rough inclined plane, or being pulled by a connected particle over a pulley), use Newton's second law, F = ma, resolving forces parallel and perpendicular to the relevant surface, to find its speed at the point where it becomes a projectile, and then apply projectile motion to the remainder of its path. Vectors are given in terms of perpendicular unit vectors i and j; a position vector r(t) for a projectile can be written in terms of t by integrating, or by using the suvat equations componentwise, with constant acceleration -gj.

1 A particle P moves in a straight line with constant acceleration. It passes a point A with speed 17 m/s and reaches a point B with speed 35 m/s, after travelling a distance of 156 m from A.

(a) Find the acceleration. **(3)**

(b) Find the time taken to travel from A to B. **(2)**

(c) Find the distance travelled in the 3rd second after passing A. **(3)**

(Total for Question 1 is 8 marks)

2 At time t = 0 a particle P has position vector (-5j) m relative to a fixed origin O, and velocity (3i - 4j) m/s. P moves with constant acceleration (3i + 2j) m/s^2.

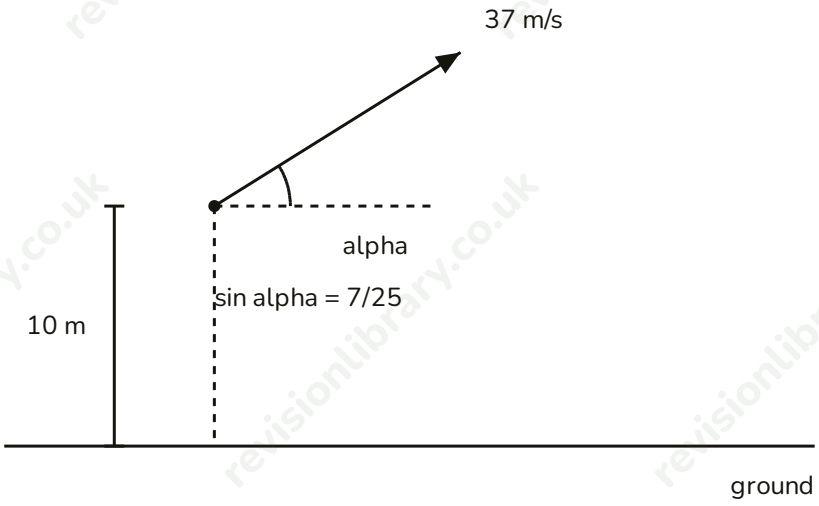
(a) Find the velocity of P when t = 4, and its speed, to 3 significant figures. **(3)**

(b) Find the position vector of P when t = 4. **(2)**

(Total for Question 2 is 5 marks)

3 A ball is projected from a point 10 m above horizontal ground with speed 37 m/s at an angle α above the horizontal, where $\sin \alpha = 7/25$. The ball moves freely under gravity and is modelled as a particle. Take $g = 9.8$ m/s^2.

Diagram NOT accurately drawn



(a) Find the horizontal and vertical components of the initial velocity. **(2)**

(b) Find the greatest height of the ball above the ground, to 3 significant figures. **(3)**

(c) Find the time of flight and the horizontal distance travelled before the ball hits the ground, each to 3 significant figures. **(4)**

(d) Find the speed of the ball 2 s after projection, and the angle its direction of motion makes with the horizontal, to 3 significant figures and the nearest degree. **(3)**

(Total for Question 3 is 12 marks)

4 A stone is projected horizontally with speed 14 m/s from the top of a vertical cliff, at a point 44.1 m above horizontal ground. The motion is modelled as that of a particle moving freely under gravity. Take $g = 9.8$ m/s^2.

(a) Find the time taken to reach the ground. **(2)**

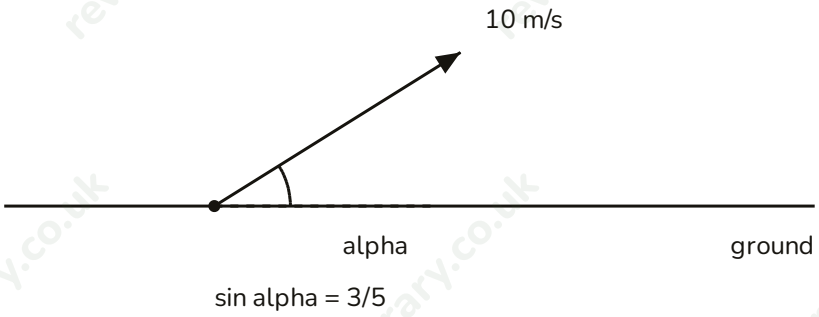
(b) Find the horizontal distance from the point of projection to the point where it lands. **(2)**

(c) Find the speed and direction of motion at the instant before it lands, to 3 significant figures and the nearest degree. **(4)**

(Total for Question 4 is 8 marks)

5 A ball is projected from ground level with speed 10 m/s at an angle α above the horizontal, where sin α = 3/5. The ball moves freely under gravity and is modelled as a particle. Take g = 9.8 m/s^2.

Diagram NOT accurately drawn



(a) Find the horizontal and vertical components of the initial velocity. **(2)**

(b) Find the greatest height of the ball above the ground, to 3 significant figures. **(3)**

(c) Find the time of flight and the horizontal distance travelled before the ball hits the ground, each to 3 significant figures. **(4)**

(d) Find the speed of the ball 1 s after projection, and the angle its direction of motion makes with the horizontal, to 3 significant figures and the nearest degree. **(3)**

(Total for Question 5 is 12 marks)

6 Find the value of U, giving your answer to 3 significant figures.

(Total for Question 6 is 3 marks)

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7 A cyclist moves in a straight line with constant acceleration. It passes a point A with speed 11 m/s and reaches a point B with speed 26 m/s, after travelling a distance of 92.5 m from A.

(a) Find the acceleration. **(3)**

(b) Find the time taken to travel from A to B. **(2)**

(c) Find the distance travelled in the 3rd second after passing A. **(3)**

(Total for Question 7 is 8 marks)

8 A stone is projected horizontally with speed 30 m/s from the top of a vertical cliff, at a point 29.4 m above horizontal ground. The motion is modelled as that of a particle moving freely under gravity. Take $g = 9.8$ m/s^2.

(a) Find the time taken to reach the ground. **(2)**

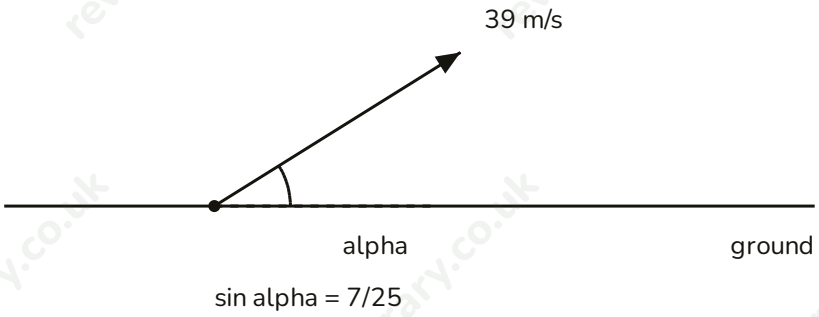
(b) Find the horizontal distance from the point of projection to the point where it lands. **(2)**

(c) Find the speed and direction of motion at the instant before it lands, to 3 significant figures and the nearest degree. **(4)**

(Total for Question 8 is 8 marks)

9 A ball is projected from ground level with speed 39 m/s at an angle α above the horizontal, where sin α = 7/25. The ball moves freely under gravity and is modelled as a particle. Take g = 9.8 m/s^2.

Diagram NOT accurately drawn



(a) Find the horizontal and vertical components of the initial velocity. **(2)**

(b) Find the greatest height of the ball above the ground, to 3 significant figures. **(3)**

(c) Find the time of flight and the horizontal distance travelled before the ball hits the ground, each to 3 significant figures. **(4)**

(d) Find the speed of the ball 1 s after projection, and the angle its direction of motion makes with the horizontal, to 3 significant figures and the nearest degree. **(3)**

(Total for Question 9 is 12 marks)

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10 A particle is projected from ground level with speed 15 m/s at an angle of 40 degrees above the horizontal. Find the horizontal component of its initial velocity, giving your answer to 3 significant figures.

(Total for Question 10 is 1 mark)

11 A particle is projected with speed 15 m/s at an angle of 40 degrees above the horizontal. Find the vertical component of its initial velocity, giving your answer to 3 significant figures.

(Total for Question 11 is 1 mark)

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12 The vertical component of a particle's initial velocity, as it is projected under gravity, is 19.6 m/s. Taking $g = 9.8$ m/s^2, find the time taken for the particle to reach its greatest height.

(Total for Question 12 is 1 mark)

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13 A particle is projected from ground level with speed 12 m/s at an angle of 30 degrees above the horizontal, and moves freely under gravity. Take $g = 9.8$ m/s^2. Find the greatest height reached by the particle, giving your answer to 3 significant figures.

(Total for Question 13 is 2 marks)

14 A particle is projected from a point O on horizontal ground with speed 22 m/s at an angle of 42 degrees above the horizontal. The particle moves freely under gravity. Take $g = 9.8$ m/s^2.

(a) Find the greatest height above the ground reached by the particle, giving your answer to 3 significant figures. **(3)**

(b) Find the time taken for the particle to reach this greatest height, giving your answer to 3 significant figures. **(2)**

(Total for Question 14 is 5 marks)

15 A particle is projected from ground level with speed 9.8 m/s at an angle of 30 degrees above the horizontal, and lands on the same horizontal ground. Take $g = 9.8$ m/s^2. You may quote the result $T = 2U\sin(\alpha)/g$ without proof. Find the time of flight, giving your answer exactly.

(Total for Question 15 is 1 mark)

16 A particle is projected from ground level with speed 14 m/s at an angle of 45 degrees above the horizontal, and lands on the same horizontal ground. Take g = 9.8 m/s^2. You may quote the result $R = U^2 \sin(2\alpha)/g$ without proof. Find the horizontal range, giving your answer exactly.

(Total for Question 16 is 1 mark)

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17 A particle is projected from ground level at an angle of 50 degrees above the horizontal, and lands on the same horizontal ground at a horizontal distance of 32 m from the point of projection. Take g = 9.8 m/s^2. You may quote the result $R = U^2 \sin(2\alpha)/g$ without proof. Find the initial speed U, giving your answer to 3 significant figures.

(Total for Question 17 is 2 marks)

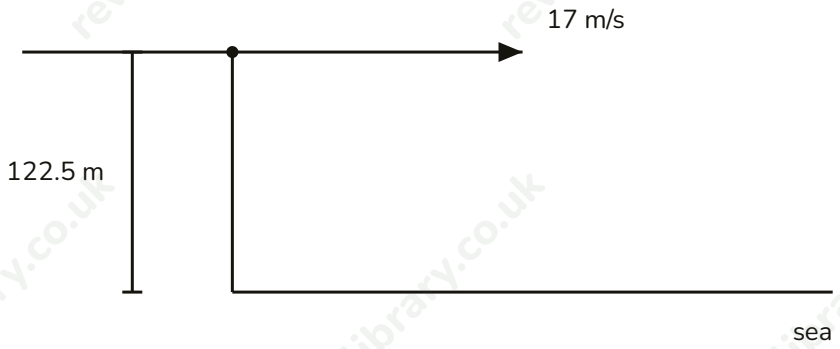
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18 A stone is thrown horizontally with speed 8 m/s from a bridge 19.6 m above a river, and moves freely under gravity until it lands in the river. Take g = 9.8 m/s^2. Find the horizontal distance from the bridge at which the stone lands, giving your answer exactly.

(Total for Question 18 is 2 marks)

19 A ball is thrown horizontally with speed 17 m/s from a point at the top of a vertical cliff. The point of projection is 122.5 m vertically above the sea, which may be modelled as a horizontal plane. The ball is modelled as a particle moving freely under gravity. Take $g = 9.8$ m/s^2.

Diagram NOT accurately drawn



(a) Show that the time taken for the ball to reach the sea is exactly 5 seconds. **(3)**

(b) Find the horizontal distance from the base of the cliff to the point where the ball lands. **(2)**

(c) Find the speed of the ball as it hits the sea, giving your answer to 3 significant figures. **(3)**

(d) Find the angle the ball's velocity makes with the horizontal as it hits the sea, giving your answer to 3 significant figures. **(2)**

(Total for Question 19 is 10 marks)

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20 A particle is projected from a point on horizontal ground with speed U at an angle α above the horizontal, and moves freely under gravity. Take $g = 9.8\ m/s^2$.

(a) By considering the vertical component of the initial velocity, show that the greatest height reached by the particle is $H = U^2 \sin^2(\alpha) / (2g)$. **(3)**

(b) Given that U = 26 m/s and α = 38 degrees, find the greatest height reached, giving your answer to 3 significant figures. **(2)**

(c) By first finding the time taken to reach the greatest height, use a different suvat equation to verify your value of H from part (b). **(3)**

(Total for Question 20 is 8 marks)

21 A particle P is projected from the origin O with velocity (5i + 9.8j) m/s, and moves freely under gravity, taking g = 9.8 m/s^2 acting in the direction -j. Find the velocity of P, in terms of i and j, when t = 1.5 seconds.

(Total for Question 21 is 2 marks)

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22 A particle is projected from the origin with velocity (6i + 12j) m/s, and moves freely under gravity, taking g = 9.8 m/s^2 acting in the direction -j. Find the speed of the particle when t = 1 second, giving your answer to 3 significant figures.

(Total for Question 22 is 2 marks)

23 A particle is projected from a point on horizontal ground with speed U at an angle of 20 degrees above the horizontal, and moves freely under gravity, landing on the same horizontal ground. The greatest height reached by the particle is 4.9 m. Take $g = 9.8$ m/s^2. Find the value of U, giving your answer to 3 significant figures.

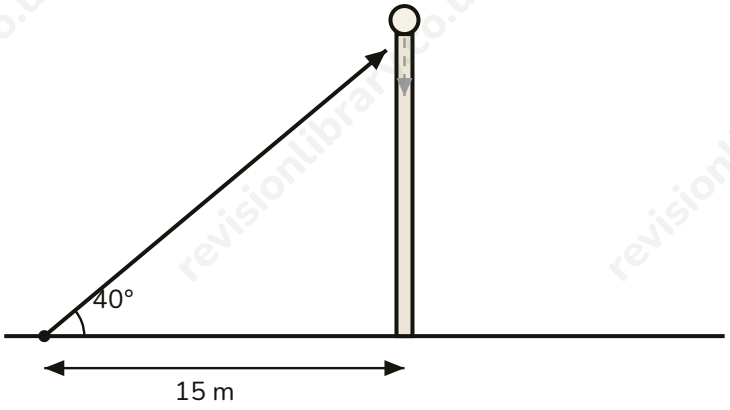
(Total for Question 23 is 2 marks)

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24 A particle is projected from ground level with speed 24.5 m/s at an angle α above the horizontal, and moves freely under gravity, landing on the same horizontal ground. Take $g = 9.8$ m/s^2. Given that the range is a maximum for this speed, find the value of the maximum range, giving your answer exactly.

(Total for Question 24 is 2 marks)

25 Two students, Ryan and Sofia, are experimenting in a field. At the instant Ryan throws a beanbag from ground level with speed 25 m/s at an angle of 40 degrees above the horizontal, aimed directly at a second beanbag balanced on top of a vertical post standing at a horizontal distance of 15 m from Ryan, in the vertical plane of the throw, Sofia releases that second beanbag from rest, so that it falls freely under gravity. Both beanbags are modelled as particles moving freely under gravity, and they collide in mid-air. Take $g = 9.8$ m/s^2.



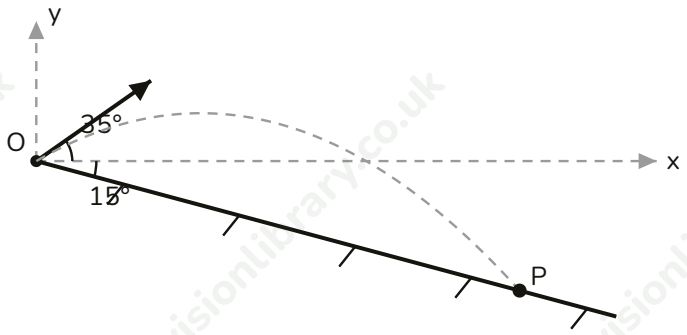
(a) By setting up expressions for the horizontal and vertical displacements of each beanbag, and using the fact that Ryan's beanbag is aimed directly at the second beanbag's initial position, find the height of the post, giving your answer to 3 significant figures. **(5)**

(b) Find the time after the beanbags are released at which they collide, giving your answer to 3 significant figures. **(2)**

(c) State two modelling assumptions, beyond treating each beanbag as a particle moving freely under gravity, that are needed for the collision to occur exactly as described. **(2)**

(Total for Question 25 is 9 marks)

26 A particle is projected from a point O with speed 21 m/s at an angle of 35 degrees above the horizontal. The point O lies at the top of a straight slope which descends at a constant angle of 15 degrees below the horizontal, in the same vertical plane as the particle's motion, so that the particle lands at a point P on the slope. Horizontal and vertically-upward displacements from O are denoted x and y respectively (both in metres, with y measured positive upward). Take g = 9.8 m/s^2.



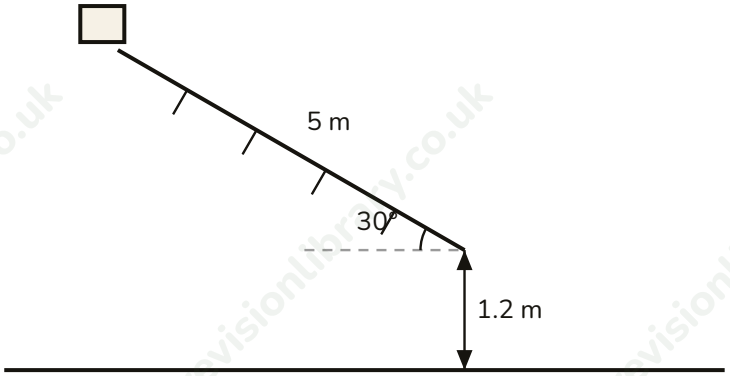
(a) Write down expressions, in terms of t, for x and y. **(2)**

(b) Given that the slope satisfies the equation y = -x tan(15), find the time of flight from O to P, giving your answer to 3 significant figures. **(5)**

(c) Hence find the distance OP, giving your answer to 3 significant figures. **(3)**

(Total for Question 26 is 10 marks)

27 A small block of mass 2 kg is released from rest at the top of a rough plane inclined at 30 degrees to the horizontal. The coefficient of friction between the block and the plane is 0.2. The plane has length 5 m, measured along a line of greatest slope, and the bottom edge of the plane is 1.2 m above horizontal ground. Beyond this edge, the block leaves the plane and moves freely under gravity as a projectile until it lands on the ground. Take $g = 9.8\ m/s^2$.



(a) By resolving forces perpendicular and parallel to the plane, show that the acceleration of the block down the plane is given by $a = g(\sin(30) - 0.2\cos(30))$. **(3)**

(b) Find the value of this acceleration, giving your answer to 3 significant figures. **(2)**

(c) Find the speed of the block at the instant it leaves the plane, giving your answer to 3 significant figures. **(2)**

(d) At the instant the block leaves the plane, its velocity is directed at 30 degrees below the horizontal, with the magnitude found in part (c). Find the horizontal distance from the point where the block leaves the plane to the point where it lands on the ground, giving your answer to 3 significant figures. **(6)**

(Total for Question 27 is 13 marks)

28 A particle P is projected from the origin O with velocity $(8\mathbf{i} + 14.7\mathbf{j})$ m/s, and moves freely under gravity, taking $g = 9.8$ m/s^2 acting in the direction $-\mathbf{j}$. At time t seconds after projection, the position vector of P relative to O is r metres.

(a) Find r in terms of t, giving your answer using the unit vectors i and j. **(2)**

(b) Find the value of t at which P is moving parallel to the vector $(2\mathbf{i} - \mathbf{j})$. **(4)**

(c) Find the speed of P at this instant. **(2)**

(Total for Question 28 is 8 marks)

29 A particle P is projected from the origin O with velocity $(12\mathbf{i} + 29.4\mathbf{j})$ m/s, and moves freely under gravity, taking $g = 9.8$ m/s^2 acting in the direction -j. At time t seconds after projection, the position vector of P relative to O is r metres.

(a) Find r in terms of t, giving your answer using the unit vectors i and j. **(2)**

(b) Find the value of t at which P is at its greatest height above the level of O. **(2)**

(c) Find, in the form ai + bj, the position vector of P at this instant. **(3)**

(d) Find the speed of P at this instant. **(2)**

(Total for Question 29 is 9 marks)

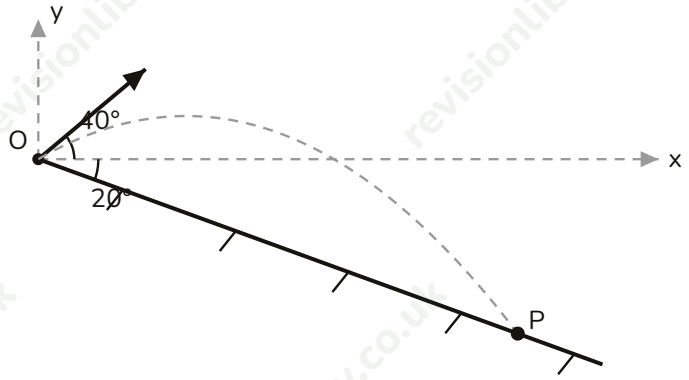
30 A particle is projected from ground level with speed 24 m/s at an angle of 55 degrees above the horizontal, and moves freely under gravity, landing on the same horizontal ground. Take $g = 9.8$ m/s^2. Find the maximum height reached and the total time of flight, giving each answer to 3 significant figures.

(Total for Question 30 is 3 marks)

31 A particle P is projected from the origin O with velocity (7i + 24.5j) m/s, and moves freely under gravity, taking g = 9.8 m/s^2 acting in the direction -j. At time t seconds after projection, the position vector of P relative to O is r metres. Find the position vector of P, in the form ai + bj, at the instant it is moving parallel to the vector (2i - 3j).

(Total for Question 31 is 3 marks)

32 A particle is projected from a point O with speed 18 m/s at an angle of 40 degrees above the horizontal. The point O lies at the top of a straight slope which descends at a constant angle of 20 degrees below the horizontal, in the same vertical plane as the particle's motion, so that the particle lands at a point P on the slope. Horizontal and vertically-upward displacements from O are denoted x and y respectively (both in metres). Take g = 9.8 m/s^2. Given that the slope satisfies the equation y = -x tan(20), find the time of flight from O to P, giving your answer to 3 significant figures.



(Total for Question 32 is 3 marks)

33 A crate of mass 4 kg is released from rest at the top of a rough plane inclined at 20 degrees to the horizontal. The coefficient of friction between the crate and the plane is 0.3. The crate slides a distance of 3.5 m down the plane before reaching the bottom. Take $g = 9.8$ m/s^2. Find the acceleration of the crate down the plane, and its speed at the bottom of the plane, giving each answer to 3 significant figures.

(Total for Question 33 is 4 marks)

Mark scheme · 3.8 Mechanics: Projectiles and Applications Depth

Question 1

(a) uses $v^2 = u^2 + 2as$ **M1**
(a) $35^2 = 17^2 + 2a(156)$ **A1**
(a) $a = 3$ m/s^2 **A1**
(a) Answer: 3 m/s^2
(b) uses $v = u + at$ **M1**
(b) $t = (35 - 17)/3 = 6$ s **A1**
(b) Answer: 6 s
(c) distance in first 3 s minus distance in first 2 s **M1**
(c) 64.5 - 40 **A1**
(c) 24.5 m **A1**
(c) Answer: 24.5 m

Question 2

(a) uses $v = u + at$ **M1**
(a) 15i + 4j **A1**
(a) speed = 15.5 m/s **A1**
(a) Answer: (15i + 4j) m/s, speed 15.5 m/s
(b) uses $r = r0 + ut + (1/2)at^2$ **M1**
(b) (36i - 5j) m **A1**
(b) Answer: (36i - 5j) m

Question 3

(a) resolves: U cos α and U sin α **M1**
(a) horizontal 35.5 m/s, vertical 10.4 m/s **A1**
(a) Answer: 35.5 m/s and 10.4 m/s
(b) uses $v^2 = u^2$ - 2gs for the vertical motion with v = 0 **M1**
(b) s = 5.476 m above the point of projection **A1**
(b) greatest height = 15.5 m **A1**
(b) Answer: 15.5 m
(c) vertical displacement $-10 = 10.360t - 4.9t^2$ **M1**
(c) T = 2.83 s **A1**
(c) horizontal distance = ux x T **M1**
(c) range = 101 m **A1**
(c) Answer: T = 2.83 s, range = 101 m
(d) vertical velocity = 10.360 - 9.8 x 2 = -9.240 **M1**
(d) speed = 36.7 m/s **A1**
(d) angle = 15 degrees below the horizontal **A1**
(d) Answer: 36.7 m/s, 15 degrees below the horizontal

Question 4

(a) uses $s = ut + (1/2)at^2$ vertically: $44.1 = (1/2)(9.8)t^2$ **M1**
(a) t = 3.00 s **A1**
(a) Answer: 3.00 s
(b) horizontal speed is constant: distance = u t **M1**
(b) 42.0 m **A1**
(b) Answer: 42.0 m
(c) vertical speed = gt = 9.8 x 3.000 = 29.400 **M1**
(c) combines the components: speed = $\sqrt{14^2 + 29.400^2}$ **M1**
(c) speed = 32.6 m/s **A1**
(c) angle 65 degrees below the horizontal from tan(angle) = 29.400/14 **A1**
(c) Answer: 32.6 m/s at 65 degrees below the horizontal

Question 5

(a) resolves: U cos α and U sin α **M1**
(a) horizontal 8.00 m/s, vertical 6.00 m/s **A1**
(a) Answer: 8.00 m/s and 6.00 m/s
(b) uses $v^2 = u^2$ - 2gs for the vertical motion with v = 0 **M1**
(b) s = 1.837 m above the point of projection **A1**
(b) greatest height = 1.84 m **A1**
(b) Answer: 1.84 m
(c) s = 0: $6.000t - 4.9t^2 = 0$ **M1**
(c) T = 1.22 s **A1**
(c) horizontal distance = ux x T **M1**
(c) range = 9.80 m **A1**
(c) Answer: T = 1.22 s, range = 9.80 m
(d) vertical velocity = 6.000 - 9.8 x 1 = -3.800 **M1**
(d) speed = 8.86 m/s **A1**
(d) angle = 25 degrees below the horizontal **A1**
(d) Answer: 8.86 m/s, 25 degrees below the horizontal

Question 6

rearranges $R = U^2 \sin(2\text{alpha})/g$ to $U^2 = Rg/\sin(2\text{alpha})$ and substitutes R = 40, α = 50 (oe) **M1**
U^2 = awrt 398 **A1**
U = awrt 20.0 (m/s), cao **A1**
Answer: U = awrt 20.0 m/s

Question 7

(a) uses $v^2 = u^2 + 2as$ **M1**
(a) $26^2 = 11^2 + 2a(92.5)$ **A1**
(a) $a = 3$ m/s^2 **A1**
(a) Answer: 3 m/s^2
(b) uses $v = u + at$ **M1**
(b) $t = (26 - 11)/3 = 5$ s **A1**
(b) Answer: 5 s
(c) distance in first 3 s minus distance in first 2 s **M1**
(c) 46.5 - 28 **A1**
(c) 18.5 m **A1**
(c) Answer: 18.5 m

Question 8

(a) uses $s = ut + (1/2)at^2$ vertically: $29.4 = (1/2)(9.8)t^2$ **M1**
(a) t = 2.45 s **A1**
(a) Answer: 2.45 s
(b) horizontal speed is constant: distance = u t **M1**
(b) 73.5 m **A1**
(b) Answer: 73.5 m
(c) vertical speed = gt = 9.8 x 2.449 = 24.005 **M1**
(c) combines the components: speed = $\sqrt{30^2 + 24.005^2}$ **M1**
(c) speed = 38.4 m/s **A1**
(c) angle 39 degrees below the horizontal from tan(angle) = 24.005/30 **A1**
(c) Answer: 38.4 m/s at 39 degrees below the horizontal

Question 9

(a) resolves: U cos α and U sin α **M1**

(a) horizontal 37.4 m/s, vertical 10.9 m/s **A1**

(a) Answer: 37.4 m/s and 10.9 m/s

(b) uses $v^2 = u^2 - 2gs$ for the vertical motion with v = 0 **M1**

(b) s = 6.084 m above the point of projection **A1**

(b) greatest height = 6.08 m **A1**

(b) Answer: 6.08 m

(c) s = 0: $10.920t - 4.9t^2 = 0$ **M1**

(c) T = 2.23 s **A1**

(c) horizontal distance = ux x T **M1**

(c) range = 83.4 m **A1**

(c) Answer: T = 2.23 s, range = 83.4 m

(d) vertical velocity = 10.920 - 9.8 x 1 = 1.120 **M1**

(d) speed = 37.5 m/s **A1**

(d) angle = 2 degrees above the horizontal **A1**

(d) Answer: 37.5 m/s, 2 degrees above the horizontal

Question 10

11.5 cao **B1**

Answer: 11.5 m/s

Question 11

9.64 cao **B1**

Answer: 9.64 m/s

Question 12

2 cao **B1**

Answer: t = 2 s

Question 13

substitutes U = 12, α = 30 into $H = U^2 \sin^2(\alpha)/(2g)$ (oe) **M1**

H = awrt 1.84 (m), cao **A1**

Answer: H = awrt 1.84 m

Question 14

(a) identifies the initial vertical component of velocity, u_y = 22sin(42) (oe) **M1**

(a) uses $v^2 = u^2 - 2gs$ with v = 0 and their u_y, rearranged for s **M1**

(a) H = awrt 11.1 (m), cao **A1**

(a) Answer: H = awrt 11.1 m

(b) uses $t = u_y/g$ with their u_y (or v = u - gt with v = 0) **M1**

(b) t = awrt 1.50 (s), cao **A1**

(b) Answer: t = awrt 1.50 s

Question 15

1 cao **B1**

Answer: T = 1 s (exact)

Question 16

20 cao **B1**

Answer: R = 20 m (exact)

Question 17

rearranges $R = U^2 \sin(2alpha)/g$ to $U^2 = Rg/\sin(2alpha)$ and substitutes R = 32, α = 50 (oe) **M1**

U = awrt 17.8 (m/s), cao **A1**

Answer: U = awrt 17.8 m/s

Question 18

uses $s = 4.9t^2 = 19.6$ to find the time of fall, t = 2 (s) **M1**

16 (m), cao **A1**

Answer: 16 m (exact)

Question 19

(a) uses $s = ut + 0.5at^2$ vertically with u = 0, a = 9.8, s = 122.5 (oe) **M1**

(a) rearranges to $t^2 = 25$ **M1**

(a) t = 5, cso **A1**

(a) Answer: t = 5 s (shown)

(b) horizontal distance = 17 x 5 (oe) **M1**

(b) 85 (m), cao **A1**

(b) Answer: 85 m

(c) vertical speed at impact $v_y = gt = 9.8 \times 5 = 49$ (oe) **M1**

(c) combines components: speed = $\sqrt{17^2 + 49^2}$ **M1**

(c) awrt 51.9 (m/s) **A1**

(c) Answer: 51.9 m/s (3sf)

(d) tan(θ) = 49/17 (oe) **M1**

(d) θ = awrt 70.9 (degrees) **A1**

(d) Answer: 70.9 degrees below the horizontal (3sf)

Question 20

(a) identifies the initial vertical component of velocity, u_y = Usin(α) **M1**

(a) uses $v^2 = u^2 - 2gs$ with v = 0 (at the greatest height) and their u_y **M1**

(a) rearranges to $H = U^2 \sin^2(\alpha)/(2g)$, cso **A1**

(a) Answer: $H = U^2 \sin^2(\alpha) / (2g)$ (shown)

(b) substitutes U = 26, α = 38 into $H = U^2 \sin^2(\alpha)/(2g)$ **M1**

(b) H = awrt 13.1 (m), cao **A1**

(b) Answer: H = awrt 13.1 m

(c) finds the time to greatest height, $t = u_y/g = 26\sin(38)/9.8$, ft **M1**

(c) uses a different suvat equation, e.g. s = ((u+v)/2)t with u = u_y, v = 0, and their t **M1**

(c) H = awrt 13.1 (m), matching (b), ft **A1**

(c) Answer: H = awrt 13.1 m (verified)

Question 21

uses v = u + at componentwise, with a = -9.8j **M1**

5i - 4.9j (m/s), cao **A1**

Answer: v = 5i - 4.9j (m/s)

Question 22

finds v(1) = 6i + (12 - 9.8)j = 6i + 2.2j, using v = u + at componentwise **M1**

awrt 6.39 (m/s), cao **A1**

Answer: awrt 6.39 m/s

Question 23

rearranges $H = U^2 \sin^2(\alpha)/(2g)$ to $U = \sqrt{2gH}/\sin(\alpha)$ and substitutes $H = 4.9$, $\alpha = 20$ (oe) **M1**

U = awrt 28.7 (m/s), cao **A1**

Answer: U = awrt 28.7 m/s

Question 24

recognises the range is greatest when $\alpha = 45$ degrees, giving $R_{max} = U^2/g$ **M1**

61.25 (m), exact, cao **A1**

Answer: R_{max} = 61.25 m (exact)

Question 25

(a) horizontal displacement of the thrown beanbag: $x = 25\cos(40)\ t$ **M1**

(a) vertical displacements: $y1 = 25\sin(40)\ t - 4.9t^2$ (thrown beanbag) and $y2 = h - 4.9t^2$ (falling beanbag, released from height h) **M1**

(a) uses the fact that Ryan aims directly at the post-top, i.e. $\tan(40) = h/15$, or equivalently finds $t = 15/(25\cos(40))$ from $x = 15$ **M1**

(a) sets $y1 = y2$ at the collision instant; the $-4.9t^2$ terms cancel, giving $h = 25\sin(40) \times t = 15\tan(40)$ **M1**

(a) h = awrt 12.6 (m) **A1**

(a) Answer: h = awrt 12.6 m

(b) $t = 15/(25\cos(40))$, ft **M1**

(b) awrt 0.783 (s) **A1**

(b) Answer: t = awrt 0.783 s

(c) the second beanbag is released from rest with no horizontal velocity, so it falls vertically downward only **B1**

(c) nothing obstructs the path of either beanbag before the collision point, and the beanbags are released and thrown at precisely the same instant **B1**

(c) Answer: E.g. the released beanbag falls with no horizontal velocity; there is no obstruction between the two beanbags before they collide

Question 26

(a) $x = 21\cos(35)\ t$, oe **B1**

(a) $y = 21\sin(35)\ t - 4.9t^2$, oe **B1**

(a) Answer: $x = 21\cos(35)\ t$; $y = 21\sin(35)\ t - 4.9t^2$

(b) substitutes the expressions for x and y from (a) into $y = -x\tan(15)$ **M1**

(b) collects terms and factors out t, noting t is not 0 at P **M1**

(b) rearranges the resulting linear equation to make t the subject **M1**

(b) unsimplified $t = (21\sin(35) + 21\cos(35)\tan(15)) / 4.9$, oe **A1**

(b) t = awrt 3.40 (s), cao **A1**

(b) Answer: t = awrt 3.40 s

(c) x = awrt 58.5 (m), using their t from (b), ft **M1**

(c) OP = x/cos(15), oe (or uses Pythagoras with x and y) **M1**

(c) OP = awrt 60.5 (m), ft **A1**

(c) Answer: OP = awrt 60.5 m

Question 27

(a) resolves perpendicular to the plane: $R = 2g\cos(30)$ **M1**

(a) resolves along the plane (down positive), using $F = 0.2R$: $2g\sin(30) - 0.2(2g\cos(30)) = 2a$ **M1**

(a) mass cancels throughout, giving $a = g(\sin(30) - 0.2\cos(30))$, cso **A1**

(a) Answer: $a = g(\sin(30) - 0.2\cos(30))$ (shown)

(b) substitutes g = 9.8 and evaluates sin(30), cos(30) **M1**

(b) a = awrt 3.20 (m/s^2), cao **A1**

(b) Answer: a = awrt 3.20 m/s^2

(c) uses $v^2 = u^2 + 2as$ with $u = 0$, $s = 5$, and their a from (a)/(b), ft **M1**

(c) v = awrt 5.66 (m/s), cao **A1**

(c) Answer: v = awrt 5.66 m/s

(d) resolves the velocity into components: $u_x = v\cos(30)$, $u_y = v\sin(30)$, ft their v from (c) **M1**

(d) u_x = awrt 4.90 (m/s) and u_y = awrt 2.83 (m/s) **A1**

(d) sets up the vertical equation of motion for the 1.2 m fall: $1.2 = u_y\ t + 4.9t^2$, oe **M1**

(d) t = awrt 0.284 (s) **A1**

(d) horizontal distance = $u_x \times t$, ft **M1**

(d) awrt 1.39 (m) **A1**

(d) Answer: awrt 1.39 m

Question 28

(a) uses $r = ut + 0.5at^2$ componentwise, with $a = -9.8j$ **M1**

(a) $r = 8ti + (14.7t - 4.9t^2)j$, oe **A1**

(a) Answer: $r = 8ti + (14.7t - 4.9t^2)j$ (m)

(b) differentiates r, or uses $v = u + at$, to obtain $v(t) = 8i + (14.7 - 9.8t)j$ **M1**

(b) sets up the condition for v to be parallel to $2i - j$, e.g. $(14.7 - 9.8t)/8 = -1/2$ (oe) **M1**

(b) rearranges to $9.8t = 18.7$ **M1**

(b) t = awrt 1.91 (s), oe **A1**

(b) Answer: t = awrt 1.91 s

(c) $v = 8i - 4j$, speed = $\sqrt{8^2 + (-4)^2}$ **M1**

(c) awrt 8.94 (m/s), oe (accept $4\sqrt{5}$) **A1**

(c) Answer: awrt 8.94 m/s (= $4\sqrt{5}$)

Question 29

(a) uses $r = ut + 0.5at^2$ componentwise, with $a = -9.8j$ **M1**

(a) $r = 12ti + (29.4t - 4.9t^2)j$, oe **A1**

(a) Answer: $r = 12ti + (29.4t - 4.9t^2)j$ (m)

(b) sets the vertical component of velocity to zero: $29.4 - 9.8t = 0$ (oe, e.g. differentiates the j-component of r) **M1**

(b) t = 3 (s), cao **A1**

(b) Answer: t = 3 s

(c) substitutes t = 3 into the i-component of r, ft **M1**

(c) substitutes t = 3 into the j-component of r, ft **M1**

(c) 36i + 44.1j (m), cao **A1**

(c) Answer: 36i + 44.1j (m)

(d) recognises the vertical component of velocity is zero at the greatest height, so the speed equals the horizontal component of velocity **M1**

(d) 12 (m/s), cao **A1**

(d) Answer: 12 m/s

Question 30

finds the initial vertical component of velocity, $u_y = 24 \sin(55)$ (oe)	**M1**
H = awrt 19.7 (m), using $H = u_y^2/(2g)$	**A1**
T = awrt 4.01 (s), using $T = 2u_y/g$	**A1**

Answer: H = awrt 19.7 m; T = awrt 4.01 s

Question 31

uses v = u + at to find v(t) = 7i + (24.5 - 9.8t)j	**M1**
sets up the condition for v to be parallel to 2i - 3j, e.g. 7(-3) - (24.5 - 9.8t)(2) = 0, and solves to find t = 25/7 (or awrt 3.57)	**M1**
substitutes their t into $r(t) = 7ti + (24.5t - 4.9t^2)j$ to find r = 25i + 25j (m), cao (exact)	**A1**

Answer: r = 25i + 25j (m), at t = 25/7 s (awrt 3.57 s)

Question 32

substitutes $x = 18\cos(40)\, t$ and $y = 18\sin(40)\, t - 4.9t^2$ into the slope equation y = -x tan(20)	**M1**
collects terms, factors out t (noting t is not 0 at P) and rearranges to make t the subject, e.g. t = (18sin40 + 18cos40 tan20)/4.9	**M1**
t = awrt 3.39 (s), cao	**A1**

Answer: t = awrt 3.39 s

Question 33

resolves perpendicular to the plane to find R = 4g cos(20), and parallel to the plane to find 4g sin(20) - 0.3(4g cos(20)) = 4a	**M1**
a = g(sin20 - 0.3cos20) = awrt 0.589 (m/s^2), cao	**A1**
uses $v^2 = u^2 + 2as$ with u = 0, s = 3.5, and their a	**M1**
v = awrt 2.03 (m/s), cao	**A1**

Answer: a = awrt 0.589 m/s^2; v = awrt 2.03 m/s