1.1 Pure: Proof: 1-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Prove that $n^2 - n + 3$ is odd for every integer n.

(Total for Question 1 is 3 marks)

---

2 Priya claims: "For every positive integer n, the value of $n^2 + n + 2$ is even."

(a) Verify Priya's claim for n = 1, n = 2 and n = 3. **(2)**

(b) Prove that Priya's claim is true for every positive integer n. **(2)**

(Total for Question 2 is 4 marks)

---

3 $f(x) = x^2 + 2x + 9$.

(a) Write f(x) in the form $a(x + p)^2 + q$, where a, p and q are constants to be found. **(3)**

(b) Hence prove that $f(x) > 0$ for all real values of x. **(2)**

(c) State the coordinates of the minimum point of the curve y = f(x), and explain why the curve does not meet the x-axis. **(2)**

(Total for Question 3 is 7 marks)

---

4 This question is about proof by contradiction.

(a) Prove by contradiction that there are no integers x and y such that $4x + 10y = 5$. **(4)**

(b) Find one pair of integers x and y such that $4x + 10y = 6$. **(2)**

(Total for Question 4 is 6 marks)

---

5 This question is about disproof by counterexample and proof by deduction.

(a) A student claims: 'If $x > y$ then $x^2 > y^2$ for all real numbers x and y.' Find a counterexample to this claim. **(3)**

(b) State, with a reason, a condition on x and y under which the claim is true. **(2)**

(Total for Question 5 is 5 marks)

© Revision Library | revisionlibrary.co.uk

6 $f(x) = 3x^2 + 30x + 82$.

(a) Write $f(x)$ in the form $a(x + p)^2 + q$, where a, p and q are constants to be found. **(3)**

(b) Hence prove that $f(x) > 0$ for all real values of x. **(2)**

(c) State the coordinates of the minimum point of the curve $y = f(x)$, and explain why the curve does not meet the x-axis. **(2)**

(Total for Question 6 is 7 marks)

---

7 This question is about disproof by counterexample and proof by deduction.

(a) A student claims that $5n + 11$ is prime for every positive integer n. Show that this is not true. **(3)**

(b) Prove that $5n + 11$ is divisible by 11 whenever n is a multiple of 11. **(3)**

(Total for Question 7 is 6 marks)

---

8 This question involves an identity established using the compound angle formulae.

(a) Show that $\sin(\theta+60) + \sin(\theta-60) = \sin(\theta)$, where θ is measured in degrees. **(4)**

(b) Hence solve, for $0 \leq \theta < 360$ (degrees), the equation $\sin(\theta+60) + \sin(\theta-60) = 0.5$. **(3)**

(Total for Question 8 is 7 marks)

---

9 This question builds a proof that $\sqrt{2}$ is irrational.

(a) Prove that if n^2 is even, where n is an integer, then n is even. **(2)**

(b) Prove, by contradiction, that $\sqrt{2}$ is irrational. **(7)**

(Total for Question 9 is 9 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.1 Pure: Proof: 1-hour Pack

Question 1

factorises $n^2 - n = n(n - 1)$	**M1**
states that n and n-1 are consecutive integers, so exactly one of them is even, hence n(n-1) is always even	**A1**
cso: since n(n-1) is even, n(n-1)+3 = even + odd = odd for every integer n	**A1**

Answer: Proof: $n^2 - n + 3 = n(n-1) + 3$ = even + 3, which is odd

Question 2

(a) correctly evaluates n=1: 4, n=2: 8, n=3: 14	**B1**
(a) states that all three values are even	**B1**

(a) Answer: n=1: 4, n=2: 8, n=3: 14, all even

(b) recognises $n^2 + n = n(n+1)$, the product of two consecutive integers, so n(n+1) is always even	**M1**
(b) cso: $n^2+n+2 = n(n+1) + 2$ = even + even = even for every positive integer n	**A1**

(b) Answer: Proof: $n^2+n+2 = n(n+1)+2$, and n(n+1) is always even, so the sum is always even

Question 3

(a) begins to complete the square: $(x + p)^2$ with p = b/2	**M1**
(a) a = 1, p = 1	**A1**
(a) q = 8	**A1**

(a) Answer: $f(x) = (x + 1)^2 + 8$

(b) states that a squared term is non-negative	**M1**
(b) so $f(x) \geq 8 > 0$ (cso)	**A1**

(b) Answer: minimum value 8 > 0

(c) (-1, 8)	**B1**
(c) the minimum value 8 is above the x-axis, so there are no real roots	**B1**

(c) Answer: (-1, 8)

Question 4

(a) assumes there are integers x and y with 4x + 10y = 5	**M1**
(a) notes the left side is even, since 4x + 10y = 2(2x + 5y)	**M1**
(a) but 5 is odd, a contradiction	**A1**
(a) so no such integers exist (cso)	**A1**

(a) Answer: no integer solutions

(b) systematic trial or use of the Euclidean algorithm	**M1**
(b) a valid pair, for example x = -6, y = 3	**A1**

(b) Answer: x = -6, y = 3

Question 5

(a) chooses a negative value for y, or negative values for both	**M1**
(a) for example x = -3, y = -4: x > y since -3 > -4	**M1**
(a) $x^2 = 9$ and $y^2 = 16$, and 9 < 16, so the claim is false	**A1**

(a) Answer: x = -3, y = -4: x > y but $x^2 = 9 < y^2 = 16$

(b) x > y > 0 (or both non-negative)	**B1**
(b) since squaring is an increasing function for positive numbers $(x^2 - y^2 = (x - y)(x + y) > 0)$	**B1**

(b) Answer: true when x and y are both positive (x + y > 0)

Question 6

(a) takes out the factor 3	**M1**
(a) a = 3, p = 5	**A1**
(a) q = 7	**A1**

(a) Answer: $f(x) = 3(x + 5)^2 + 7$

(b) states that a squared term is non-negative	**M1**
(b) so $f(x) \geq 7 > 0$ (cso)	**A1**

(b) Answer: minimum value 7 > 0

(c) (-5, 7)	**B1**
(c) the minimum value 7 is above the x-axis, so there are no real roots	**B1**

(c) Answer: (-5, 7)

Question 7

(a) tries n = 11 (or other values until a composite appears)	**M1**
(a) 5(11) + 11 = 66	**A1**
(a) 66 = 11 x 6, which is not prime, so the claim is false	**A1**

(a) Answer: n = 11 gives 66 = 11 x 6

(b) writes n = 11m	**M1**
(b) 5(11m) + 11 = 11(5m + 1)	**A1**
(b) which is a multiple of 11 (cso)	**A1**

(b) Answer: 11(5m + 1)

Question 8

(a) expands $\sin(\theta+60) = \sin(\theta)\cos60 + \cos(\theta)\sin60$	**M1**
(a) expands $\sin(\theta-60) = \sin(\theta)\cos60 - \cos(\theta)\sin60$	**M1**
(a) adds the two expansions so the $\cos(\theta)\sin60$ terms cancel, leaving $2\sin(\theta)\cos60$	**A1**
(a) substitutes the exact value $\cos60=1/2$ to obtain $2\sin(\theta)x(1/2)=\sin(\theta)$, cso	**A1**

(a) Answer: Proof: $\sin(\theta+60)+\sin(\theta-60) = \sin(\theta)$

(b) uses the identity from part (a) to rewrite the equation as $\sin(\theta)=0.5$, ft	**M1**
(b) $\theta=30$	**A1**
(b) $\theta=150$, cao, both values and no extras in range	**A1**

(b) Answer: $\theta = 30$ or $\theta = 150$

© Revision Library | revisionlibrary.co.uk

Question 9

(a) assumes n is odd, writes n=2k+1 for integer k, and expands $n^2=(2k+1)^2=4k^2+4k+1$ **M1**

(a) writes $n^2=2(2k^2+2k)+1$, which is odd; since "n odd implies n^2 odd" has been shown, its contrapositive "n^2 even implies n even" must also be true, cso **A1**

(a) Answer: Proof: if n^2 is even then n is even

(b) assumes, for contradiction, that $\sqrt{2}$ is rational, so $\sqrt{2}=p/q$ where p and q are integers with no common factor (q not equal 0), i.e. p/q is in its lowest terms **B1**

(b) squares both sides to obtain $p^2=2q^2$ **M1**

(b) deduces p^2 is even, so by part (a), p is even; writes p=2m for integer m, ft **A1**

(b) substitutes p=2m into $p^2=2q^2$ to get $4m^2=2q^2$, so $q^2=2m^2$, dependent on the previous M1 **dM1**

(b) deduces q^2 is even, so by part (a), q is even, ft **A1**

(b) states that p and q are both even, so they share a common factor of 2, contradicting the assumption that p/q has no common factor **B1**

(b) concludes that the original assumption is false, hence $\sqrt{2}$ is irrational, cso **B1**

(b) Answer: Proof by contradiction: $\sqrt{2}$ is irrational

© Revision Library | revisionlibrary.co.uk