1.11 Pure: Algebra and Functions Depth: Test Pack

EDEXCEL 9MA0 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $h(x) = x^3 - 2x^2 + kx - 5$, where k is a constant. When h(x) is divided by (x - 2), the remainder is 3.

(a) Find the value of k. **(3)**

(b) Using this value of k, find the remainder when h(x) is divided by (x + 1). **(3)**

(Total for Question 1 is 6 marks)

---

2 $f(x) = x^3 - 4x + 9$. Find f(-2).

(Total for Question 2 is 1 mark)

---

3 $f(x) = 2x^3 - 3x^2 - 11x + 6$.

(a) Show that (x - 3) is a factor of f(x). **(2)**

(b) Hence factorise f(x) completely. **(3)**

(c) Solve f(x) = 0. **(2)**

(Total for Question 3 is 7 marks)

---

4 State the domain of $f(x) = 1/(x-4)$, where x is real.

(Total for Question 4 is 1 mark)

---

5 Solve the equation $|2x + 1| = 3x - 5$.

(a) By considering the equation $(2x + 1) = (3x - 5)$ and the equation $(2x + 1) = -(3x - 5)$, find two possible values of x. **(4)**

(b) By checking each value in the original equation $|2x + 1| = 3x - 5$, determine which value(s) are valid solutions, justifying any rejection. **(4)**

(Total for Question 5 is 8 marks)

---

6 Express $(5x + 7) / ((x-1)(x+2))$ in the form $A/(x-1) + B/(x+2)$, stating the values of A and B.

(Total for Question 6 is 2 marks)

© Revision Library | revisionlibrary.co.uk

7 Functions f and g are defined by $f(x) = 3x/(x-2)$, x is real, x not equal to 2, and $g(x) = x - 1$, x is real.

(a) Find fg(x), simplifying your answer fully. **(2)**

(b) Find $f^{-1}(x)$, stating its domain. **(2)**

(Total for Question 7 is 4 marks)

---

8 $f(x) = x^3 + px^2 + qx - 4$, where p and q are constants. When f(x) is divided by (x + 2) the remainder is -24, and when f(x) is divided by (x - 4) the remainder is 12.

(a) Show that p and q satisfy the simultaneous equations $4p - 2q = -12$ and $16p + 4q = -48$. **(3)**

(b) Hence find the values of p and q. **(3)**

(c) Find the remainder when f(x) is divided by (x - 1). **(2)**

(Total for Question 8 is 8 marks)

---

9 $f(x) = (x^3 + x^2 + x + 3) / (x^2 + x - 2)$, x not equal to -2 or 1.

(a) Show that f(x) can be written in the form $x + A/(x + 2) + B/(x - 1)$, stating the values of A and B. **(7)**

(b) Hence write down the equation of the oblique asymptote of the curve y = f(x). **(2)**

(c) Find f(3) as an exact fraction, and verify it agrees with the value obtained by direct substitution into the original expression for f(x). **(3)**

(Total for Question 9 is 12 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 1.11 Pure: Algebra and Functions Depth: Test Pack

Question 1

(a) applies the remainder theorem: h(2) = 3 **M1**

(a) forms 8 - 8 + 2k - 5 = 3, i.e. 2k - 5 = 3 **M1**

(a) k = 4 cao **A1**

(a) Answer: k = 4

(b) applies the remainder theorem with x = -1: remainder = h(-1) **M1**

(b) h(-1) = -1 - 2 - 4 - 5 oe **M1**

(b) remainder = -12 cao **A1**

(b) Answer: Remainder = -12

Question 2

f(-2) = 9 cao **B1**

Answer: 9

Question 3

(a) substitutes x = 3 into f(x), or attempts algebraic division by (x - 3) **M1**

(a) f(3) = 2(27) - 3(9) - 11(3) + 6 = 0, so (x - 3) is a factor, cso **A1**

(a) Answer: f(3) = 0, so (x - 3) is a factor of f(x) (shown)

(b) divides f(x) by (x - 3) to obtain a quadratic quotient **M1**

(b) quotient $2x^2 + 3x - 2$ oe **A1**

(b) f(x) = (x - 3)(2x - 1)(x + 2) oe, fully factorised **A1**

(b) Answer: f(x) = (x - 3)(2x - 1)(x + 2)

(c) x = 3 and x = -2 stated **B1**

(c) x = 1/2 stated, all three roots oe **B1**

(c) Answer: x = 3, x = 1/2, x = -2

Question 4

x is real, x ≠ 4 **B1**

Answer: x real, x ≠ 4

Question 5

(a) solves 2x+1 = 3x-5 **M1**

(a) x = 6 **A1**

(a) solves 2x+1 = -(3x-5) **M1**

(a) x = 0.8 oe (4/5) **A1**

(a) Answer: x = 6 or x = 0.8

(b) substitutes x = 6: LHS = |13| = 13, RHS = 3(6)-5 = 13, equal so valid **M1**

(b) x = 6 is a valid solution **A1**

(b) substitutes x = 0.8: RHS = 3(0.8)-5 = -2.6, and notes RHS < 0 so cannot equal a modulus (which is always ≥ 0) **M1**

(b) x = 0.8 is rejected, cso **A1**

(b) Answer: x = 6 is the only valid solution; x = 0.8 is rejected since it gives a negative value for 3x - 5, which cannot equal a modulus

Question 6

sets up A(x+2) + B(x-1) = 5x + 7 and substitutes x=1 and x=-2 (or compares coefficients) **M1**

A = 4, B = 1 **A1**

Answer: A = 4, B = 1

Question 7

(a) substitutes g(x) into f, forming 3(x-1)/((x-1)-2) **M1**

(a) (3x-3)/(x-3) oe, x ≠ 3 **A1**

(a) Answer: fg(x) = (3x-3)/(x-3)

(b) rearranges y = 3x/(x-2) to make x the subject **M1**

(b) $f^{-1}(x) = 2x/(x-3)$, x is real, x ≠ 3 **A1**

(b) Answer: $f^{-1}(x) = 2x/(x-3)$, x real, x ≠ 3

Question 8

(a) uses the remainder theorem: f(-2) = -24 **M1**

(a) 4p - 2q = -12 **A1**

(a) f(4) = 12 gives 16p + 4q = -48 (cso) **A1**

(a) Answer: equations as stated

(b) solves the pair of simultaneous equations (elimination or substitution) **M1**

(b) p = -3 **A1**

(b) q = 0 **A1**

(b) Answer: p = -3, q = 0

(c) evaluates f(1) **M1**

(c) remainder = -6 **A1**

(c) Answer: -6

Question 9

(a) attempts algebraic division of x^3+x^2+x+3 by x^2+x-2 **M1**

(a) quotient x, remainder 3x + 3 obtained oe **A1**

(a) writes (3x+3)/((x+2)(x-1)) = A/(x+2) + B/(x-1) and forms the identity 3x+3 = A(x-1) + B(x+2) **M1**

(a) substitutes x = 1 (or x = -2) to find one constant **M1**

(a) B = 2 **A1**

(a) A = 1 **A1**

(a) states the required form f(x) = x + 1/(x+2) + 2/(x-1) with A = 1 and B = 2, cso **A1**

(a) Answer: f(x) = x + 1/(x + 2) + 2/(x - 1), so A = 1, B = 2

(b) recognises that 1/(x+2) tends to 0 and 2/(x-1) tends to 0 as x tends to ± infinity **M1**

(b) y = x oe **A1**

(b) Answer: y = x

(c) substitutes x = 3 into f(x) = x + 1/(x+2) + 2/(x-1) **M1**

(c) f(3) = 21/5 cao **A1**

(c) verifies using the original expression: (27+9+3+3)/(9+3-2) = 42/10 = 21/5 **B1**

(c) Answer: 21/5

© Revision Library | revisionlibrary.co.uk