1.1 Pure: Proof: 30-minute Pack

EDEXCEL 9MA0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question explores proof by exhaustion and its limitations, using the statement: for every positive integer n, n^3+n is even.

(a) Prove by exhaustion that n^3+n is even for every integer n such that $1 \leq n \leq 4$. **(4)**

(b) Prove, by considering the cases n even and n odd, that n^3+n is even for every positive integer n. **(4)**

(c) Explain why the working in part (a) alone does not prove that n^3+n is even for every positive integer n. **(2)**

(Total for Question 1 is 10 marks)

2 In each part, find a counter-example to disprove the given statement.

(a) Disprove the statement: "For every real number x, $x^2 \geq x$." **(2)**

(b) Disprove the statement: "For every integer n, if n is prime then n is odd." **(2)**

(c) Disprove the statement: "For all real numbers a and b, $(a+b)^2 = a^2+b^2$." **(2)**

(d) Disprove the statement: "For $0 \leq \theta \leq 360$ (degrees), $\sin(\theta) \leq \cos(\theta)$ implies $\theta \leq 45$." **(2)**

(Total for Question 2 is 8 marks)

3 This question involves proof by deduction using cases.

(a) Prove that the product of any two consecutive integers, n(n+1), is always even. **(3)**

(b) Hence prove that n^2+n+1 is always odd, for any integer n. **(3)**

(Total for Question 3 is 6 marks)

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Mark scheme · 1.1 Pure: Proof: 30-minute Pack

Question 1

(a) n=1: $1^3+1=2$, even **B1**

(a) n=2: $2^3+2=10$, even **B1**

(a) n=3: $3^3+3=30$, even **B1**

(a) n=4: $4^3+4=68$, even, and all four cases $1 \le n \le 4$ have been checked, cao **B1**

(a) Answer: n=1,2,3,4 give 2, 10, 30, 68, all even

(b) case n=2k: substitutes to get $n^3+n=8k^3+2k$ **M1**

(b) writes $8k^3+2k=2(4k^3+k)$, a multiple of 2, hence even **A1**

(b) case n=2k+1: substitutes and factorises $n^3+n=(2k+1)[(2k+1)^2+1]=(2k+1)(4k^2+4k+2)$ **M1**

(b) writes this as $2(2k+1)(2k^2+2k+1)$, a multiple of 2, hence even; since every integer is even or odd, this proves the result for all positive integers, cso **A1**

(b) Answer: Proof: n^3+n is even for all positive integers n

(c) states that part (a) only checks a finite number of cases (n=1 to 4), but the claim is about all positive integers, an infinite set **B1**

(c) states that a proof by exhaustion must check every case in the set, which is impossible for an infinite set, so a general argument (as in part (b)) is needed instead **B1**

(c) Answer: Checking 4 cases cannot establish a result for infinitely many integers

Question 2

(a) chooses a valid value of x with $0<x<1$, e.g. x=0.5, oe **B1**

(a) shows $x^2<x$ for this value, e.g. $0.5^2=0.25<0.5$, so the statement is false, cao **A1**

(a) Answer: x=0.5 gives 0.25 < 0.5, so the statement is false

(b) states n=2 **B1**

(b) justifies that 2 is prime and even, contradicting the statement, cao **A1**

(b) Answer: n=2 is prime and even, so the statement is false

(c) chooses valid values with a and b both non-zero, e.g. a=1, b=1, oe **B1**

(c) shows $(1+1)^2=4$ but $1^2+1^2=2$, and 4 is not equal to 2, so the statement is false, cao **A1**

(c) Answer: a=1, b=1 gives 4 not equal to 2, so the statement is false

(d) chooses a valid value of θ with θ>45 in the given range, e.g. θ=350, oe **B1**

(d) shows sin(350) is awrt -0.17 and cos(350) is awrt 0.98, so $\sin(\theta) \le \cos(\theta)$ holds, yet θ=350 is not ≤45, so the statement is false, cao **A1**

(d) Answer: θ=350 gives sin(θ) awrt -0.17 ≤ cos(θ) awrt 0.98, but θ>45, so the statement is false

Question 3

(a) considers the case n=2k (n even) and writes $n(n+1)=2k(2k+1)$, a multiple of 2 **M1**

(a) considers the case n=2k+1 (n odd) and writes $n(n+1)=(2k+1)(2k+2)=2(2k+1)(k+1)$, a multiple of 2 **A1**

(a) since every integer n is even or odd, and the product is a multiple of 2 in both cases, n(n+1) is always even, cso **A1**

(a) Answer: Proof: n(n+1) is always even

(b) recognises $n^2+n=n(n+1)$, oe **M1**

(b) uses the result from part (a) that n(n+1) is even, ft **A1**

(b) states that even + 1 is odd, so n^2+n+1 is always odd, cso **A1**

(b) Answer: Proof: $n^2+n+1 = n(n+1)+1 = \text{even}+1 = \text{odd}$

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