1.1 Pure: Proof: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 This question involves proof by deduction using algebraic representations of odd and even integers.

(a) Prove that the sum of any two odd integers is always even. **(3)**

(b) Prove that the square of any odd integer is always odd. **(3)**

(c) Prove that the sum of any two consecutive integers is always odd. **(2)**

(Total for Question 1 is 8 marks)

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2 Write the integer 23 in the form $2k + 1$, stating the value of k.

(Total for Question 2 is 1 mark)

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3 Two odd integers are written as $2a + 1$ and $2b + 1$, for integers a and b. State, without proof, whether their sum is always odd or always even.

(Total for Question 3 is 1 mark)

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4 Expand and fully simplify $(n + 1)^2 - n^2$.

(Total for Question 4 is 1 mark)

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5 Simplify $(n + 3) + (n + 4)$, and state whether the result is odd or even for every integer n.

(Total for Question 5 is 1 mark)

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6 Show that $(n + 2)^2 - n^2 = 4n + 4$, and state whether this expression is always a multiple of 4.

(Total for Question 6 is 2 marks)

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7 Fully factorise $n^3 - n$, and hence explain why $n^3 - n$ is the product of three consecutive integers.

(Total for Question 7 is 2 marks)

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8 State a value of x, with $0 < x < 1$, that could be used as a counter-example to disprove "$x^2 > x$ for every non-zero real number x."

(Total for Question 8 is 1 mark)

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9 Priya defines $f(n) = n^2 - n + 11$ for positive integers n. She calculates f(1)=11, f(2)=13, f(3)=17, f(4)=23 and f(5)=31, all of which are prime, and concludes: "f(n) is prime for every positive integer n."

(a) Show that f(11) = 121, and explain why this disproves Priya's conclusion. **(3)**

(b) Explain why testing values of n from 1 to 5, as Priya did, could never have proved her conclusion, even before a counter-example was found. **(3)**

(Total for Question 9 is 6 marks)

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10 Priya claims: "$n^2 - n + 41$ is prime for every positive integer n." Use n = 41 to disprove Priya's claim.

(Total for Question 10 is 3 marks)

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11 This question is about disproof by counterexample and proof by deduction.

(a) A student claims that $n^2 + n + 7$ is a prime number for every positive integer n. Show that the claim is false. **(3)**

(b) Prove that $n^2 + n + 7$ is divisible by 7 when $n = 7k - 1$ for any positive integer k. **(4)**

(Total for Question 11 is 7 marks)

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12 $f(x) = 2x^2 - 20x + 55$.

(a) Write $f(x)$ in the form $a(x + p)^2 + q$, where a, p and q are constants to be found. **(3)**

(b) Hence prove that $f(x) > 0$ for all real values of x. **(2)**

(c) State the coordinates of the minimum point of the curve $y = f(x)$, and explain why the curve does not meet the x-axis. **(1)**

(Total for Question 12 is 6 marks)

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13 Prove by contradiction that if n^2 is a multiple of 3, where n is an integer, then n is a multiple of 3.

(Total for Question 13 is 4 marks)

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14 A student claims: "$n^2 + n + 5$ is prime for every positive integer n."

(a) Verify the student's claim for $n = 1$, $n = 2$ and $n = 3$. **(2)**

(b) By finding a suitable value of n, prove that the student's claim is false. **(3)**

(Total for Question 14 is 5 marks)

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Mark scheme · 1.1 Pure: Proof: Test Pack

Question 1

(a) represents two odd integers as 2m+1 and 2n+1, where m and n are integers, oe **B1**

(a) forms the sum (2m+1)+(2n+1) and simplifies to 2m+2n+2 **M1**

(a) writes the sum as 2(m+n+1), a multiple of 2, hence even, cso **A1**

(a) Answer: Proof: sum = 2(m+n+1), which is even

(b) represents an odd integer as 2k+1, where k is an integer, oe **B1**

(b) expands $(2k+1)^2$ to $4k^2+4k+1$ **M1**

(b) writes $4k^2+4k+1$ as $2(2k^2+2k)+1$, a multiple of 2 plus 1, hence odd, cso **A1**

(b) Answer: Proof: $(2k+1)^2 = 2(2k^2+2k)+1$, which is odd

(c) lets the consecutive integers be n and n+1 and forms the sum n+(n+1)=2n+1, oe **M1**

(c) states 2n is a multiple of 2, so 2n+1 is odd for every integer n, cso **A1**

(c) Answer: Proof: sum = 2n+1, which is odd

Question 2

k = 11, so 23 = 2(11) + 1 **B1**

Answer: k = 11

Question 3

always even **B1**

Answer: Always even

Question 4

2n + 1 cao **B1**

Answer: 2n + 1

Question 5

2n + 7, which is odd for every integer n **B1**

Answer: 2n + 7, odd

Question 6

expands $(n + 2)^2$ to $n^2 + 4n + 4$ and subtracts n^2 **M1**

cso: obtains 4n + 4 = 4(n + 1), which is always a multiple of 4 **A1**

Answer: 4n + 4 = 4(n + 1), always a multiple of 4

Question 7

takes out a factor of n, $n^3 - n = n(n^2 - 1)$, then factorises the difference of squares to n(n-1)(n+1) **M1**

identifies (n-1), n and (n+1) as three consecutive integers, cao **A1**

Answer: $n^3 - n = (n-1)n(n+1)$, the product of three consecutive integers

Question 8

any value with $0 < x < 1$, e.g. x = 0.5 **B1**

Answer: x = 0.5 (any value with $0 < x < 1$)

Question 9

(a) substitutes n=11 into f(n): $11^2-11+11$ **M1**

(a) simplifies to 121, cso **A1**

(a) states 121=11x11, which is not prime, so n=11 is a counter-example that disproves the conclusion, cao **B1**

(a) Answer: f(11)=121=11x11, not prime, so the conclusion is false

(b) states that Priya's conclusion is a claim about every positive integer n, an infinite set, but she only checked 5 cases **B1**

(b) states that finitely many successful checks (proof by exhaustion of only some cases) cannot establish a universal statement over an infinite set **B1**

(b) states that a single valid counter-example, such as n=11, is sufficient to disprove a universal claim, whereas no finite number of confirming cases could ever prove it, oe **B1**

(b) Answer: Finitely many confirming cases can never prove a claim about all positive integers, but one counter-example disproves it

Question 10

substitutes n = 41 **B1**

computes $41^2 - 41 + 41 = 1681$ **M1**

cao: states 1681 = 41 x 41, which is not prime, so Priya's claim is false **A1**

Answer: n = 41 gives 1681 = 41 x 41, not prime, so the claim is false

Question 11

(a) tries values of n, or looks for a value of n making the expression a multiple of the constant term **M1**

(a) any valid counterexample with its value, e.g. n = 6: $6^2 + 6 + 7 = 49$ (also n = 1: 9, n = 4: 27, n = 7: 63) **A1**

(a) shows that value is not prime, e.g. 49 = 7 x 7, so the claim is false (cso) **A1**

(a) Answer: Any counterexample, e.g. n = 6 gives $49 = 7^2$, which is not prime (n = 1, 4 and 7 also work)

(b) substitutes n = 7k - 1: $(7k - 1)^2 + (7k - 1) + 7$ **M1**

(b) expands to $49k^2 - 14k + 1 + 7k - 1 + 7$ **M1**

(b) simplifies to $49k^2 - 7k + 7$ **A1**

(b) factorises as $7(7k^2 - k + 1)$, a multiple of 7 (cso) **A1**

(b) Answer: $7(7k^2 - k + 1)$

Question 12

(a) takes out the factor 2 **M1**

(a) a = 2, p = -5 **A1**

(a) q = 5 **A1**

(a) Answer: $f(x) = 2(x - 5)^2 + 5$

(b) states that a squared term is non-negative **M1**

(b) so $f(x) \geq 5 > 0$ (cso) **A1**

(b) Answer: minimum value $5 > 0$

(c) the minimum value 5 is above the x-axis, so there are no real roots **B1**

(c) Answer: (5, 5)

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Question 13

assumes, for contradiction, that n is not a multiple of 3, so n = 3k+1 or n = 3k+2 for an integer k **M1**

case n = 3k+1: $n^2 = 9k^2+6k+1 = 3(3k^2+2k)+1$, not a multiple of 3 **A1**

case n = 3k+2: $n^2 = 9k^2+12k+4 = 3(3k^2+4k+1)+1$, not a multiple of 3 **A1**

cso: in both cases n^2 is not a multiple of 3, contradicting the assumption that n^2 is a multiple of 3, so n must be a multiple of 3 **A1**

Answer: Proof by contradiction: if n^2 is a multiple of 3 then n is a multiple of 3

Question 14

(a) correctly evaluates n=1: 7, n=2: 11, n=3: 17 **B1**

(a) states that 7, 11 and 17 are all prime **B1**

(a) Answer: n=1: 7, n=2: 11, n=3: 17, all prime

(b) evaluates n = 4: 16 + 4 + 5 = 25 **M1**

(b) states 25 = 5 x 5, which is not prime **A1**

(b) cso: concludes n = 4 is a counter-example, so the student's claim is false **A1**

(b) Answer: n = 4 gives 25 = 5 x 5, not prime, so the claim is false

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