1.11 Pure: Algebra and Functions Depth: 2-hour Pack

EDEXCEL 9MA0 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Find the remainder when $f(x) = x^3 + 2x^2 - 5x + 1$ is divided by $(x - 1)$.

(Total for Question 1 is 1 mark)

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2 $f(x) = 2x - 5$, $g(x) = x + 3$. Find $fg(x)$.

(Total for Question 2 is 1 mark)

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3 $f(x) = 2x - 5$. Find $f^{-1}(x)$.

(Total for Question 3 is 1 mark)

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4 $f(x) = x^2 - 6x + 5$, $x \geq 3$.

(a) Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. **(3)**

(b) State the range of f, given the domain $x \geq 3$. **(2)**

(c) Find $f^{-1}(x)$ and state its domain. **(4)**

(Total for Question 4 is 9 marks)

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5 $f(x) = 2x^3 + ax^2 - 3x + b$, where a and b are constants. When $f(x)$ is divided by $(x - 2)$, the remainder is 15. When $f(x)$ is divided by $(x + 1)$, the remainder is -6. Find the values of a and b.

(Total for Question 5 is 4 marks)

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6 Solve the following involving the modulus function.

(a) Solve $|4x - 3| = 2x - 2$. **(4)**

(b) Solve $|4x - 3| < 3$. **(2)**

(Total for Question 6 is 6 marks)

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7 The equation $x^2 + 2kx + (3k + 10) = 0$, where k is a constant, is to be considered.

(a) Find the discriminant of the equation, simplifying fully. **(3)**

(b) Find the set of values of k for which the equation has two distinct real roots. **(3)**

(c) Find the value(s) of k for which the equation has equal roots, and find the repeated root in each case. **(3)**

(Total for Question 7 is 9 marks)

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8 $f(x) = (8x + 26)/((x + 2)(x + 4))$, $x \neq -2$, $x \neq -4$.

(a) Express f(x) in partial fractions. **(4)**

(b) Hence find the exact value of f(x) when $x = 0$. **(2)**

(Total for Question 8 is 6 marks)

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9 The function f is defined by $f(x) = (3x + 8)/(x - 4)$, x in R, $x \neq 4$. The function g is defined by $g(x) = x^2 + 5$, x in R.

(a) Find $f^{-1}(x)$, stating its domain. **(4)**

(b) Write down the range of f. **(1)**

(c) Find fg(x) and hence solve $fg(x) = 71/17$. **(4)**

(Total for Question 9 is 9 marks)

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10 $g(x) = (3x^2 - 17x + 26)/((x + 2)(x - 4)^2)$, $x \neq -2$, $x \neq 4$.

(a) Express g(x) in the form $A/(x + 2) + B/(x - 4) + C/(x - 4)^2$. **(4)**

(b) Find the exact value of g(6). **(2)**

(Total for Question 10 is 6 marks)

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11 The equation $x^2 + 2kx + (-5k - 4) = 0$, where k is a constant, is to be considered.

(a) Find the discriminant of the equation, simplifying fully. **(3)**

(b) Find the set of values of k for which the equation has two distinct real roots. **(3)**

(c) Find the value(s) of k for which the equation has equal roots, and find the repeated root in each case. **(3)**

(Total for Question 11 is 9 marks)

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12 Solve the following involving the modulus function.

(a) Solve $|3x - 4| = 2x - 3$. **(4)**

(b) Solve $|3x - 4| < 3$. **(2)**

(Total for Question 12 is 6 marks)

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13 The function f is defined by $f(x) = (6x + 7)/(x - 3)$, x in R, $x \neq 3$. The function g is defined by $g(x) = x^2 + 3$, x in R.

(a) Find $f^{-1}(x)$, stating its domain. **(4)**

(b) Write down the range of f. **(1)**

(c) Find fg(x) and hence solve $fg(x) = 49/4$. **(4)**

(Total for Question 13 is 9 marks)

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14 $h(x) = (2x^2 + 14x + 18)/((x + 4)(x + 1))$, $x \neq -4$, $x \neq -1$.

(a) Show that h(x) can be written in the form $2 + A/(x + 4) + B/(x + 1)$, and find the values of A and B. **(4)**

(b) Hence find h(1). **(2)**

(Total for Question 14 is 6 marks)

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15 Functions f and g are defined by $f(x) = 2x/(x - 3)$, x is real, x not equal to 3, and $g(x) = x + 1$, x is real.

(a) Find fg(x), simplifying your answer fully. **(3)**

(b) Find $f^{-1}(x)$, stating its domain. **(4)**

(c) Solve the equation $fg(x) = f^{-1}(x)$, showing that there is no solution. **(3)**

(Total for Question 15 is 10 marks)

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16 $f(x) = (5 + x) / ((1 + 2x)(1 - x))$, $|x| < 1/2$.

(a) Express f(x) in the form $A/(1 + 2x) + B/(1 - x)$, stating the values of A and B. **(5)**

(b) Hence expand f(x) in ascending powers of x, up to and including the term in x^2, giving each coefficient as an exact number. State the range of values of x for which the expansion is valid. **(4)**

(Total for Question 16 is 9 marks)

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Mark scheme · 1.11 Pure: Algebra and Functions Depth: 2-hour Pack

Question 1

remainder = -1 cao	**B1**

Answer: -1

Question 2

2x + 1 cao	**B1**

Answer: fg(x) = 2x + 1

Question 3

(x+5)/2 cao	**B1**

Answer: $f^{-1}(x) = (x+5)/2$

Question 4

(a) attempts to complete the square on $x^2 - 6x + 5$	**M1**
(a) a = 3	**A1**
(a) b = 4 oe, i.e. $f(x) = (x-3)^2 - 4$	**A1**

(a) Answer: $f(x) = (x - 3)^2 - 4$, so a = 3, b = 4

(b) recognises that for $x \geq 3$, $(x-3) \geq 0$ so $(x-3)^2 \geq 0$, and f is increasing on this domain	**M1**
(b) $f(x) \geq -4$ oe	**A1**

(b) Answer: $f(x) \geq -4$

(c) sets $y = (x-3)^2 - 4$ and rearranges to $(x-3)^2 = y+4$	**M1**
(c) takes the square root and selects the positive root, consistent with domain $x \geq 3$: $x-3 = \sqrt{y+4}$	**M1**
(c) $f^{-1}(x) = 3 + \sqrt{x+4}$ oe cao	**A1**
(c) domain $x \geq -4$, ft from the range of f found in part (b)	**B1ft**

(c) Answer: $f^{-1}(x) = 3 + \sqrt{x + 4}$, domain: $x \geq -4$

Question 5

forms the equation f(2) = 15, i.e. 4a + b = 5 oe	**M1**
forms the equation f(-1) = -6, i.e. a + b = -7 oe	**M1**
solves the two equations simultaneously	**M1**
a = 4, b = -11 cao	**A1**

Answer: a = 4, b = -11

Question 6

(a) squares both sides, or considers 4x - 3 = 2x - 2 and 4x - 3 = -(2x - 2)	**M1**
(a) obtains the candidate solutions 1/2, 5/6	**A1**
(a) checks each candidate in the original equation (the right side must be non-negative)	**M1**
(a) no solutions	**A1**

(a) Answer: no solutions

(b) -3 < 4x - 3 < 3	**M1**
(b) 0 < x < 3/2	**A1**

(b) Answer: 0 < x < 3/2

Question 7

(a) uses $b^2 - 4ac$ with a = 1, b = 2k	**M1**
(a) $4k^2 - 4(3k + 10)$	**A1**
(a) = $4(k^2 - 3k - 10)$	**A1**

(a) Answer: discriminant = 4(k + 2)(k - 5)

(b) discriminant > 0 and finds the critical values k = -2 and k = 5	**M1**
(b) chooses the outside region for a positive quadratic in k	**M1**
(b) k < -2 or k > 5	**A1**

(b) Answer: k < -2 or k > 5

(c) sets the discriminant to zero	**M1**
(c) k = -2 or k = 5	**A1**
(c) repeated root x = -k, so x = 2 or x = -5	**A1**

(c) Answer: k = -2, x = 2; k = 5, x = -5

Question 8

(a) writes f(x) = A/(x + 2) + B/(x + 4) and equates numerators	**M1**
(a) substitutes x = -2 (or equates coefficients) to find A	**M1**
(a) A = 5	**A1**
(a) B = 3	**A1**

(a) Answer: f(x) = 5/(x + 2) + 3/(x + 4)

(b) substitutes into either form	**M1**
(b) 13/4	**A1**

(b) Answer: 13/4

Question 9

(a) sets y = (3x + 8)/(x - 4) and rearranges: y(x - 4) = 3x + 8	**M1**
(a) collects x terms: x(y - 3) = 4y + 8	**M1**
(a) $f^{-1}(x) = (4x + 8)/(x - 3)$	**A1**
(a) domain x in R, $x \neq 3$	**B1**

(a) Answer: $f^{-1}(x) = (4x + 8)/(x - 3)$, $x \neq 3$

(b) $f(x) \neq 3$, i.e. y in R, $y \neq 3$	**B1**

(b) Answer: $y \neq 3$

(c) $fg(x) = f(x^2 + 5) = (3(x^2 + 5) + 8)/(x^2 + 5 - 4)$	**M1**
(c) sets equal to 71/17 and rearranges to a quadratic in x^2 (or x)	**M1**
(c) $x^2 = 16$	**A1**
(c) x = 4 or x = -4	**A1**

(c) Answer: x = ±4

Question 10

(a) writes the identity $A(x - q)^2 + B(x - p)(x - q) + C(x - p) =$ numerator	**M1**
(a) substitutes x = -2 and x = 4 to find A and C	**M1**
(a) A = 2, C = 1	**A1**
(a) compares coefficients of x^2 to find B = 1	**A1**

(a) Answer: A = 2, B = 1, C = 1

(b) substitutes into the partial fraction form or the original	**M1**
(b) 1	**A1**

(b) Answer: 1

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Question 11

(a) uses $b^2 - 4ac$ with $a = 1$, $b = 2k$ **M1**

(a) $4k^2 - 4(-5k - 4)$ **A1**

(a) $= 4(k^2 + 5k + 4)$ **A1**

(a) Answer: discriminant = 4(k + 4)(k + 1)

(b) discriminant > 0 and finds the critical values k = -4 and k = -1 **M1**

(b) chooses the outside region for a positive quadratic in k **M1**

(b) k < -4 or k > -1 **A1**

(b) Answer: k < -4 or k > -1

(c) sets the discriminant to zero **M1**

(c) k = -4 or k = -1 **A1**

(c) repeated root x = -k, so x = 4 or x = 1 **A1**

(c) Answer: k = -4, x = 4; k = -1, x = 1

Question 12

(a) squares both sides, or considers 3x - 4 = 2x - 3 and 3x - 4 = -(2x - 3) **M1**

(a) obtains the candidate solutions 1, 7/5 **A1**

(a) checks each candidate in the original equation (the right side must be non-negative) **M1**

(a) no solutions **A1**

(a) Answer: no solutions

(b) -3 < 3x - 4 < 3 **M1**

(b) 1/3 < x < 7/3 **A1**

(b) Answer: 1/3 < x < 7/3

Question 13

(a) sets y = (6x + 7)/(x - 3) and rearranges: y(x - 3) = 6x + 7 **M1**

(a) collects x terms: x(y - 6) = 3y + 7 **M1**

(a) $f^{-1}(x) = (3x + 7)/(x - 6)$ **A1**

(a) domain x in R, $x \neq 6$ **B1**

(a) Answer: $f^{-1}(x) = (3x + 7)/(x - 6)$, $x \neq 6$

(b) $f(x) \neq 6$, i.e. y in R, $y \neq 6$ **B1**

(b) Answer: $y \neq 6$

(c) $fg(x) = f(x^2 + 3) = (6(x^2 + 3) + 7)/(x^2 + 3 - 3)$ **M1**

(c) sets equal to 49/4 and rearranges to a quadratic in x^2 (or x) **M1**

(c) $x^2 = 4$ **A1**

(c) x = 2 or x = -2 **A1**

(c) Answer: x = ±2

Question 14

(a) recognises the numerator and denominator have the same degree and divides, quotient 2 **M1**

(a) forms the identity for the remainder over the two linear factors **M1**

(a) A = 2 **A1**

(a) B = 2 **A1**

(a) Answer: h(x) = 2 + 2/(x + 4) + 2/(x + 1)

(b) substitutes x = 1 **M1**

(b) 17/5 **A1**

(b) Answer: 17/5

Question 15

(a) attempts f(g(x)) = f(x+1) = 2(x+1)/((x+1)-3) **M1**

(a) simplifies the denominator to (x - 2) **A1**

(a) fg(x) = 2(x+1)/(x-2), x not equal to 2, oe cao **A1**

(a) Answer: fg(x) = 2(x + 1)/(x - 2), x not equal to 2

(b) sets y = 2x/(x-3) and rearranges: y(x-3) = 2x **M1**

(b) collects x terms: x(y-2) = 3y oe **M1**

(b) $f^{-1}(x) = 3x/(x-2)$ oe cao **A1**

(b) domain: x is real, x not equal to 2, ft from the denominator of $f^{-1}(x)$ **B1ft**

(b) Answer: $f^{-1}(x) = 3x/(x - 2)$, domain: x is real, x not equal to 2

(c) sets 2(x+1)/(x-2) = 3x/(x-2) **M1**

(c) multiplies both sides by (x-2), x not equal to 2, to obtain 2(x+1) = 3x, giving x = 2 **M1**

(c) rejects x = 2 as it is excluded from the domain of both functions, so there is no solution, cso **A1**

(c) Answer: No solution (the only candidate, x = 2, is excluded from the domain)

Question 16

(a) forms the identity 5 + x = A(1 - x) + B(1 + 2x) **M1**

(a) substitutes x = 1 (or x = -1/2) to find one constant **M1**

(a) B = 2 **A1**

(a) A = 3 **A1**

(a) f(x) = 3/(1+2x) + 2/(1-x) oe cao **A1**

(a) Answer: 3/(1 + 2x) + 2/(1 - x)

(b) expands $3(1+2x)^{-1} = 3(1 - 2x + 4x^2 - ...)$ using the binomial series **M1**

(b) expands $2(1-x)^{-1} = 2(1 + x + x^2 + ...)$ using the binomial series **M1**

(b) $f(x) = 5 - 4x + 14x^2 + ...$ oe cao **A1**

(b) valid for |x| < 1/2 oe, identified as the more restrictive of the two ranges **B1**

(b) Answer: $f(x) = 5 - 4x + 14x^2 + ...$, valid for $|x| < 1/2$

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